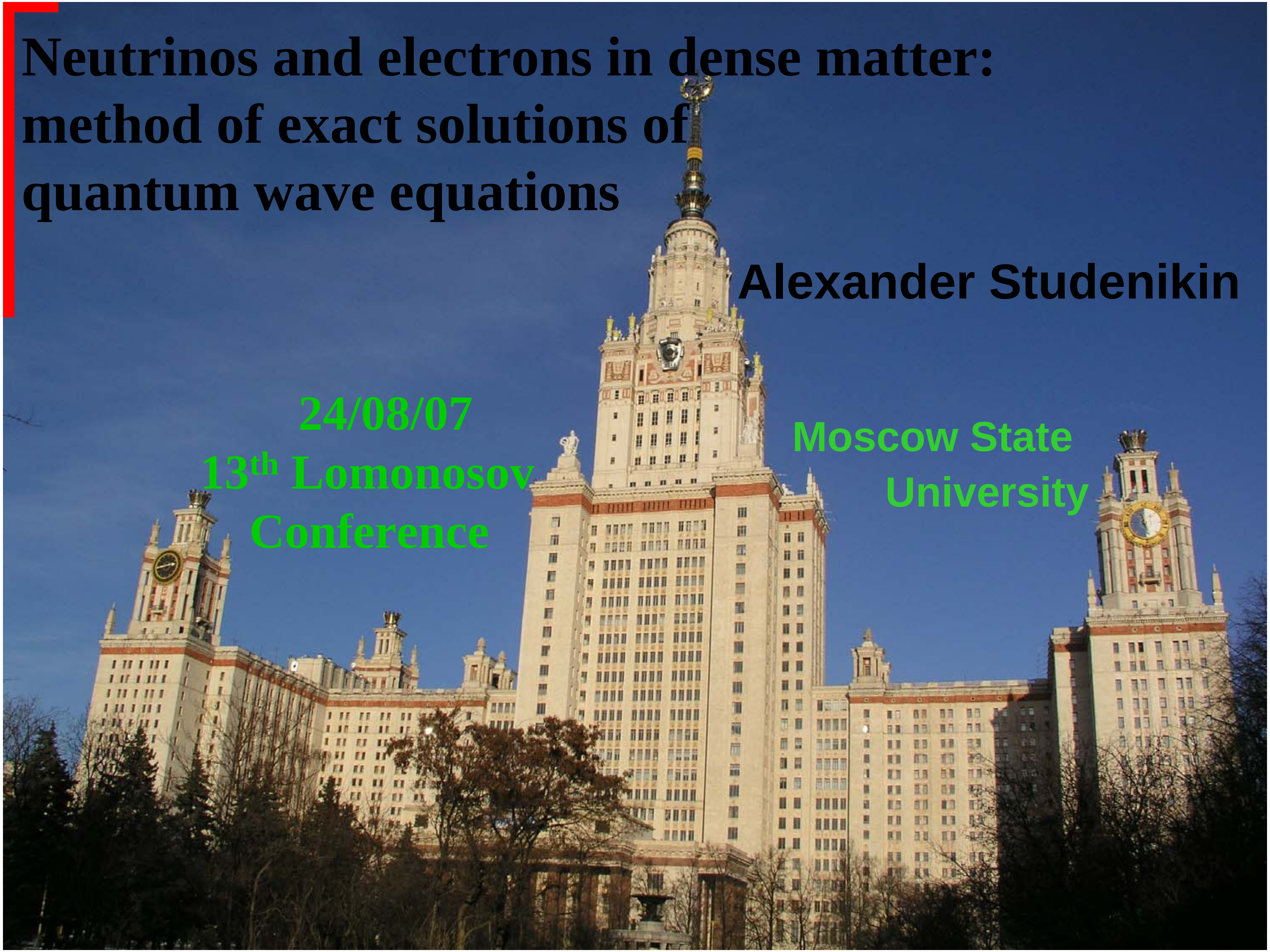
Neutrinos and electrons in dense matter: method of exact solutions of quantum wave equations

Alexander Studenikin



24/08/07
13th Lomonosov
Conference

Moscow State
University

Crucial role of neutrino

ν is a “tiny” particle :

- very light $m_{\nu_f} \ll m_f, \quad f = e, \mu, \tau$
- electrically neutral $q_\nu = 0$ $q_\nu < 4 \times 10^{-17} e$
- with very small magnetic moment μ_ν ?
- weak interactions are indeed weak
  - $\sigma_{\nu_e N} \sim 10^{-39}\ cm^2$ ν-N scattering
  - $\sigma_{\bar{\nu}_e p} \sim 10^{-40}\ cm^2$ inverse β-decay
  - $\sigma_{\nu_e e} \sim 10^{-43}\ cm^2$ ν-e scattering

at the final stages of development of particular elementary particle physics framework

ν manifests itself most vividly under the influence of external conditions:

- bakground matter

and

- external (electromagnetic etc) fields

A.Studenikin,**J.Phys.A:Math.Gen.39**(2006)6769; **Ann.Fond. de Broglie 31** # 2-3 (2006)

A.Studenikin, **Phys.Atom.Nucl. 70** (2007) 1275; *ibid* **67** (2004)1014

A.Grigoriev, S.Shinkevich, A.Studenikin, A.Ternov, I.Trofimov, **Phys. J. 6** (2007) 66;

A.Grigoriev, A.Savochkin, A.Studenikin, **Phys. J. 8** (2007) 66;

A.Studenikin, A.Ternov, **Phys.Lett.B 608** (2005) 107

A.Grigoriev, A.Studenikin, A.Ternov, **Phys.Lett.B 622** (2005) 199;

