The Discovery of Color
A Participant Viewpoint

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Outline

I Perspective on color.

II Discovery of color as a global quantum number.

III Introduction of gauged SU(3) color.

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Perspective on Color

Main aspects of color

- Color — hidden 3-valued degree of freedom for quarks.
- Colored quarks—fractional electric charges.
- Hidden degree of freedom is gauged.
- Hidden color gauge group commutes with electromagnetism.

Color, hidden 3-valued charge (1964)

- **Quarks, 1964 — fractional electric charges.**

  M. Gell-Mann, 1964, Phys. Lett. 8, 214;

  G. Zweig, 1964, CERN Reps. 8182/TH401, 8419TH412.

- **SU(6) theory, 1964—baryons in symmetric representation—violates spin-statistics theorem.**

  F. Gürsey and L.A. Radicati, 1964, Phys. Rev. Lett. 13, 173.

- **Spin-statistics conflict resolved, 1964.**

  O.W. Greenberg, 1964, Phys. Rev. Lett. 13, 598.

  - **Introduced hidden 3-valued degree of freedom — color**

Early papers on color

- O.W. Greenberg, 1964
  - parafermi order 3
- M.Y. Han & Y. Nambu, 1965
  - 3 integer charged triplets
  - octet of gauge vector fields
- Y. Nambu, 1965
  - 3 integer charged triplets

More early papers on color

- A. Tavkhelidze, 1965, High Energy Phys. and El. Part., Vienna, p753. 3 integer charged triplets (*cites Greenberg*).

- A. Tavkhelidze, 1965, High Energy Phys. and El. Part., Vienna, p763. 3 integer charged triplets (*cites Nambu*).

- Y. Miyamoto, 1965, 3 integer charged triplets

- T. Tati, 1965, SO(3) theory

Fractional vs. integer electric charges for quarks

- Fractional charges not seen—led to doubt
- Fractional charges allow exact color symmetry
- Integer charges for quarks conflict with exact color symmetry and with direct experimental evidence

Color & electromagnetism commute

- Identical fractional electric charges allow color & electromagnetism to commute.
- Allows color to be an exact, unbroken, symmetry.
- Crucial part of understanding of quantum chromodynamics, QCD.

Color as charge
Color as gauge symmetry

- Color as charge - analogous to electric charge in electromagnetism.
- Color as gauge symmetry - analogous to U(1) symmetry of electromagnetism.
- Two independent discoveries.

Type of observables determines type of gauge theory

- **If only currents such as**

$$[\bar{\psi}(x), \gamma^\mu \psi(x)]_-$$

are observable, symmetry is SU(3).

- **With additional observables, such as**

$$[\psi(x), \gamma^\mu \psi(x)]_-$$

symmetry is SO(3).

Only baryon number zero currents should be observable

- **Currents** $$[\bar{\psi}(x), \gamma^{\mu}\psi(x)]_{-}$$

  have baryon number zero.

- **Currents** $$[\psi(x), \gamma^{\mu}\psi(x)]_{-}$$

  have baryon number ±2/3.

SU(N) or SO(N)

- Depends on choice of observables.
- In context of quark model, baryon number zero observables lead to SU(3) symmetry.
- Baryon number ±2/3 observables lead to SO(3) symmetry (as discussed above).

Ohnuki, et al; Druhl, et al

- Use Klein transformation to convert parafields to sets of normal fields.

- Can have either SU(N) or SO(N) symmetry.

Possible symmetries connected with color

- The parastatistics of H.S. Green cannot be gauged.
- Greenberg & Macrae showed how to gauge version of parastatistics with Grassmann numbers.
- If Grassmann numbers obey an SU(N) algebra, get SU(N) gauge theory.
- If Grassmann numbers obey an SO(N) algebra, get SO(N) gauge theory.

References on symmetries connected with parastatistics

- Y. Ohnuki and S. Kamefuchi, Phys. Rev. 170, 1279 (1968); Ann. Phys. (N.Y.) 51, 337 (1969).
- K. Druhl, R. Haag and J.E. Roberts, Commun. Math. Phys. 18, 204 (1970).
- O.W. Greenberg and K.I. Macrae, Nucl. Phys. B219, 358 (1983).

