Plasma induced neutrino spin-flip in a supernova and new bounds on the neutrino magnetic moment

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Neutrino spin-flip in the supernova core

Neutrino magnetic moment $\Rightarrow$ spin-flipping processes

in the supernova core:

$$\nu_L \to \nu_R$$

ν_R's being sterile fly away from the core $\Rightarrow$ *leaving no enough energy to explain the observed luminosity of the supernova* $\Rightarrow$ upper bound on the neutrino magnetic moment.

SN1987A, R. Barbieri and R. N. Mohapatra (1988): the neutrino spin-flip via both $\nu_L e^- \to \nu_R e^-$ and $\nu_L p \to \nu_R p$ scattering processes.

From the ν_R luminosity upper limit $Q_{\nu_R} < 10^{53}$ erg/s, the upper bound on the neutrino magnetic moment was established :

$$\mu_\nu < (0.2 - 0.8) \times 10^{-11} \, \mu_{\rm B} \, .$$

However, the essential plasma polarization effects in the photon propagator were not considered comprehensively. An *ad hoc* photon thermal mass was inserted instead.

Neutrino spin-flip in the supernova core

Later on, A. Ayala, J. C. D'Olivo and M. Torres (1999) used the formalism of the **Thermal Field Theory** to take into account the influence of hot dense astrophysical plasma on the photon propagator.

The upper bound for the neutrino magnetic moment was improved by them in the factor of 2:

$$\mu_\nu < (0.1 - 0.4) \times 10^{-11} \, \mu_{\rm B} \, .$$

However, looking at the intermediate analytical results by the authors, we conclude that only the contribution of plasma **electrons** was taken into account there, while the **proton** fraction was omitted.

Thus, the reason exists to reconsider the neutrino spin-flip processes in the supernova core more attentively.

We confirm in part, that the neutrino scattering on plasma **protons** is essential, as well as the scattering on plasma **electrons**.

The photon dispersion

The functions $\Pi_{(\lambda)}$, defining the photon dispersion law:

$$\omega^2 - k^2 - \Pi_{(\lambda)}(\omega, k) = 0 \,,$$

where $\lambda = t, \ell$ mean transversal and longitudinal photon polarizations, are the eigenvalues of the photon polarization tensor $\Pi_{\alpha\beta}$.

In general, the functions $\Pi_{(\lambda)}$ have imaginary parts. This means, that the "photon" is unstable in plasma, and can not be treated as a real photon.

It would be more self-consistent to consider the vertex $\nu_L \nu_R \gamma^*$ in the neutrino scattering via the *intermediate virtual plasmon* γ^* on plasma particles.

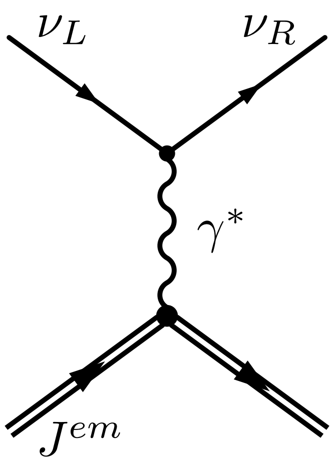
The Lagrangian of the interaction of a neutrino with a magnetic moment μ_ν with photons is:

$$\mathcal{L} = -\frac{\mathrm{i}}{2} \, \mu_\nu \, (\bar{\nu} \sigma_{\alpha\beta} \nu) \, F^{\alpha\beta} \,,$$

where $\sigma_{\alpha\beta} = (1/2) \, (\gamma_\alpha \gamma_\beta - \gamma_\beta \gamma_\alpha)$, $F^{\alpha\beta}$ is the tensor of the photon electromagnetic field.

Neutrino interaction with background

The neutrino chirality flip process of the neutrino scattering via the *intermediate virtual plasmon* γ^* on the plasma electromagnetic current presented by electrons, $\nu_L e^- \longrightarrow \nu_R e^-$, protons, $\nu_L p \longrightarrow \nu_R p$, etc., is shown in the diagram:



Here, J^{em} is an electromagnetic current in the general sense, formed by different components of the medium, i.e. free electrons and positrons, free ions, neutral atoms, etc.