Grav. & Cosm. 11 (2005) 132 **Phys.Atom.Nucl. 69** (2006)1940

K.Kouzakov, A.Studenikin, **Phys.Rev.C 72** (2005) 015502

M.Dvornikov, A.Grigoriev, A.Studenikin, **Int.J Mod.Phys.D 14** (2005) 309

S.Shinkevich, A.Studenikin, **Pramana 64** (2005) 124

A.Studenikin, **Nucl.Phys.B** (Proc.Suppl.) **143** (2005) 570

M.Dvornikov, A.Studenikin, **Phys.Rev.D 69** (2004) 073001

Phys.Atom.Nucl. 64 (2001) 1624

Phys.Atom.Nucl. 67 (2004) 719

JETP 99 (2004) 254 **JHEP 09** (2002) 016

A.Lobanov, A.Studenikin, **Phys.Lett.B 601** (2004) 171

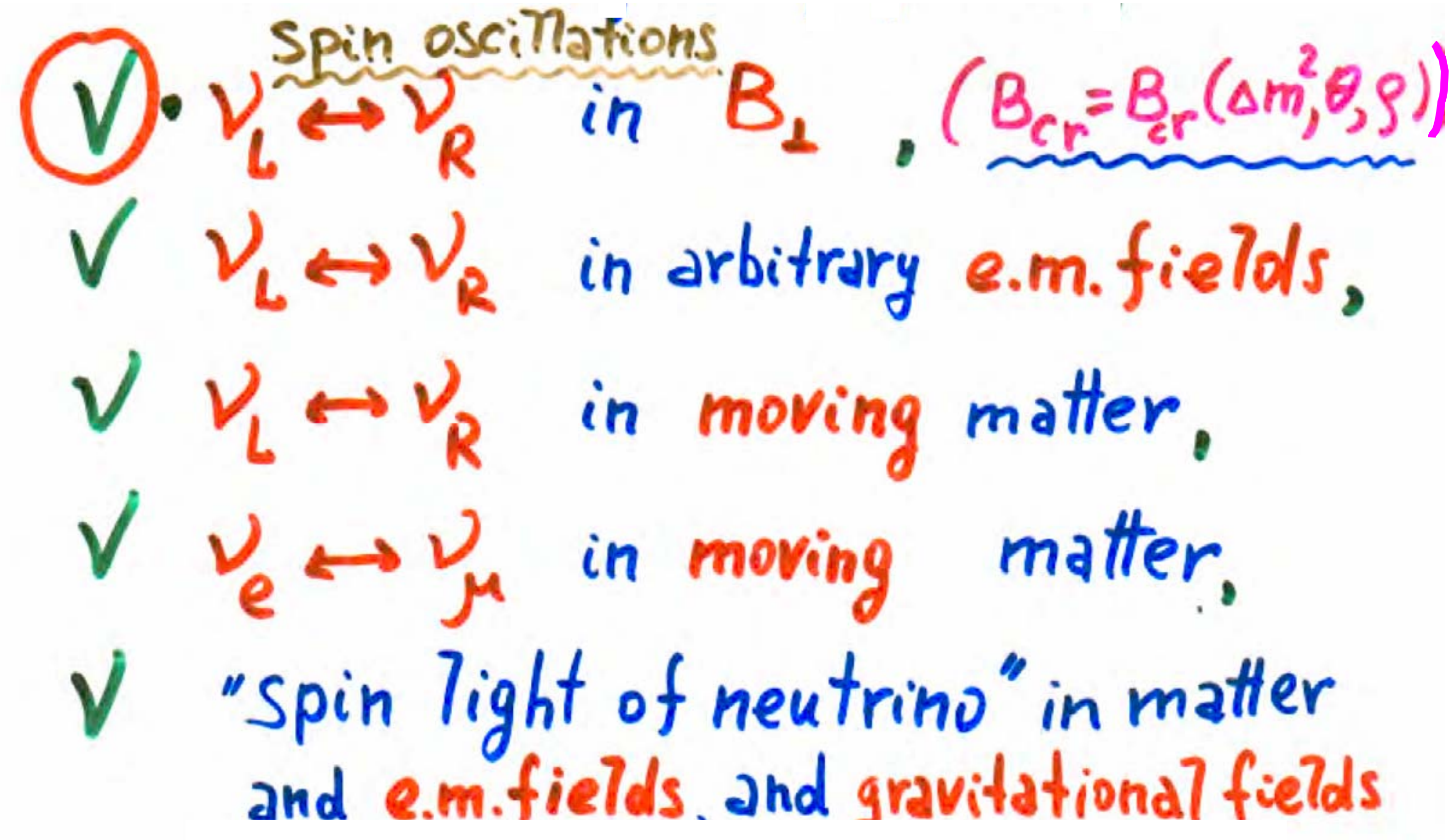
Phys.Lett.B 564 (2003) 27

Phys.Lett.B 515 (2001) 94

A.Grigoriev, A.Lobanov, A.Studenikin, **Phys.Lett.B 535** (2002) 187

A.Egorov, A.Lobanov, A.Studenikin, **Phys.Lett.B 491** (2000) 137

Main results of our previous studies



1994-1997 ✓ • $\overset{\text{Spin oscillations}}{\nu_L \leftrightarrow \nu_R}$ in $B_\perp$, $(B_{cr}=B_{cr}(\Delta m^2,\theta,\rho))$

1998-2000 ✓ $\nu_L \leftrightarrow \nu_R$ in arbitrary e.m. fields,

2000-2002 ✓ $\nu_L \leftrightarrow \nu_R$ in moving matter,

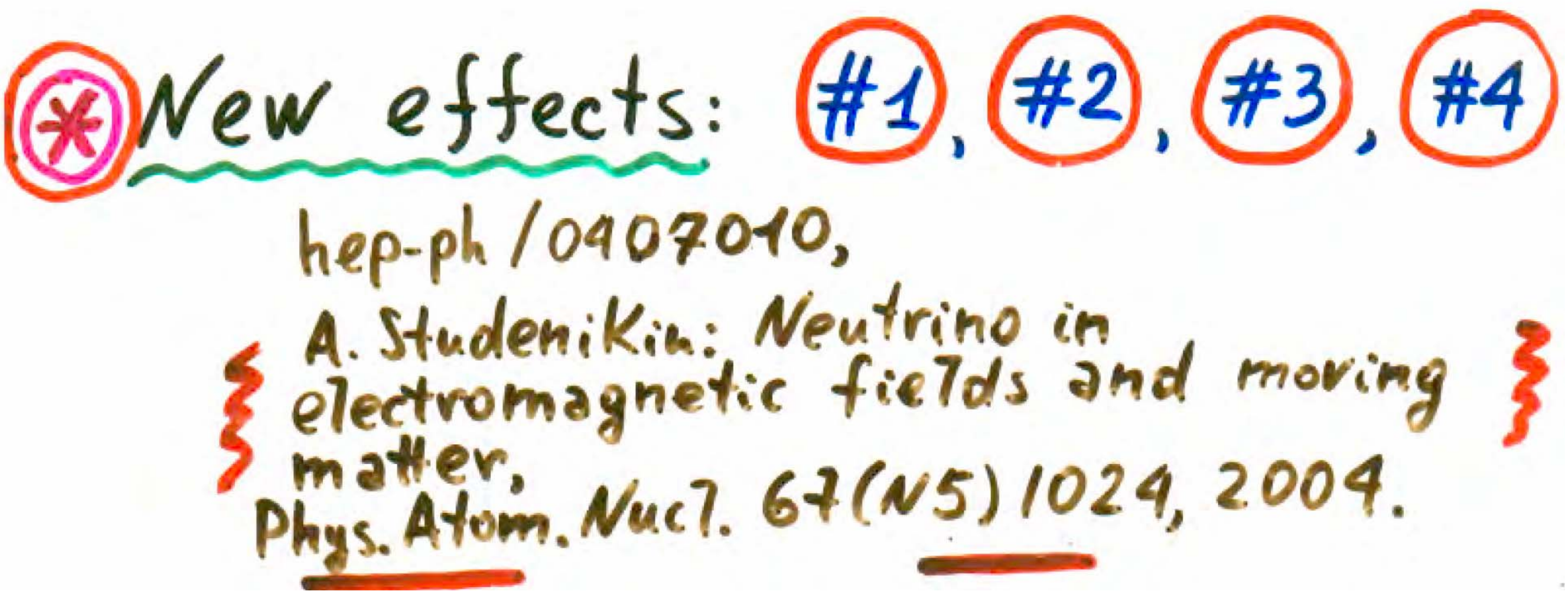
1995-2002 ✓ $\nu_e \leftrightarrow \nu_\mu$ in moving matter,

2003-2005 ✓ "spin light of neutrino" in matter and e.m. fields and gravitational fields

2004-2006… ✓ quantum theory of neutrino motion in background matter

NB !

These studies are performed within the Standard Model of interaction



* New effects: #1, #2, #3, #4

hep-ph/0407010,

A. Studenikin: Neutrino in electromagnetic fields and moving matter,

Phys. Atom. Nucl. 67 (N5) 1024, 2004.

The four new effects in neutrino oscillations,

Nucl.Phys.B (Proc.Suppl.) **143** (2005) 570

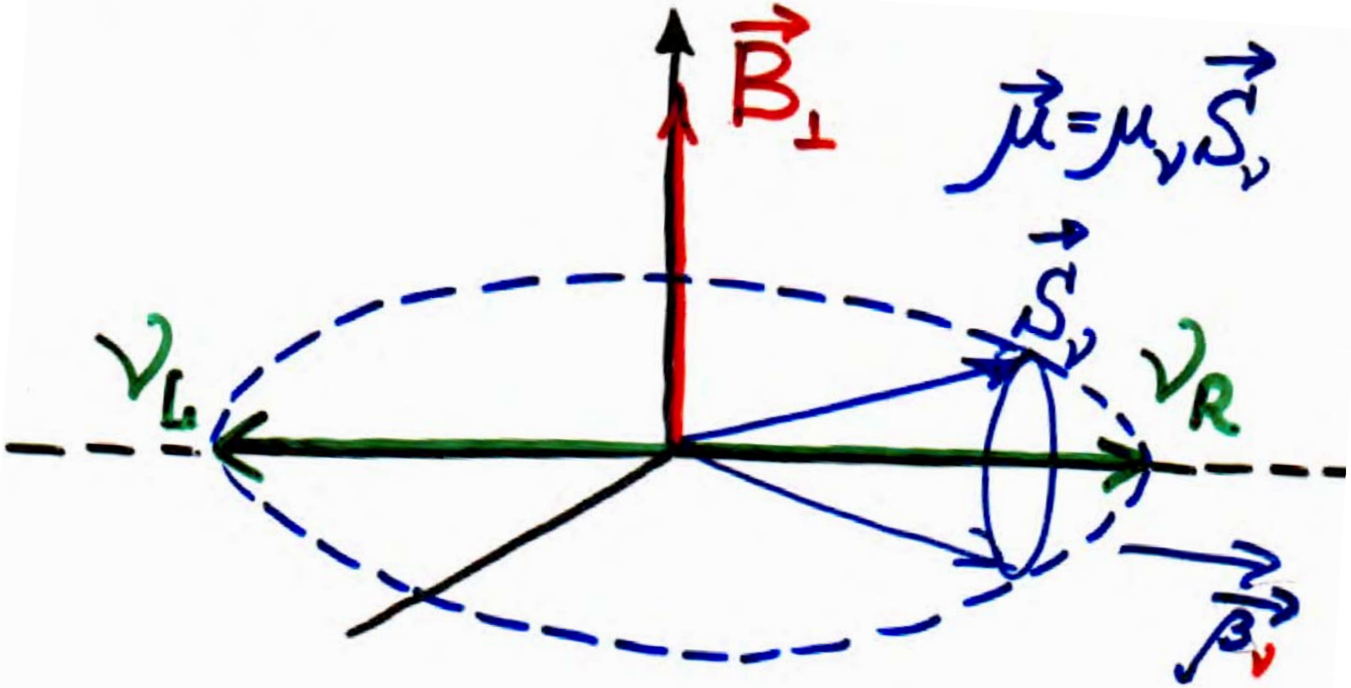
#1 Lorentz invariant approach to
ν spin evolution in
arbitrary e.m. field $F_{\mu\nu}$
(only $B_{\perp}$ was considered before) $\circledast$

$\Longrightarrow$

predictions for new resonances in
$\nu_L \leftrightarrow \nu_R$ in various configurations
of e.m. fields (e.m. wave etc ...)