Observables & gauge symmetries

- Correct observables lead to correct gauge symmetry.

- Choose correct gauge symmetry by correct choice of observables.

Full understanding of color

- **Full understanding of color emerged from work of:**
  - **Greenberg, 1964**
  - **Han and Nambu, 1965**
- Neither got everything in final form at the beginning.

II

DISCOVERY OF HIDDEN 3-VALUED COLOR DEGREE OF FREEDOM

Discovery of hidden color degree of freedom for quarks

Background to discovery

Disparate influences

- Very simple ideas used to classify newly discovered particles.
- Sophisticated mathematical techniques based on quantum field theory.

Wightman, Axiomatic Quantum Field Theory

- My PhD thesis: Asymptotic condition in quantum field theory
  - Formalization of LSZ scattering theory.
  - Purely theoretical—no numbers, except to label pages & equations.
- Operator-valued distributions, relative mathematical rigor.

Interest in identical particles

- Why only bosons or fermions?
- Are there other possibilities?

Parastatistics

- H.S. Green's parastatistics (1953) as a generalization of each type.
- Boson—paraboson, order p,
- Fermion—parafermion, order p;
- p=1 is Bose or Fermi.

Primer on parastatistics

$$q = \sum_{\alpha=1}^{p} q^{(\alpha)},$$

$$[q^{(\alpha)}(x), q^{(\alpha)\ \dagger}(y)]_{+} = \delta(\mathbf{x} - \mathbf{y}), x^0 = y^0$$

$$[q^{(\alpha)}(x), q^{(\beta)\ \dagger}(y)]_{-} = 0, x^0 = y^0, \alpha \neq \beta$$

Klein transformation

- Schematic form of Klein transformation is

$$Q^{(\alpha)}(x) = K_{(\alpha)} q^{(\alpha)}(x)$$

$$K_{(\alpha)}|0\rangle = |0\rangle$$

- Changes the anomalous anticommutation relations to the normal ones.

1962: Naples, Istanbul, SACLAY

- Axiomatic version of parastatistics with Dell'Antonio & Sudarshan in Naples.

- Presented at NATO summer school in Bebek, near Istanbul.

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Istanbul

- NATO summer school at Robert College in Bebek, 1962
  - organized by Feza Gürsey
- Sidney Coleman, Shelly Glashow, Louis Michel, Giulio Racah, Eugene Wigner

SACLAY with Messiah

- Worked with Messiah at SACLAY 1962 - 1964

(Messiah fought with Free French Army of General Leclerc)

Generalized statistics

- First quantized theory allows all representations of symmetric group.

A.M.L. Messiah and O.W. Greenberg, Phys. Rev. 136, B248, (1964)

- Theorems show generality of parastatistics
  - Green's ansatz not necessary.

O.W. Greenberg and A.M.L. Messiah, Phys. Rev.138, B1155 (1965)

1964

- Crucial year for discovery of quarks and color.

The crucial year, 1964

- Gell-Mann—”quarks”—current quarks.
  M. Gell-Mann, 1964, Phys. Lett. 8, 214.

- Zweig—”aces”—constituent quarks.
  G. Zweig, 1964, CERN articles.

- Why only qqq & q-qbar?
  – No reason in original models.

Background to paradox, 1964

- Relativistic SU(6), Gürsey & Radicati
  F. Gürsey and L.A. Radicati, 1964, Phys. Rev. Lett. 13, 173.

- Generalize Wigner's nonrelativistic nuclear physics idea
  - to combine $SU(2)_I$ with $SU(2)_S$ to get an SU(4) to classify nuclear states.

- Gürsey & Radicati combined $SU(3)_f$ with $SU(2)_S$ to get an SU(6) to classify particle states.