The rate of creation of the right-handed neutrino

The most interesting value is the rate $\Gamma_{\nu_R}(E')$ of creation of the right-handed neutrino with the fixed energy E' by all the left-handed neutrinos. It can be obtained by integration of the amplitude squared over the states of particles forming the electromagnetic current and over the states of the initial left-handed neutrinos.

Given $\Gamma_{\nu_R}(E')$, one can calculate both the right-handed neutrino flux and the right-handed neutrino luminosity.

The technics of calculations of the neutrino spin-flip rate is rather standard. The only principal point is to use the photon propagator $G^{\alpha\beta}(q)$ with taking account of the plasma polarization effects.

The rate of creation of the right-handed neutrino

We take the photon propagator in the form:

$$G^{\alpha\beta}(q) = \frac{\mathrm{i}\, \varrho^{\alpha\beta}_{(t)}}{q^2 - \Pi_{(t)}} + \frac{\mathrm{i}\, \varrho^{\alpha\beta}_{(\ell)}}{q^2 - \Pi_{(\ell)}} \,,$$

where $\varrho^{\alpha\beta}_{(t,\ell)}$ are the density matrices for the transversal and longitudinal photon polarizations,

$$\varrho^{\alpha\beta}_{(t)} = -\left(g^{\alpha\beta} - \frac{q^\alpha q^\beta}{q^2} - \frac{\ell^\alpha \ell^\beta}{\ell^2} \right), \qquad \varrho^{\alpha\beta}_{(\ell)} = -\frac{\ell^\alpha \ell^\beta}{\ell^2} \,,$$

$\ell_\alpha = q_\alpha\, (u\, q) - u_\alpha\, q^2$, and u_α is the four-vector of the plasma velocity. The propagator has no ambiguity when the functions $\Pi_{(t,\ell)}$ are *real*. Our generalization to the case of *complex* functions is based on using the same form of the propagator with the *retarded* functions $\Pi_{(t,\ell)}$.

The rate of creation of the right-handed neutrino

There is also such a subtle effect as the additional energy W acquired by a left-handed neutrino in plasma. With this effect, the general expression for the rate of creation of the right-handed neutrino is:

$$\Gamma_{\nu_R}(E') = \frac{\mu_\nu^2}{16\,\pi^2\,E'^2} \int\limits_D \frac{\mathrm{d}q_0\,\mathrm{d}k}{k}\, f_\nu(E'+q_0)\,[1+f_\gamma(q_0)]\,(2E'+q_0)^2\,q^4$$

$$\times \left\{ \left(1 - \frac{k^2}{(2E'+q_0)^2}\right) \left[1 - \frac{2q_0 W}{q^2} + \frac{8E'(E'+q_0)W^2}{q^4\,[(2E'+q_0)^2/k^2 - 1]}\right] \rho_{(t)}(q_0,k) \right.$$

$$\left. - \left(1 - \frac{2q_0 W}{q^2}\right) \rho_{(\ell)}(q_0,k) \right\},$$

where $q^2 = q_0^2 - k^2$, $f_\nu(E'+q_0)$ and $f_\gamma(q_0)$ are the neutrino and photon distribution functions, and the photon spectral density functions are introduced:

$$\rho_{(\lambda)} = \frac{2\left(-\mathrm{Im}\,\Pi_{(\lambda)}\right)}{\left(q^2 - \mathrm{Re}\,\Pi_{(\lambda)}\right)^2 + \left(\mathrm{Im}\,\Pi_{(\lambda)}\right)^2}.$$

The rate of creation of the right-handed neutrino

We note that our result is in agreement with the rate obtained by P. Elmfors et al. (1997).

However, extracting *the electron contribution* from our general expression, we obtain the result which is larger by the factor of 2 than the corresponding formula in the papers by A. Ayala et al. It can be seen that an error was made there just in the first formula defining the production rate Γ of a right-handed neutrino.