#2 … matter effect included …

ν spin precession can be stimulated not only by e.m. interactions with e.m. field $F_{\mu\nu}$ but also by ν weak interactions with matter!



$$\frac{d\vec{S}_\nu}{dt} = 2\mu_\nu[\vec{S}_\nu \times \underline{\underline{\vec{B}}}] + \boxed{2\mu_\nu[\vec{S}_\nu \times \vec{G}]}$$

electromagnetic interacton with e.m. field

weak interaction with matter

ν spin evolution in presence of general external fields

M.Dvornikov, A.Studenikin,
JHEP 09 (2002) 016

General types non-derivative interaction with external fields

$$\boxed{\begin{gathered}-\mathcal{L} = g_s s(x)\bar{\nu}\nu + g_p \pi(x)\bar{\nu}\gamma^5\nu + g_v V^\mu(x)\bar{\nu}\gamma_\mu\nu + g_a A^\mu(x)\bar{\nu}\gamma_\mu\gamma^5\nu + \\ + \frac{g_t}{2} T^{\mu\nu}\bar{\nu}\sigma_{\mu\nu}\nu + \frac{g'_t}{2}\Pi^{\mu\nu}\bar{\nu}\sigma_{\mu\nu}\gamma_5\nu,\end{gathered}}$$

scalar, pseudoscalar, vector, axial-vector, tensor and pseudotensor fields:

$$s, \pi, V^\mu = (V^0, \vec{V}), A^\mu = (A^0, \vec{A}),$$
$$T_{\mu\nu} = (\vec{a}, \vec{b}), \Pi_{\mu\nu} = (\vec{c}, \vec{d})$$

Relativistic equation (quasiclassical) for ν spin vector:

$$\begin{aligned}\dot{\vec{\zeta}}_\nu = 2g_a\Big\{&A^0[\vec{\zeta}_\nu \times \vec{\beta}] - \frac{m_\nu}{E_\nu}[\vec{\zeta}_\nu \times \vec{A}] - \frac{E_\nu}{E_\nu+m_\nu}(\vec{A}\vec{\beta})[\vec{\zeta}_\nu \times \vec{\beta}]\Big\} \\ &+2g_t\Big\{[\vec{\zeta}_\nu \times \vec{b}] - \frac{E_\nu}{E_\nu+m_\nu}(\vec{\beta}\vec{b})[\vec{\zeta}_\nu \times \vec{\beta}] + [\vec{\zeta}_\nu \times [\vec{a} \times \vec{\beta}]]\Big\} + \\ &+2ig'_t\Big\{[\vec{\zeta}_\nu \times \vec{c}] - \frac{E_\nu}{E_\nu+m_\nu}(\vec{\beta}\vec{c})[\vec{\zeta}_\nu \times \vec{\beta}] - [\vec{\zeta}_\nu \times [\vec{d} \times \vec{\beta}]]\Big\}.\end{aligned}$$

- *Neither **s** nor **π** nor **V** contributes to spin evolution*

- **Electromagnetic interaction**

$$T_{\mu\nu} = F_{\mu\nu} = (\vec{E}, \vec{B})$$

- **SM weak interaction**

$$G_{\mu\nu} = (-\vec{P}, \vec{M})$$
$$\vec{M} = \gamma(A^0\vec{\beta} - \vec{A})$$
$$\vec{P} = -\gamma[\vec{\beta} \times \vec{A}],$$

#3 $\nu_L \leftrightarrow \nu_R$ and $\nu_\ell \leftrightarrow \nu_{\ell'}$, $\ell \neq \ell'$

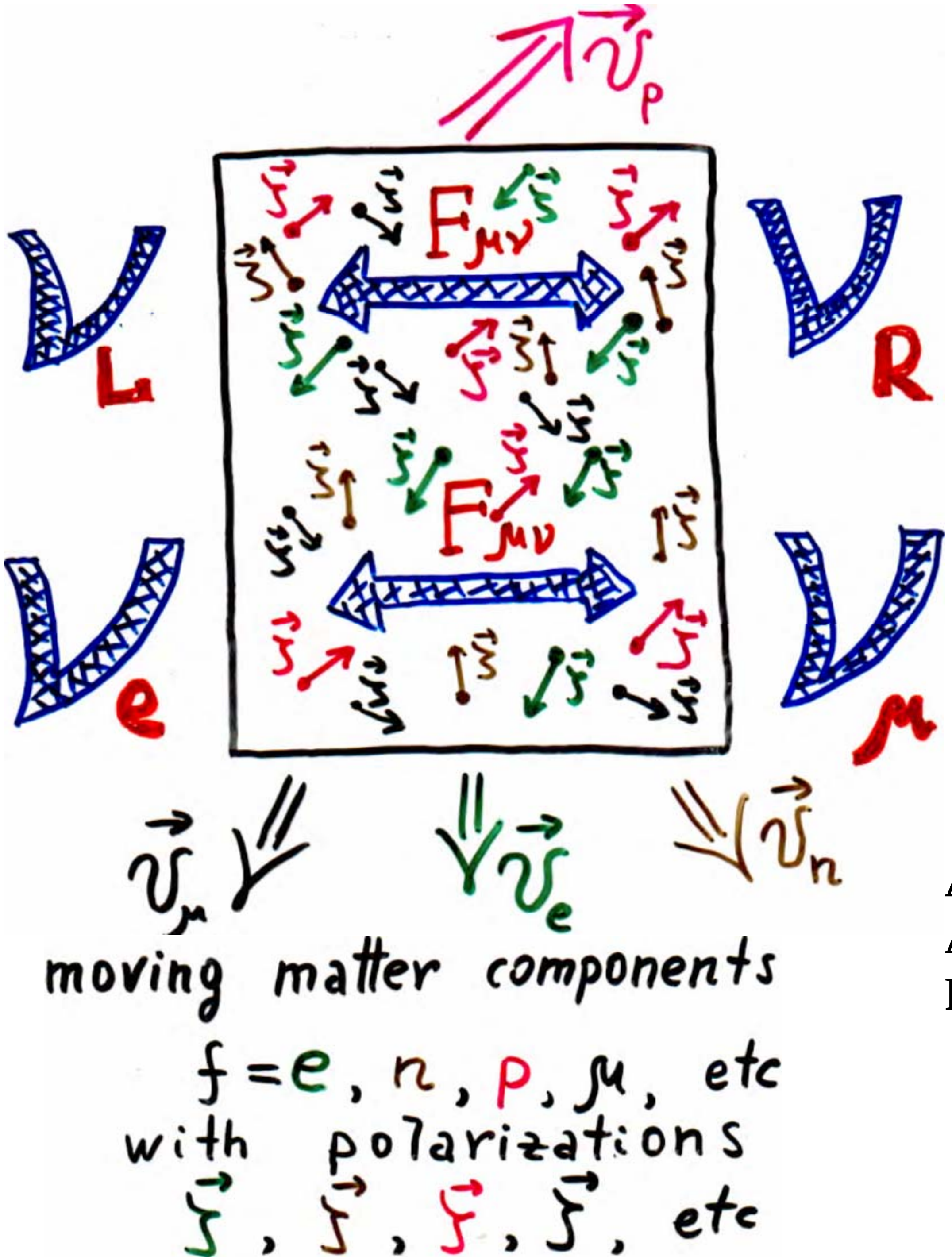
(neutrino spin and flavour oscillations)

in moving and polarized matter

$\Longrightarrow$

matter motion can significantly
change the neutrino oscillation pattern

✓



G.Likhachev,
A.Studenikin,
1995

A.Egorov, A.Lobanov,
A.Studenikin,
Phys.Lett.B 491 (2000) 137

A.Lobanov, A.Studenikin,
Phys.Lett.B 515 (2001) 94

A.Lobanov, A.Grigoriev,
A.Studenikin,
Phys.Lett.B 535 (2002) 187

Unpolarized but moving matter

$(\vec{\zeta}_e = 0,\ v_e \neq 0)$

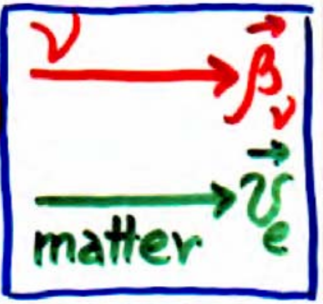
Resonance condition:

- $$\boxed{\frac{\delta m_\nu^2}{2|\vec{p}|}\cos 2\theta = \sqrt{2}\, G_F\, n_e^{(0)} \frac{1-\vec{\beta}_\nu \vec{v}_e}{\sqrt{1-v_e^2}}}$$

$n_e^{(0)}$ — invariant matter density in r.f.