SU(6) classifications

$$q \sim (u, d, s), \text{ in } SU(3)_f$$

$$(1/2) \sim (\uparrow, \downarrow)$$

$$\mathbf{6} = (q, 1/2) \sim (u_\uparrow, u_\downarrow, d_\uparrow, d_\downarrow, s_\uparrow, s_\downarrow) \text{ in } SU(6)_{fS}$$

$$SU(6)_{fS} \rightarrow SU(3)_f \times SU(2)_S = (u, d, s,) \times (\uparrow, \downarrow)$$

Mesons

$$\mathbf{6} \otimes \mathbf{6}^{\star} = \mathbf{1} + \mathbf{35}$$

$$\mathbf{35} \rightarrow (\mathbf{8}, \mathbf{0}) + (\mathbf{1} + \mathbf{8}, \mathbf{1})$$

$$(K^{+}, K^{0}, \pi^{+}, \pi^{0}, \pi^{-}, \eta^{0}, \bar{K}^{0}, \bar{K}^{-})$$

$$(K^{\star\,+}, K^{\star\,0}, \phi^{0}, \rho^{+}, \rho^{0}, \rho^{-}, \omega^{0}, \bar{K}^{\star\,0} \bar{K}^{\star\,-})$$

Baryons

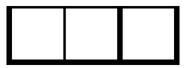
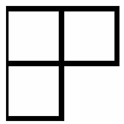
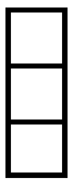
$$\mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6} = \mathbf{56} + \mathbf{70} + \mathbf{70} + \mathbf{20}$$

$$\mathbf{56} \to (\mathbf{8}, 1/2) + (\mathbf{10}, 3/2)$$

$$(p^+, n^0, \Lambda^0, \Sigma^+, \Sigma^0, \Sigma^-, \Xi^0, \Xi^-)$$

$$(\Delta^{++}, \Delta^+, \Delta^0, \Delta^-, Y_1^{\star\,+}, Y_1^{\star\,0}, Y_1^{\star\,-}, \Xi^{\star\,0}, \Xi^{\star\,-}, \Omega^-)$$

Statistics paradox

- 56 
- 70 
- 20 

Attempts to make higher dimensional relativistic theory

- Pais, Rev. Mod. Physics 38, 215 (1966).

- U(6,6)
- U(12)
- GL(12,C)

- Pais, Salam, et al, Freund, et al.

Magnetic moment ratio, 1964

- Beg, Lee, & Pais μ_p/μ_n

$$|p^+_\uparrow\rangle = \frac{1}{\sqrt{3}} u^\dagger_\uparrow (u^\dagger_\uparrow d^\dagger_\downarrow - u^\dagger_\downarrow d^\dagger_\uparrow)|0\rangle$$

$$|n^0_\uparrow\rangle = \frac{1}{\sqrt{3}} d^\dagger_\uparrow (u^\dagger_\uparrow d^\dagger_\downarrow - u^\dagger_\downarrow d^\dagger_\uparrow)|0\rangle$$

$$\mu_B = \langle B_\uparrow|\mu_3|B_\uparrow\rangle$$

$$\mu_3 = 2\mu_0 \sum_q Q_q S_q, \quad Q_q = (\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$$

$$\mu_p = 2\mu_0 \cdot \frac{1}{3}\{2[\frac{2}{3} \cdot 1 + (-\frac{1}{3}) \cdot (-\frac{1}{2})] + [(-\frac{1}{3}) \cdot \frac{1}{2}]\} = \mu_0$$

$$\mu_n = -\frac{2}{3}\mu_0 \qquad m_N/2.79 = 336 MeV$$

Previous calculations of magnetic moments

- Complicated calculations using pion clouds failed.
- Nobody even realized that ratio was so simple.

Significance of magnetic moment calculation

- Simple one-line calculation gave ratio accurate to 3%.
- Very convincing additional argument for quark model.
- Quarks have concrete reality.

Spin-statistics theorem

- Particles that have integer spin must obey Bose statistics

- Particles that have odd-half-integer spin must obey Fermi statistics.

Generalized spin-statistics theorem

- Not part of general knowledge in 1964:
- Particles that have integer spin must obey parabose statistics:
- Particles that have odd-half-integer spin must obey parafermi statistics.
- Each family labeled by an integer p; p=1 is ordinary Bose or Fermi statistics.

Parafermi quark model, 1964

- Suggested model in which quarks carry order-3 parafermi statistics.
- Allows up to 3 quarks in same space-spin-flavor state without violating Pauli principle, so statistics paradox is resolved.
- **This was the introduction of hidden degree of freedom now called color**.