Our formula having the most general form can be used for neutrino-photon processes ($\nu_L \to \nu_R \gamma^*$) in any optically active medium. We only need to identify the photon spectral density functions $\rho_{(\lambda)}$. For example, in the medium where $\mathrm{Im}\, \Pi_{(t)} \to 0$ in the space-like region $q^2 < 0$ corresponding to the refractive index values $n > 1$, the spectral density function is transformed to δ-function, and we reproduce the result of the paper by W. Grimus and H. Neufeld (1993) devoted to the study of the Cherenkov radiation of **transversal** photons by neutrinos.

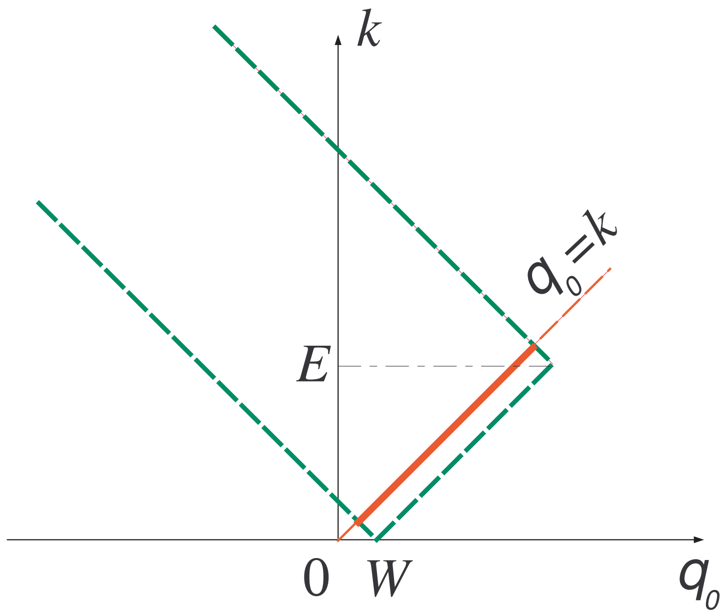
The rate of creation of the right-handed neutrino

If one **formally** takes the limit $\mathrm{Im}\,\Pi_{(\ell)} \to 0$, the result obtained by S. Mohanty and S. Sahu (1997) can be reproduced, namely, the width of the Cherenkov radiation and absorption of **longitudinal** photons by neutrinos in the space-like region $q^2 < 0$.

However, the limit $\mathrm{Im}\,\Pi_{(\ell)} \to 0$ itself is unphysical in the real astrophysical plasma conditions considered by those authors and leads to the strong overestimation of a result.

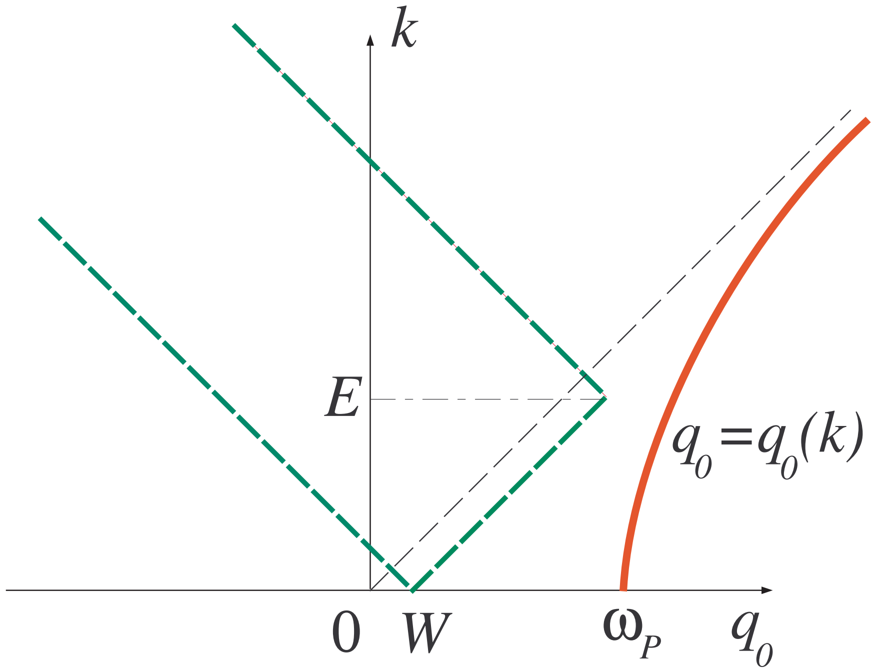
"Neutrino spin light"