If $\longleftrightarrow \vec{\beta}_\nu$, $\longleftrightarrow \vec{v}_e$:

- $$\left.\frac{1-\vec{\beta}_\nu \vec{v}_e}{\sqrt{1-v_e^2}}\right|_{\beta_\nu \approx 1} = \begin{cases} \left.\sqrt{\dfrac{1-v_e}{1+v_e}}\right|_{v_e \approx 1} \approx \boxed{\dfrac{\sqrt{1-v_e}}{\sqrt{2}}} \\ \left.\sqrt{\dfrac{1+v_e}{1-v_e}}\right|_{v_e \approx 1} \approx \boxed{\dfrac{\sqrt{2}}{\sqrt{1-v_e}}} \end{cases}$$



#4

New mechanism of e.m. radiation by ν in matter and e.m. fields, and gravitational fields

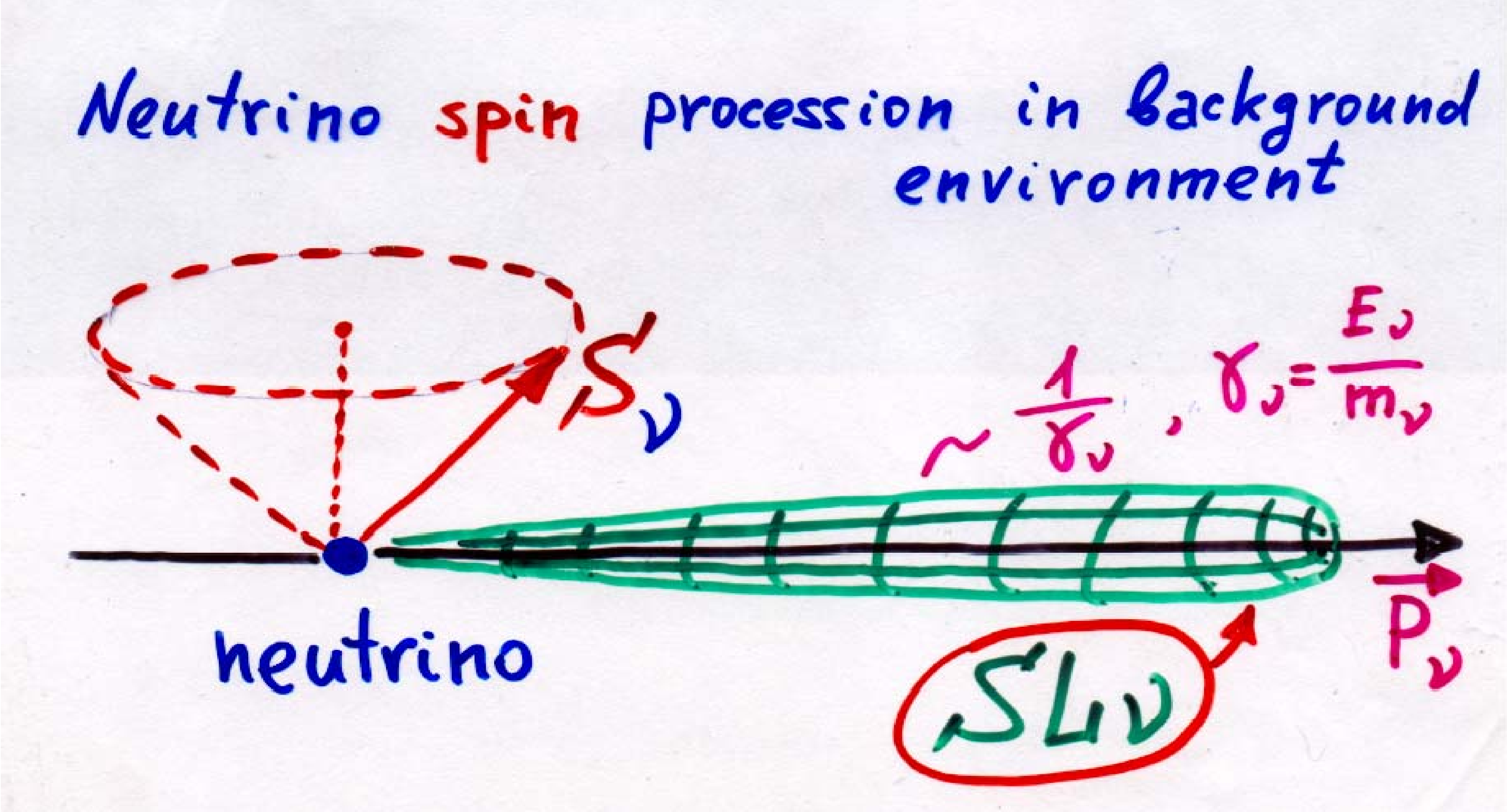
$$\boxed{\nu \xrightarrow[\text{environment}]{\text{background}} \nu + \gamma ,}$$

‖ "Spin Light of Neutrino": "$SL\nu$"

Quasi-classical theory of spin light of neutrino in matter and gravitational field

$SL\nu$

A.Lobanov, A.Studenikin, Phys.Lett. B 564 (2003) 27,
Phys.Lett. B 601 (2004) 171;
M.Dvornikov, A.Grigoriev, A.Studenikin, Int.J.Mod.Phys. D 14 (2005) 309



Now we know:

#4 new mechanism of e.m. radiation by ν in matter (with or without e.m. field being superimposed)

– spin light of neutrino –

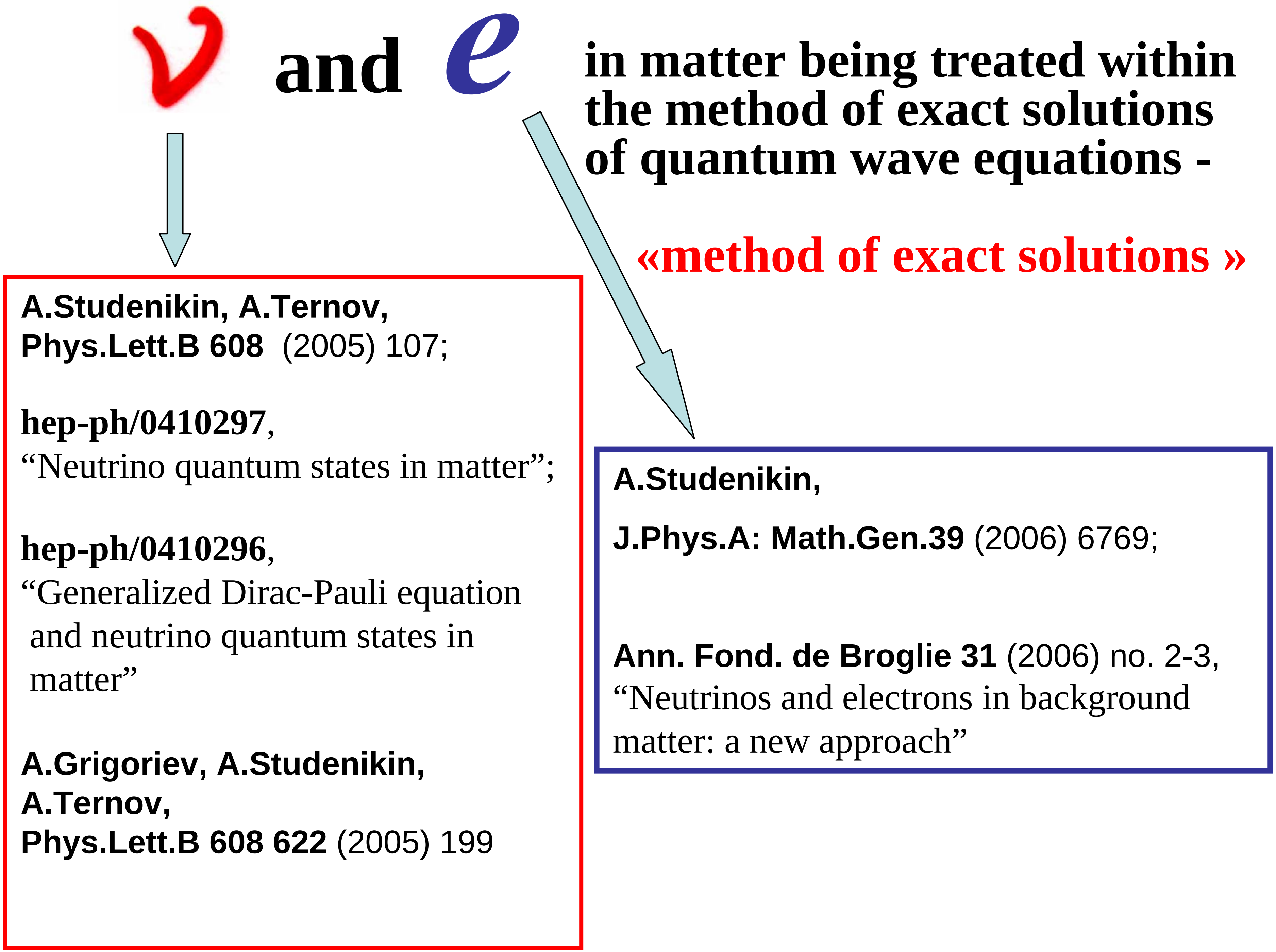
that must be important for dense astrophysical (SN,… gamma-ray bursts) cosmological (the early Universe) environments.

…however !!!