  O.W. Greenberg, 1964, Phys. Rev. Lett. 13, 598.

My response to quark model

- Exhilarated—resolving statistics problem seemed of lasting value.

- Not interested in higher relativistic groups;

  – from O'Raifeartaigh's & my own work I knew that combining internal & spacetime symmetries is difficult or impossible.

No-go theorems

- Supersymmetry is only way to combine internal & spacetime symmetries in a larger group—
  - O'Raifeartaigh, Coleman & Mandula, & Haag, Lopuszanski & Sohnius.

Baryon spectroscopy

- Hidden parafermi (color) degree of freedom takes care of required antisymmetry of Pauli principle.
- Quarks can be treated as bosons in visible space, spin & flavor degrees of freedom.

Table of excited baryons

- Developed simple bound state model with s & p state quarks in 56, L=0+ & 70, L=1- supermultiplets.

Table II. Low-lying states in paraquark model of baryons.[a]

Orbitals configuration	L	Parity	Young diagram	(SU(3), J) decomposition	Total multiplicity	Total no. of I multiplets
s^3 (pure)	0	+	(3)	($\underline{8}$, $J = 1/2$) ($\underline{10}$, $J = 3/2$)	56	8
s^2p^1 (spurious)	1	−	(3)	($\underline{8}$, $J = 1/2, 3/2$) ($\underline{10}$, $J = 1/2, 3/2, 5/2$)	168	20
s^2p^1 (pure)	1	−	(2, 1)	($\underline{1}$, $J = 1/2, 3/2$) ($\underline{8}$, $J = 1/2, 3/2, 5/2$) ($\underline{10}$, $J = 1/2, 3/2$) ($\underline{8}$, $J = 1/2, 3/2$)	210	30
s^1p^2, s^2d^1 (mixed)	2	+	(3)	($\underline{8}$, $J = 3/2, 5/2$) ($\underline{10}$, $J = 1/2, 3/2, 5/2, 7/2$)	280	24
s^1p^2, s^2d^1 (mixed)	2	+	(2, 1)	($\underline{1}$, $J = 3/2, 5/2$) ($\underline{8}$, $J = 1/2, 3/2, 5/2, 7/2$) ($\underline{10}$, $J = 3/2, 5/2$) ($\underline{8}$, $J = 3/2, 5/2$)	350	34
s^1p^2, s^2d^1 (mixed)	1	+	(2, 1)	($\underline{1}$, $J = 1/2, 3/2$) ($\underline{8}$, $J = 1/2, 3/2, 5/2$) ($\underline{10}$, $J = 1/2, 3/2$) ($\underline{8}$, $J = 1/2, 3/2$)	210	30
s^1p^2 (pure)	1	+	(1, 1, 1)	($\underline{8}$, $J = 1/2, 3/2$) ($\underline{1}$, $J = 1/2, 3/2, 5/2$)	60	11

Later developments of baryon spectroscopy

- Greenberg & Resnikoff
- Dalitz & collaborators
- Isgur & Karl
- Riska & collaborators

Disbelief in physics community

- J. Robert Oppenheimer (1964)
- Steven Weinberg (1967)

Gave Oppenheimer preprint in Princeton, 1964

- At conference at University of Maryland
- ***"Greenberg, it's beautiful!"***
  - I was very excited.

Oppenheimer's response, *(continued)*

- ***"but I don't believe a word of it."***

  – I came down to earth.

Response to Oppenheimer’s comment

- Not discouraged.
- But did not ask why he did not believe it.

Weinberg: The making of the standard model

- ***"At that time I did not have any faith in the existence of quarks."*** (1967)

Sources of skepticism about color (1964 - 1968)

- Quarks had just been suggested (1964).
- Fractional electric charges had never been seen (1964).
- Gell-Mann himself was ambiguous (1964).

Gell-Mann's comments

- *"It is fun to speculate …if they were physical particles of finite mass (instead of purely mathematical entities as they would be in the limit of infinite mass)...A search … would help to reassure us of the non-existence of real quarks." (1964)*

Skepticism *(continued)*

- Assuming hidden degree of freedom on top of fractionally charged unseen quarks.
- Seemed to stretch credibility to the breaking point.
- Parastatistics was unfamiliar.