One more unphysical case, the so-called "neutrino spin light", was considered in the papers by A. Studenikin et al. (2003-2006), where the photon dispersion in medium was ignored. The region of integration for the width $\Gamma^{\rm tot}_{\nu_L \to \nu_R}$ with the *fixed initial neutrino energy* E would contain the vacuum dispersion line $q_0 = k$ (the **red bold** line in the integration plot).



However, the photon dispersion in plasma is not the vacuum one!

“Neutrino spin light”



In general, the photon dispersion curve does not enter the allowed kinematical region. For the fixed plasma parameters the threshold neutrino energy $E_{\rm min}$ exists for the process $\nu_L \to \nu_R \gamma^*$ to be possible.

It is useful to compare the numerical values in the figure.

“Neutrino spin light”

For the interior of a neutron star, the additional energy acquired by a left-handed neutrino in plasma (N_B is the barion density):

$$W \simeq 6\,\mathrm{eV} \left(\frac{N_B}{10^{38}\,\mathrm{cm}^{-3}} \right),$$

while the plasmon frequency, defining the photon dispersion:

$$\omega_P \simeq 10^7\,\mathrm{eV} \left(\frac{N_B}{10^{38}\,\mathrm{cm}^{-3}} \right)^{1/3}.$$

The threshold neutrino energy in this case:

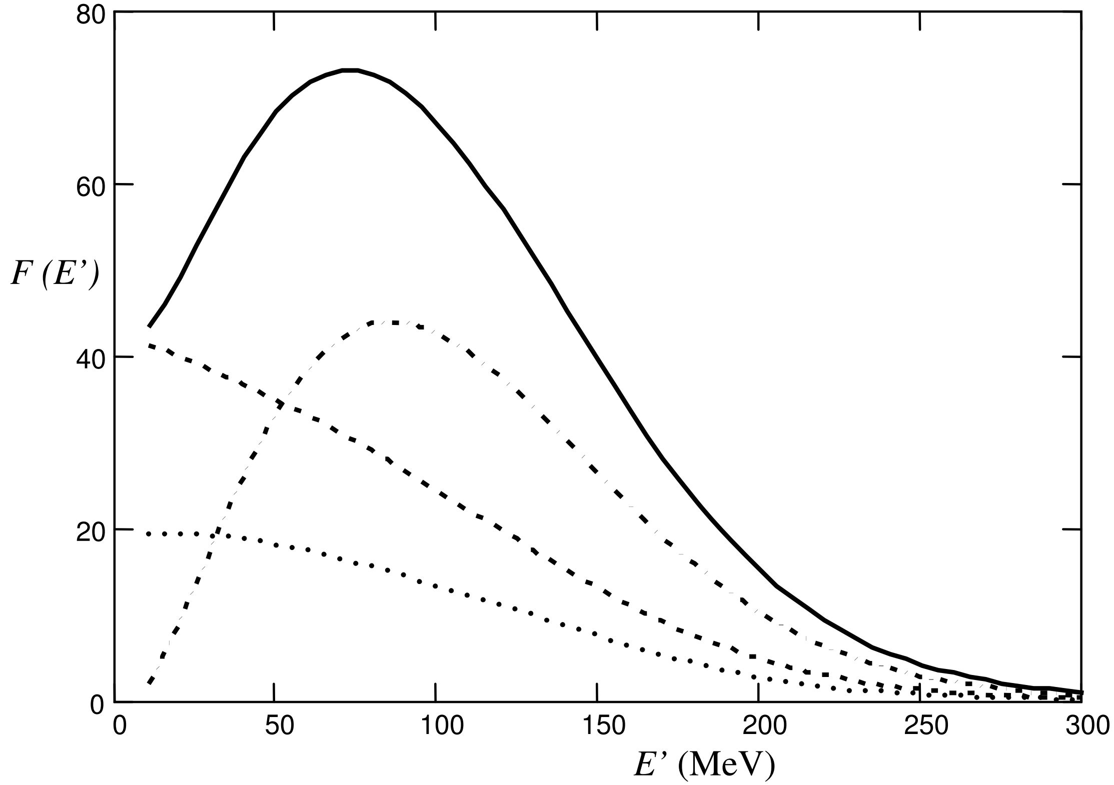
$$E_{\min} \simeq \frac{\omega_P^2}{2\,W} \simeq 10\,\mathrm{TeV}\,.$$