… Consistent approach to $SL\nu$

A.S., "Neutrinos and electrons in background matter: a new approach",
Ann.Fond. de Broglie 31 (2006) no. 2-3

- We present a rather **powerful method** for description of **neutrinos** (and **electrons**) motion in **background matter** which implies the use of **modified Dirac equations** with effective matter potentials being included.



ν and e in matter being treated within the method of exact solutions of quantum wave equations -

«method of exact solutions »

A.Studenikin, A.Ternov,
Phys.Lett.B 608 (2005) 107;

hep-ph/0410297,
"Neutrino quantum states in matter";

hep-ph/0410296,
"Generalized Dirac-Pauli equation and neutrino quantum states in matter"

A.Grigoriev, A.Studenikin, A.Ternov,
Phys.Lett.B 608 622 (2005) 199

A.Studenikin,

J.Phys.A: Math.Gen.39 (2006) 6769;

Ann. Fond. de Broglie 31 (2006) no. 2-3,
"Neutrinos and electrons in background matter: a new approach"

Interaction of particles in external electromagnetic fields

(**Furry representation** in quantum electrodynamics)

$$e \xrightarrow{\mathrm{B}_\perp} e + \gamma$$

synchrotron radiation

Potential of electromagnetic field

$$\boxed{A_\mu(x) = A^q_\mu(x) + A^{ext}_\mu(x)},$$

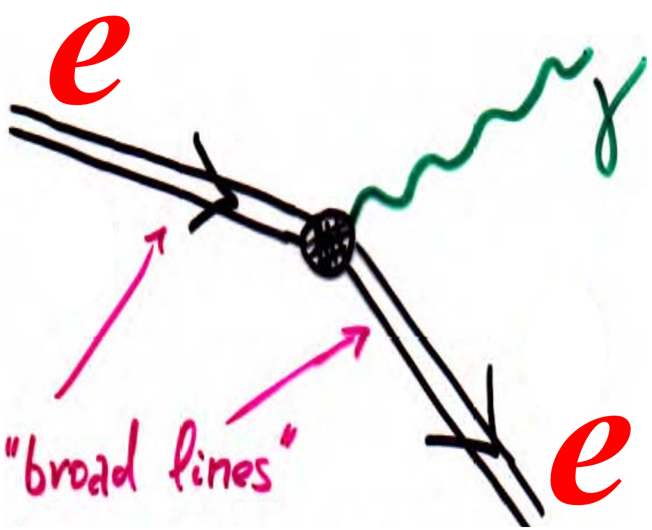
quantized part of potential

evolution **operator**

$$\boxed{U_F(t_1, t_2) = Texp\Big[-i\int_{t_1}^{t_2} j^\mu(x) A^q_\mu(x) dx \Big]},$$

charged particles **current**

$$\boxed{j_\mu(x) = \frac{e}{2}\left[\bar{\Psi}_F\gamma_\mu, \Psi_F\right]},$$



Dirac equation in **external classical** **(non-quantized)** **field** $A^{ext}_\mu(x)$

$$\boxed{\Big\{\gamma^\mu\Big(i\partial_\mu - eA^{ext}_\mu(x)\Big) - m_e\Big\}\Psi_F(x) = 0}$$

Modified **Dirac equations** for e and ν

(containing the correspondent effective matter potentials)

+

exact solutions (particles wave functions)

↓

a basis for investigation of different phenomena which can proceed when **neutrinos** and **electrons** move in dense media

(**astrophysical** and **cosmological** **environments**).

2003-2004-2005

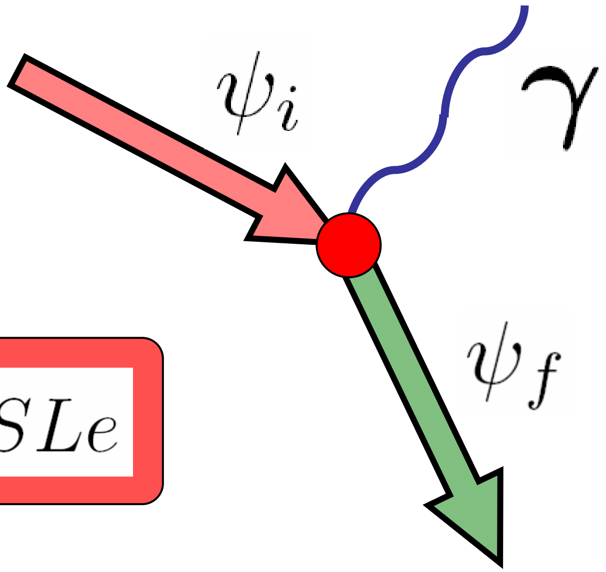
Spin light of neutrino in matter

$SL\nu$

2005-2006

Spin light of electron in matter

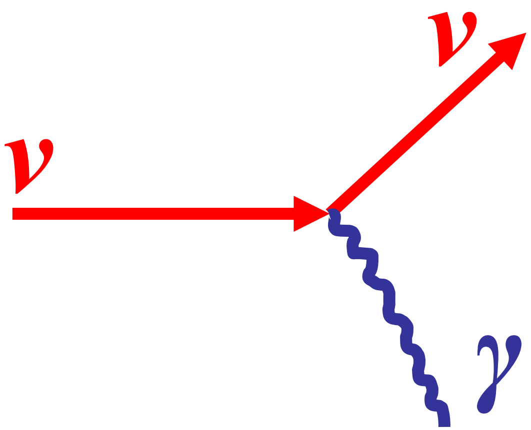
SLe



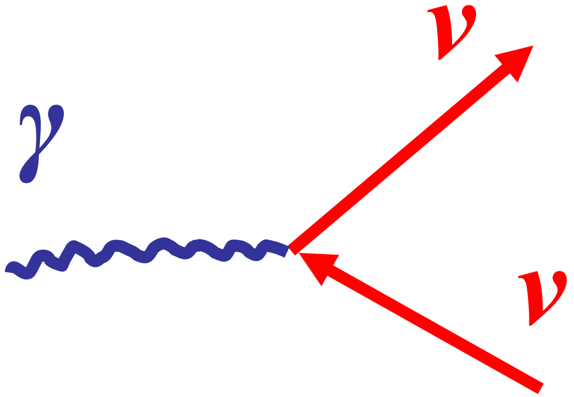
- We predict the existence of a **new mechanism** of the electromagnetic process stimulated by the presence of matter, in which a neutrino or electron change their initial states and light be can emitted.

New mechanism of electromagnetic radiation

Neutrino – photon couplings (I)

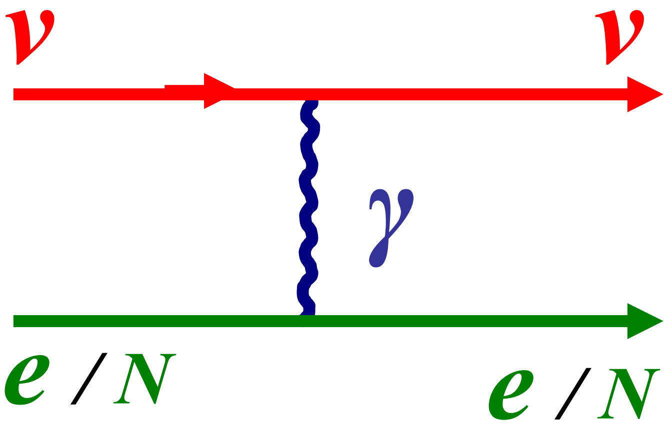


ν decay, Cherenkov radiation

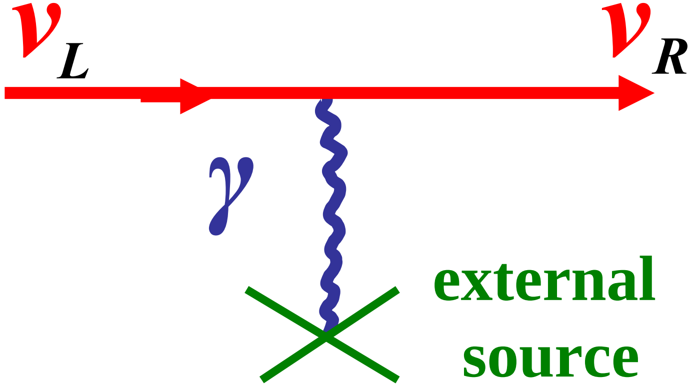


γ decay in plasma

!!!