Evidence for color

- 1964, O.W. Greenberg, baryon spectra
- 1969, S. Adler, J. Bell & R. Jackiw explained pi to 2 gamma decay rate.
- From 1964 to 1969 baryon spectroscopy was the only experimental evidence for color.

III

GAUGE THEORY OF COLOR

Gauge theory of color

- Explicit color SU(3)—Han-Nambu, 1965.

- Used 3 dissimilar triplets in order to have integer charges-not correct.

- *“Introduce now eight gauge vector fields which behave as (1,8), namely as an octet in SU(3)".* **This was the introduction of the gauge theory of color.**

Wrong solutions to statistics paradox

- Complicated antisymmetric ground state
- Quarks are not real anyway
- Other models, baryonettes, etc.

Attempt to avoid new degree of freedom

- Dalitz preferred complicated ground state that would avoid statistics problem.
- As rapporteur, Dalitz always put model with Fermi quarks first.

Arguments for simple ground state

- General theorems lead to an s-wave ground state.

- Simplest antisymmetric polynomial in quark coordinates is

$$(\mathbf{x}_1^2 - \mathbf{x}_2^2)(\mathbf{x}_2^2 - \mathbf{x}_3^2)(\mathbf{x}_3^2 - \mathbf{x}_1^2)$$

Arguments for simple ground state (*continued*)

- Then not clear what to choose for excited states.

$$\mathbf{x}_1 \times \mathbf{x}_2 \cdot \mathbf{x}_3$$

- Polynomial vanishes because coordinates are linearly dependent.

- Adding $q\bar{q}$ pairs leads to unseen “exploding SU(3) states” that are not seen.

Arguments for simple ground state (*continued*)

- Zeroes in ground state wave function leads to zeroes in proton electric & magnetic form factors, which are not seen.

If quarks are not "real?"

- If quarks are just mathematical constructs, then their statistics is irrelevant.

First rapporteur
who preferred the
parastatistics model was
Harari, Vienna
1968.

Saturation

- Why are hadrons made from just two combinations,

$$qqq \text{ and } q\bar{q}$$

Work with Zwanziger, 1966

- Surveyed existing models, constructed new models to account for saturation.
- Only models that worked were parafermi model, order 3, and equivalent 3-triplet or color SU(3) models.

Equivalence as classification symmetry

States that are bosons or fermions in parafermi model, order 3,

are in

1-to-1 correspondence with states that are color singlets in SU(3) color model.

Relations & differences between models

$$\pi^0 \to \gamma\gamma$$

$$\sigma(e^+e^- \to hadrons)/\sigma(e^+e^- \to \mu^+\mu^-)$$

Properties that require gauge theory

- Confinement

- S. Weinberg, 1973
- D.J. Gross & F. Wilczek, 1973
- H. Fritzsch, M. Gell-Mann & H. Leutwyler, 1973

Properties that require gauge theory *(continued)*

- Asymptotic freedom,
  Gross, Wilczek, 1973
  Politzer, 1973

- Reconciles quasi-free quarks of parton model with confined quarks of low-energy hadrons

Properties that require gauge theory *(continued)*

- Running of coupling constants & precision tests of QCD.
- Jets in high-energy collisions.

Summary

Two facets of strong interaction

1 Color as classification symmetry & global quantum number

- parafermi model (1964)
- was first introduction of color as global quantum number.

Two facets of strong interaction

2 SU(3) color as local gauge theory

- Han-Nambu model (1965) was first introduction of gauged SU(3) color.

Thank you!

Спасибо!

Early work on color

- O.W. Greenberg, 1964, parafermi order 3
- M.Y. Han & Y. Nambu, 1965, 3 integer charged triplets, octet of gauge vector fields
- Y. Nambu, 1965, 3 integer charged triplets

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- Y. Miyamoto, 1965, 3 integer charged triplets
- T. Tati. 1965, SO(3) theory

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SU(N) or SO(N)

- Depends on choice of observables.
- In context of quark model, baryon number zero observables lead to SU(3) symmetry ; baryon number $\pm 2/3$ observables lead to SO(3) symmetry, as discussed above.

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Nucl. Phys. B219. 358, (1983).