The details can be found in our papers:

- Mod. Phys. Lett. A **21**, 1769 (2006), hep-ph/0606262;
- Int. J. Mod. Phys. A **22**, 3211 (2007), hep-ph/0701228.

The rate of creation of the right-handed neutrino

The production rate of ν_R: the electron contribution (dashed line), the proton contribution (dash-dotted line), the total rate (solid line) for T =30 MeV. The dotted line shows the result by A. Ayala et al.



The function $F(E')$ is defined by: $\Gamma_{\nu_R}(E') = (\mu_\nu^2 \, T^3/(32\,\pi))\, F(E')$.

Bound on μ_ν from the right-handed neutrino luminosity

The supernova core luminosity for ν_R emission can be computed as

$$Q_{\nu_R} = V \int \frac{\mathrm{d}^3 p'}{(2\pi)^3} \, E' \, \Gamma_{\nu_R}(E') \,,$$

where V is the plasma volume.

For the same supernova core conditions as in the paper by Ayala et al. (plasma volume $V \sim 8 \times 10^{18} \mathrm{cm}^3$, temperature range $T = 30 - 60$ MeV, electron chemical potential range $\mu_e = 280 - 307$ MeV), we obtain

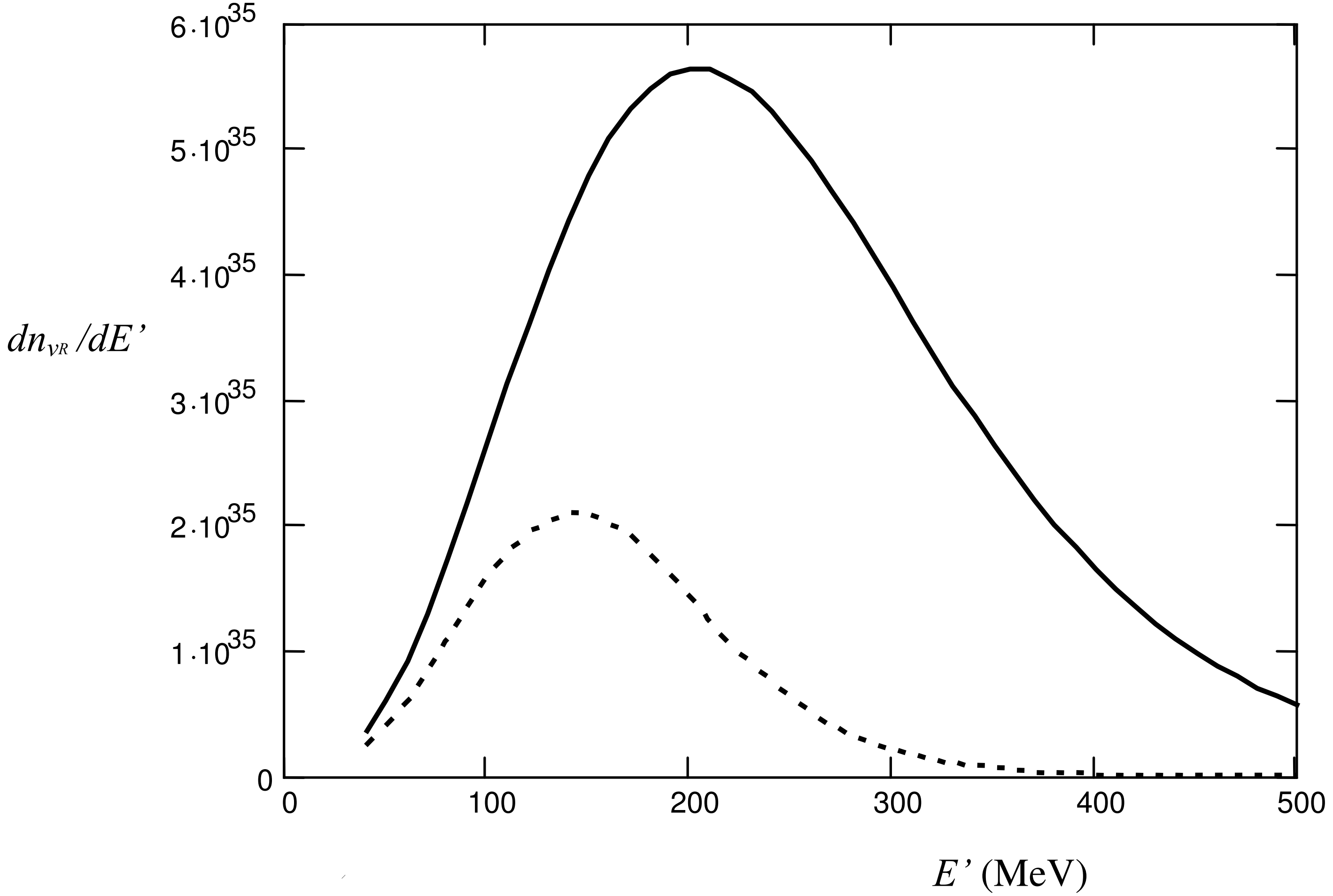
$$Q_{\nu_R} = \left(\frac{\mu_\nu}{\mu_\mathrm{B}} \right)^2 (0.76 - 4.4) \times 10^{77} \, \text{erg/s} \,.$$

Assuming that $Q_{\nu_R} < 10^{53}$ erg/s, we obtain the upper limit on the neutrino magnetic moment: $\mu_\nu < (0.5 - 1.1) \, \times 10^{-12} \, \mu_\mathrm{B}$.

Remind that the result by A. Ayala et al. was: $\mu_\nu < (1 - 4) \, \times 10^{-12} \, \mu_\mathrm{B}$.