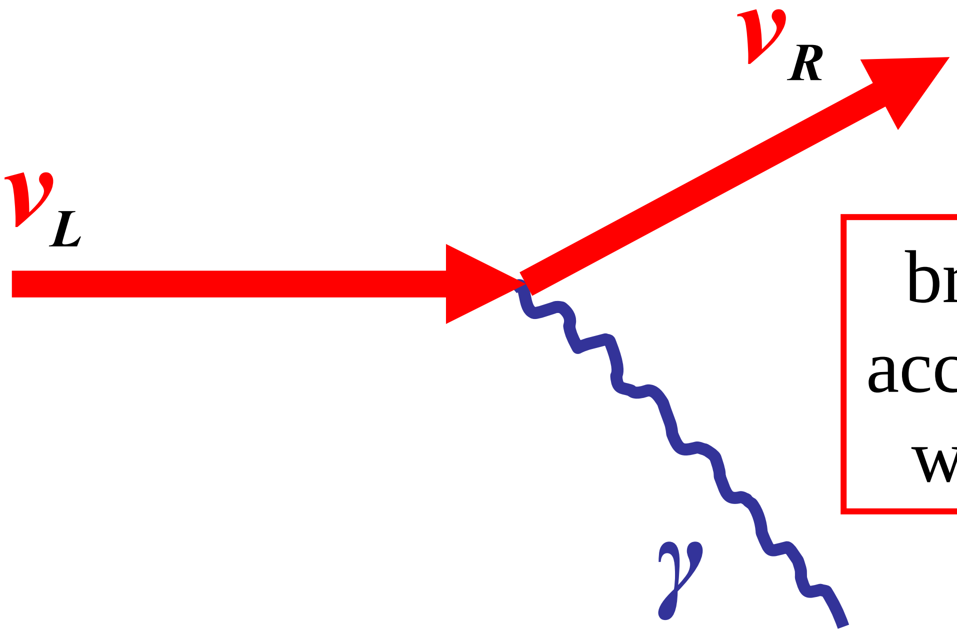


Scattering



Spin procession

Neutrino – photon couplings (II)



broad neutrino lines
account for interaction
with environment

“Spin light of neutrino in matter”

…within the quantum treatment…

Quantum treatment of neutrino in matter

A.Studenikin, J.Phys.A: Math.Gen 39 (2006) 6769

A.Grigoriev, A.Studenikin, A.Ternov, Phys.Atom.Nucl. 69 (2006) 1940

A.Studenikin, A.Ternov, Phys.Lett.B 608 (2005) 107

A.Grigoriev, A.Studenikin, A.Ternov, Phys.Lett.B 622 (2005) 199

Grav. & Cosm. 11 (2005) 132

I.Pivovarov, A.Studenikin, PoS (HEP2005) 191

Standard model electroweak interaction of a flavour neutrino in matter (*f* = *e*)

Interaction Lagrangian (it is supposed that **matter contains only electrons**)

$$
\boxed{\begin{aligned} L_{int} = &-\frac{g}{4\cos\theta_W}\left[\bar{\nu}_e\gamma^\mu(1+\gamma_5)\nu_e - \bar{e}\gamma^\mu(1-4\sin^2\theta_W+\gamma_5)e\right]Z_\mu \\ &-\frac{g}{2\sqrt{2}}\bar{\nu}_e\gamma^\mu(1+\gamma_5)eW_\mu^+ - \frac{g}{2\sqrt{2}}\bar{e}\gamma^\mu(1+\gamma_5)\nu_eW_\mu^- \end{aligned}}
$$

➡ **Charged current** interactions contribution to neutrino potential in matter

✸ $$\Delta L_{eff}^{CC} = \sqrt{2}\,G_F\left\langle\bar{e}\gamma^\mu(1+\gamma_5)e\right\rangle\left(\bar{\nu}_e\gamma^\mu\frac{1+\gamma_5}{2}\nu_e\right)$$

➡ **Neutral current** interactions contribution to neutrino potential in matter

✸ $$\Delta L_{eff}^{NC} = -\frac{G_F}{\sqrt{2}}\left\langle\bar{e}\gamma^\mu\left[(1-4\sin\theta_W^2)+\gamma_5\right]e\right\rangle\left(\bar{\nu}_e\gamma^\mu\frac{1+\gamma_5}{2}\nu_e\right)$$

Modified Dirac equation for neutrino in matter

Addition to the vacuum neutrino Lagrangian

$$\Delta L_{eff} = \Delta L^{CC}_{eff} + \Delta L^{NC}_{eff} = -f^{\mu}\Big(\bar{\nu}\gamma_{\mu}\frac{1+\gamma^{5}}{2}\nu\Big)$$

matter current

where $$f^{\mu} = \frac{G_F}{\sqrt{2}}\Big((1+4\sin^2\theta_W)j^{\mu} - \lambda^{\mu}\Big)$$

matter polarization

$$\Big\{i\gamma_{\mu}\partial^{\mu} - \frac{1}{2}\gamma_{\mu}(1+\gamma_5)f^{\mu} - m\Big\}\Psi(x) = 0$$

It is suppose that there is a macroscopic amount of electrons in the scale of a neutrino de Broglie wave length. Therefore, **the interaction of a neutrino with the matter (electrons) is coherent.**

L.Chang, R.Zia,'88; J.Panteleone,'91; K.Kiers, N.Weiss, M.Tytgat,'97-'98; P.Manheim,'88; D.Nötzold, G.Raffelt,'88; J.Nieves,'89; V.Oraevsky, V.Semikoz, Ya.Smorodinsky,89; W.Naxton, W-M.Zhang'91; M.Kachelriess,'98; A.Kusenko, M.Postma,'02.

A.Studenikin, A.Ternov, hep-ph/0410297;
***Phys.Lett.B* 608 (2005) 107**

This is the most general equation of motion of a neutrino in which the effective potential accounts for both the **charged** and **neutral-current** interactions with the background matter and also for the possible effects of the matter **motion** and **polarization**.

Stationary states

$$\Psi(\mathbf{r},t) = e^{-i(E_\varepsilon t - \mathbf{pr})} u(\mathbf{p}, E_\varepsilon),$$

neutrino wave function in matter

and

$$E_\varepsilon = \varepsilon\sqrt{\mathbf{p}^2\Big(1 - s\alpha\frac{m}{p}\Big)^2 + m^2} + \alpha m$$

neutrino energy spectrum in matter

$s = \pm 1$ for two **helicity** states ,

where the **matter density parameter**

$$\alpha = \frac{1}{2\sqrt{2}}\tilde{G}_F\frac{n}{m},$$

J.Panteleone, 1991
(if NC interaction
were left out)

$\frac{1}{2\sqrt{2}}\tilde{G}_F n \sim 1\ eV$ for $n = 10^{37}\ cm^{-3}$

density of matter
in a neutron star

Neutrino energy in the background matter depends on the state of the neutrino longitudinal polarization (helicity), i.e. in the relativistic case the left-handed and right-handed neutrinos with equal momenta have different energies.

Neutrino wave function in matter (II)

$$\Psi_{\varepsilon,\mathbf{p},s}(\mathbf{r},t)=\frac{e^{-i(E_\varepsilon t-\mathbf{p}\mathbf{r})}}{2L^{\frac{3}{2}}}\begin{pmatrix}\sqrt{1+\frac{m}{E_\varepsilon-\alpha m}}\sqrt{1+s\frac{p_3}{p}}\\ s\sqrt{1+\frac{m}{E_\varepsilon-\alpha m}}\sqrt{1-s\frac{p_3}{p}}\;\;e^{i\delta}\\ s\varepsilon\eta\sqrt{1-\frac{m}{E_\varepsilon-\alpha m}}\sqrt{1+s\frac{p_3}{p}}\\ \varepsilon\eta\sqrt{1-\frac{m}{E_\varepsilon-\alpha m}}\sqrt{1-s\frac{p_3}{p}}\;\;e^{i\delta}\end{pmatrix}$$

A.Studenikin, A.Ternov, hep-ph/0410297;
***Phys.Lett.B* 608 (2005) 107;**

$$\eta=\mathrm{sign}\left(1-s\alpha\frac{m}{p}\right),\ \delta=\arctan\left(p_2/p_1\right)$$

A.Grigoriev, A.Studenikin, A.Ternov,
***Phys.Lett.B* 622 (2005) 199**

$$\boxed{E_\varepsilon-\alpha m=\varepsilon\sqrt{\mathbf{p}^2\left(1-s\alpha\frac{m}{p}\right)^2+m^2}}$$

The quantity $\boxed{\varepsilon=\pm 1}$ splits the solutions into the two branches that in the limit of **vanishing matter density,** $\boxed{\alpha\to 0,}$ reproduce the **positive** and **negative-frequency** solutions, respectively.

Spin **L**ight of **N**eutrino in matter

Quantum theory of $\boxed{SL\nu}$

- A.Studenikin, A.Ternov, *Phys. Lett.**B*** **608** (2005) 107;
- A.Grigoriev, A.Studenikin, A.Ternov, *Phys. Lett.**B*** **622** (2005) 199,
  hep-ph/0502231, hep-ph/0507200;
- A.Grigoriev, A.Studenikin, A.Ternov, *Grav. & Cosm.* **11** (2005) 132;

A.Grigoriev, A.Studenikin, A.Ternov, Phys.Atom.Nucl. 69 (2006) 1940,
hep-ph/0502210, hep-ph/0511311,
hep-ph/0511330;

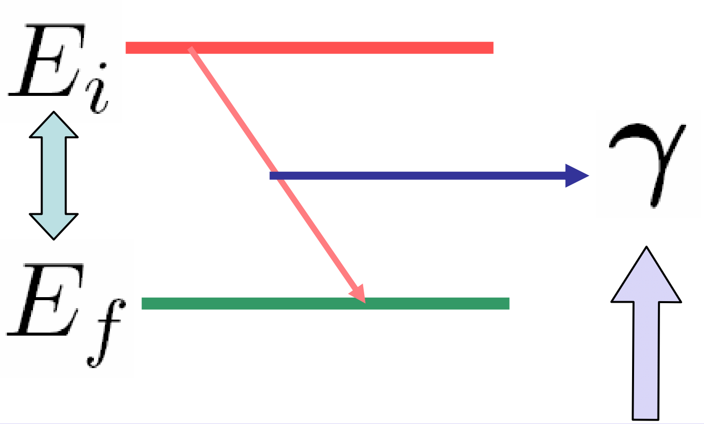
A.Studenikin, A.Ternov, hep-ph/0410296, hep-ph/0410297

Quantum theory of spin light of neutrino (I)

Quantum treatment of *spin light of neutrino* in matter
showns that this process originates from the **two subdivided phenomena:**

$SL\nu$

the **shift** of the neutrino **energy levels** in the presence of the background matter, which is different for the two opposite neutrino helicity states,



$$E=\sqrt{\mathbf{p}^2\left(1-s\alpha\frac{m}{p}\right)^2+m^2}+\alpha m$$

$$s = \pm 1$$

the radiation of the photon in the process of the neutrino transition from the **"excited" helicity state** to the **low-lying helicity state** in matter

A.Studenikin, A.Ternov, Phys.Lett.B 608 (2005) 107;
A.Grigoriev, A.Studenikin, A.Ternov, Phys.Lett.B 622 (2005) 199;
Grav. & Cosm. 14 (2005) 132;

hep-ph/0507200, hep-ph/0502210,

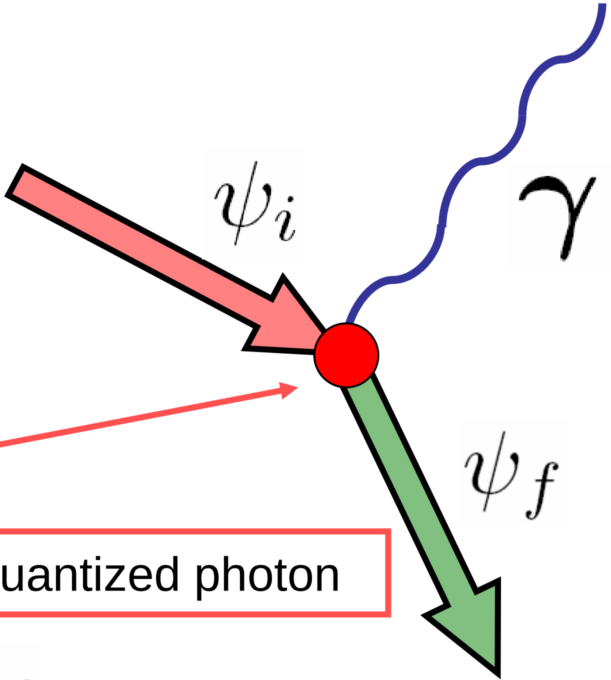
hep-ph/0502231

neutrino-spin self-polarization effect in the matter

A.Lobanov, A.Studenikin, Phys.Lett.B 564 (2003) 27;
Phys.Lett.B 601 (2004) 171

Quantum theory of spin light of neutrino $SL\nu$

Within the **quantum approach**, the corresponding Feynman diagram is the one-photon emission diagram with the **initial** and **final** neutrino states described by the **"broad lines"** that account for the neutrino interaction with matter.



Neutrino magnetic moment interaction with quantized photon

the amplitude of the transition $\psi_i \longrightarrow \psi_f$

$$S_{fi} = -\mu\sqrt{4\pi}\int d^4x\,\bar{\psi}_f(x)(\hat{\mathbf{\Gamma}}\mathbf{e}^*)\frac{e^{ikx}}{\sqrt{2\omega L^3}}\psi_i(x) ,$$

$$\hat{\mathbf{\Gamma}} = i\omega\{[\mathbf{\Sigma}\times\boldsymbol{\varkappa}] + i\gamma^5\mathbf{\Sigma}\} ,$$

$k^\mu = (\omega, \mathbf{k}), \boldsymbol{\varkappa} = \mathbf{k}/\omega$ momentum

$\mathbf{e}^*$ polarization of photon

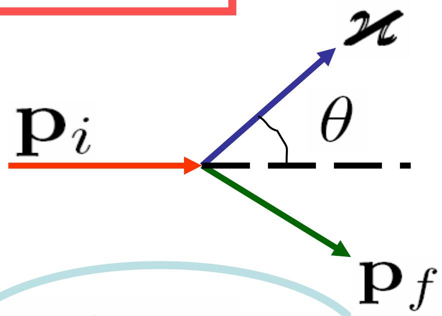
Spin light of neutrino photon’s energy

$SL\boldsymbol{\nu}$ transition amplitude after integration :

$$S_{fi} = -\mu\sqrt{\frac{2\pi}{\omega L^3}}\,2\pi\delta(E_f - E_i + \omega)\int d^3x\bar{\psi}_f(\mathbf{r})(\hat{\mathbf{\Gamma}}\mathbf{e}^*)e^{i\mathbf{kr}}\psi_i(\mathbf{r})$$

Energy-momentum conservation

$$E_i = E_f + \omega, \quad \mathbf{p}_i = \mathbf{p}_f + \boldsymbol{\varkappa}$$



For **electron neutrino** moving in matter composed of **electrons**

$$\alpha = \frac{1}{2\sqrt{2}}\tilde{G}_F\frac{n}{m} > 0$$

$$\omega = \frac{2\alpha m p_i\left[(E_i - \alpha m) - (p_i + \alpha m)\cos\theta\right]}{(E_i - \alpha m - p_i\cos\theta)^2 - (\alpha m)^2}$$

photon energy

- In the radiation process: $s_i = -1 \Longrightarrow s_f = +1$ **neutrino self-polarization**
- For not very high densities of matter , $\tilde{G}_F n/m \ll 1$, in the linear approximation over α

$$\omega = \frac{\beta}{1 - \beta\cos\theta}\omega_0 \quad , \quad \omega_0 = \frac{\tilde{G}_F}{\sqrt{2}}n\beta$$

neutrino speed in vacuum

Spin light transition rate (III)

$$\boxed{SL\nu}$$

transition rate for different neutrino momentum p
and **matter density parameter** $\alpha = \frac{1}{2\sqrt{2}}\tilde{G}_F \frac{n}{m} > 0$

“relativistic” case

$$\boxed{p \gg m}$$

$$\Gamma = \begin{cases} \frac{64}{3}\mu^2\alpha^3p^2m, & \text{for } \alpha \ll \frac{m}{p}, \\ 4\mu^2\alpha^2m^2p, & \text{for } \frac{m}{p} \ll \alpha \ll \frac{p}{m}, \\ 4\mu^2\alpha^3m^3, & \text{for } \alpha \gg \frac{p}{m}. \end{cases}$$

“non-relativistic” case

$$\boxed{p \ll m}$$

neutrino momentum

mass

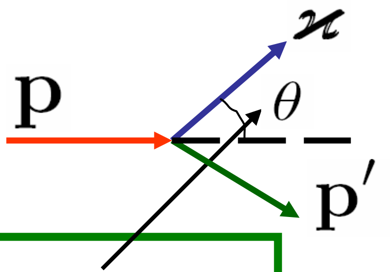
$$\Gamma = \begin{cases} \frac{64}{3}\mu^2\alpha^3p^3, & \text{for } \alpha \ll 1, \\ \frac{512}{5}\mu^2\alpha^6p^3, & \text{for } 1 \ll \alpha \ll \frac{m}{p}, \\ 4\mu^2\alpha^3m^3, & \text{for } \alpha \gg \frac{m}{p}. \end{cases}$$

neutrino magnetic moment

Spin light radiation power

$SL\nu$

radiation power angular distribution :



$$I=\mu^2\int_0^\pi \omega^4\left[(\tilde\beta\tilde\beta'+1)(1-y\cos\theta)-(\tilde\beta+\tilde\beta')(\cos\theta-y)\right]\frac{\sin\theta}{1+\tilde\beta' y}\,d\theta$$

$$\tilde\beta=\frac{p+\alpha m}{E-\alpha m},\quad \tilde\beta'=\frac{p'-\alpha m}{E'-\alpha m},\quad y=\frac{\omega-p\cos\theta}{p'},\quad K=\frac{E-\alpha m-p\cos\theta}{\alpha m},\quad \omega=\frac{2\alpha m p\left[(E-\alpha m)-(p+\alpha m)\cos\theta\right]}{(E-\alpha m-p\cos\theta)^2-(\alpha m)^2}$$