Right-handed neutrino spectrum

The number of right-handed neutrinos (for $\mu_\nu = 10^{-12}\,\mu_{\rm B}$) emitted per 1 cm^3 per 1 sec per 1 MeV of the energy spectrum for the plasma temperature T =60 MeV (solid line) and for T =30 MeV (dashed line).



Bound on μ_ν from the left-handed neutrino washing out

An additional method can be used to put a bound on the neutrino magnetic moment. Integrating the above-plotted value over all energies, one obtains the number of right-handed neutrinos emitted per 1 cm^3 per 1 sec. Dividing this to the initial left-handed neutrino number density, one can estimate the averaged time of the conversion of left-handed neutrinos to right-handed neutrinos. For the temperature range $T = 30 - 60$ MeV, and for the electron chemical potential $\mu_e \sim 300$ MeV, we obtain

$$\tau \simeq \left(\frac{\mu_\nu}{10^{-12}\, \mu_{\rm B}} \right)^2 (0.14 - 0.36)\ \text{sec}\,.$$

In order not to spoil the Kelvin—Helmholtz stage of the protoneutron star cooling ($\sim$ 10 sec), this time of the neutrino spin-flip should exceed a few seconds. Taking the conservative limit $\tau >$ 1 sec, we obtain the bound on the neutrino magnetic moment: $\mu_\nu < (0.4 - 0.6)\, \times 10^{-12}\, \mu_{\rm B}\,.$

By this means, we improve the best astrophysical upper bound on the neutrino magnetic moment obtained by A. Ayala et al. (1999) by the factor of 3 to 7.

Conclusions

- We have investigated in detail the neutrino chirality-flip process under the conditions of astrophysical plasma. The plasma polarization effects caused both by electrons and protons were taken into account in the photon propagator. The rate $\Gamma_{\nu_R}(E')$ of creation of the right-handed neutrino with the fixed energy E', the energy spectrum, and the luminosity have been calculated.
- From the limit on the supernova core luminosity for ν_R emission, we have obtained the upper bound on the neutrino magnetic moment $\mu_\nu < (0.5 - 1.1) \times 10^{-12} \, \mu_{\rm B}$.
- From the limit on the averaged time of the left-handed neutrino washing out, we have obtained the upper bound $\mu_\nu < (0.4 - 0.6) \times 10^{-12} \, \mu_{\rm B}$.
- We have improved the best astrophysical upper bound on the neutrino magnetic moment by the factor of 3 to 7.

"Neutrino spin light" at ultra-high neutrino energies?

At **ultra-high** neutrino energies the local limit of the weak interaction does not describe comprehensively the additional neutrino energy in plasma, and the **non-local** weak contribution must be taken into account.

In a general case, this non-local term *identical for both neutrinos and antineutrinos*, is

$$\Delta^{(\mathrm{nloc})} W_i = -\frac{16\, G_\mathrm{F}\, E}{3\sqrt{2}} \left[\frac{< E_{\nu_i} >}{m_Z^2} \left(N_{\nu_i} + \bar{N}_{\nu_i} \right) + \delta_{ie} \frac{< E_e >}{m_W^2} \left(N_e + \bar{N}_e \right) \right] .$$

E is the energy of a neutrino with the flavor i, propagating through plasma, $< E_{\nu_i} >$ and $< E_e >$ are the averaged energies of plasma neutrinos and electrons.

There arises the window (if exists) in the neutrino energies for the process to be kinematically opened, $E_{\mathrm{min}} < E < E_{\mathrm{max}}$. For example, in the solar interior there is no window for the process with electron neutrinos at all.

Kinematical equivalence of "neutrino spin light" and $\bar\nu_e + e^- \to \tau^- + \bar\nu_\tau$

Let us compare the processes:

$$\nu_L \to \nu_R + \gamma \qquad \bar\nu_e + e^- \to \tau^- + \bar\nu_\tau$$

The energy and momentum conservation in the lab frame:

$$E + W = E' + \omega \qquad E + m_e = E' + \omega$$

$$\mathbf{p} = \mathbf{p}' + \mathbf{k} \qquad \mathbf{p} = \mathbf{p}' + \mathbf{k}$$

The Mandelstam S variable in the lab frame:

$$S = 2\,W\,E + W^2 \qquad S = 2\,m_e\,E + m_e^2$$

The Mandelstam S variable in the center-of-mass frame:

$$S = \left(\sqrt{m_\gamma^2 + p'^2} + p'\right)^2 \geqslant m_\gamma^2 \qquad S = \left(\sqrt{m_\tau^2 + p'^2} + p'\right)^2 \geqslant m_\tau^2$$

The threshold value for the initial neutrino energy:

$$E \geqslant E_0 = \frac{m_\gamma^2 - W^2}{2\,W} \simeq \frac{m_\gamma^2}{2\,W} \qquad E \geqslant E_0 = \frac{m_\tau^2 - m_e^2}{2\,m_e} \simeq \frac{m_\tau^2}{2\,m_e}$$

“Neutrino spin light” has a famous precursor?

Why the radiation of a relativistic charged particle in an external magnetic field, termed “spin light” does exist, while the “neutrino spin light” does not ?

Because the influence of a **weak magnetic field** and of **dense matter** on the photon dispersion is rather different.

In **dense matter** giving an additional energy to the left-handed neutrino, a photon acquires the effective mass, while in a **laboratory magnetic field** where the “spin light” was investigated, the photon effective mass is negligibly small.