“relativistic” case

$$\boxed{p\gg m}$$

$$I=\begin{cases}\frac{128}{3}\mu^2\alpha^4p^4, & \text{for } \alpha\ll\frac{m}{p},\\ \frac{4}{3}\mu^2\alpha^2m^2p^2, & \text{for } \frac{m}{p}\ll\alpha\ll\frac{p}{m},\\ 4\mu^2\alpha^4m^4, & \text{for } \alpha\gg\frac{p}{m}.\end{cases}$$

“non-relativistic” case

$$\boxed{p\ll m}$$

$$I=\begin{cases}\frac{128}{3}\mu^2\alpha^4p^4, & \text{for } \alpha\ll 1,\\ \frac{1024}{3}\mu^2\alpha^8p^4, & \text{for } 1\ll\alpha\ll\frac{m}{p},\\ 4\mu^2\alpha^4m^4, & \text{for } \alpha\gg\frac{m}{p}.\end{cases}$$

Spin light photon average energy

$$\langle\omega\rangle = \frac{\text{radiation power}}{\text{transition rate}} = \frac{I}{\Gamma}$$

See also:
A.Lobanov,
Phys.Lett.B 619
(2005) 136

“relativistic” case

$\boxed{p \gg m}$

$$\langle\omega\rangle \simeq \begin{cases} 2\alpha\frac{p^2}{m}, & \text{for } \alpha \ll \frac{m}{p}, \\ \frac{1}{3}p, & \text{for } \frac{m}{p} \ll \alpha \ll \frac{p}{m}, \\ \alpha m, & \text{for } \alpha \gg \frac{p}{m}. \end{cases}$$

“non-relativistic” case

$\boxed{p \ll m}$

$$\langle\omega\rangle \simeq \begin{cases} 2p\alpha, & \text{for } \alpha \ll 1, \\ \frac{10}{3}p\alpha^2, & \text{for } 1 \ll \alpha \ll \frac{m}{p}, \\ \alpha m, & \text{for } \alpha \gg \frac{m}{p}. \end{cases}$$

$\alpha \ll \frac{m}{p}$

$$\omega = 2.37 \times 10^{-7} \left(\frac{n}{10^{30} cm^{-3}}\right) \left(\frac{E}{m_\nu}\right)^2 eV$$

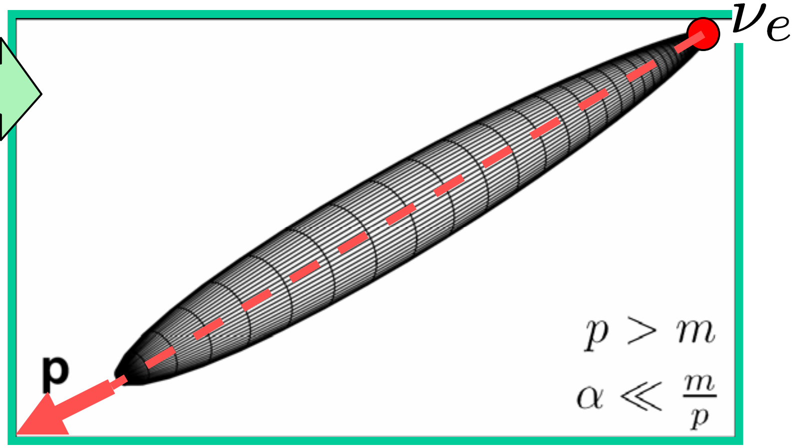
energy range of $\boxed{SL\nu}$

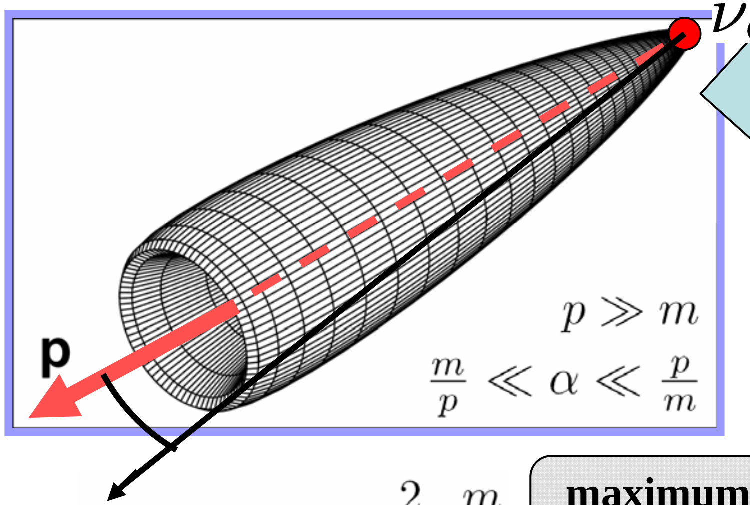
span up to **gamma-rays**

Spatial distribution of radiation power

From the **angular distribution** of $\boxed{SL\nu}$

$$I = \mu^2 \int_0^\pi \omega^4 \left[(\tilde\beta\tilde\beta' + 1)(1 - y\cos\theta) - (\tilde\beta + \tilde\beta')(\cos\theta - y)\right] \frac{\sin\theta}{1+\tilde\beta' y}\, d\theta$$





for $p/m = 5$ **and** $\alpha = 0.01$

$n \approx 10^{35} cm^{-3}$

neutrino momentum

mass

matter density

$$\cos\theta_{max} \simeq 1 - \frac{2}{3}\alpha\frac{m}{p}$$

maximum in radiation power distribution

for $p/m = 10^3$ **and** $\alpha = 100$

$n \approx 10^{39} cm^{-3}$

increase of matter density

projector-like distribution → **cap-like distribution**

Propagation of spin light photon in plasma

A.Grigoriev, A.Studenikin, A.Ternov, Phys.Lett.B 622 (2005) 199;
Grav. & Cosm. 14 (2005) 132

$$\boxed{\omega_{pl} = \sqrt{\frac{4\pi e^2}{m_e} n}}$$

$e^2 = \alpha_{QED}$ **fine-structure constant**

Only **photons with energy that exceeds plasmon frequency** can propagate in electron plasma.

The case of relativistic **neutrino** $p \gg m$
and rather dense **plasma** $\frac{m}{p} \ll \alpha \ll \frac{p}{m}$:

$$\omega_{max} \simeq p \quad \Longleftrightarrow \quad \omega(\theta_{max}) \simeq \frac{3}{4} p$$

maximal value of photon's energy

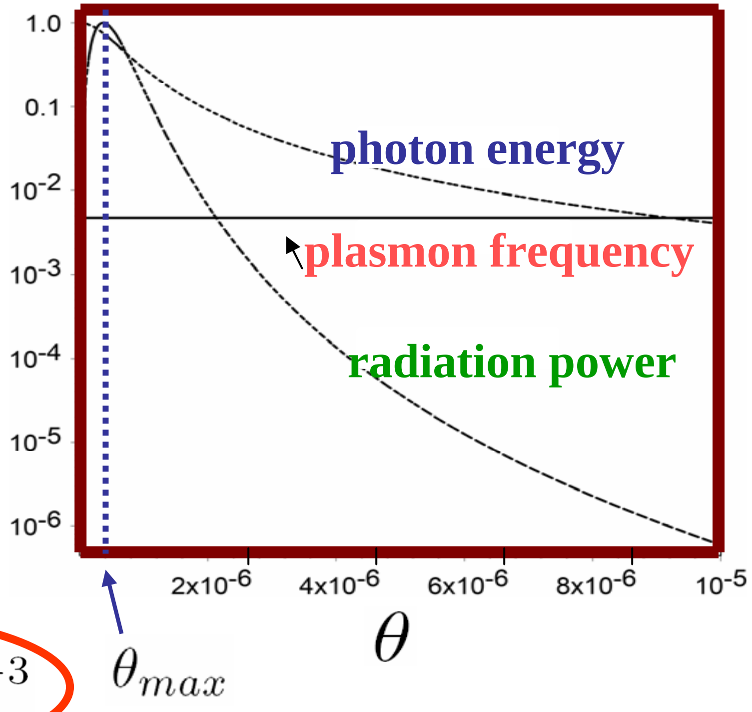
photon's energy in direction of radiation power maximum

$$\cos\theta_{\omega_{max}} = 1 \quad \Longleftrightarrow \quad \cos\theta_{max} \simeq 1 - \frac{2}{3}\alpha\frac{m}{p}$$

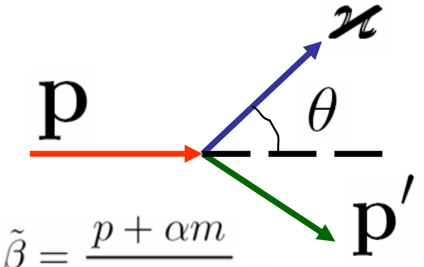
$$p \gg p_{min} = 3.5 \times 10^4 \left(\frac{n}{10^{30} cm^{-3}}\right)^{1/2} eV$$

$p_{min} \sim 1\ MeV$ for $n \sim 10^{33}\ cm^{-3}$

Angular distributions
of **photon energy**
and **radiation power**



Polarization properties of $SL\nu$ photons (II)



- Radiation power of **circularly polarized** photons:

$$\boxed{I^{(l)} = \mu^2 \int_0^{\pi} \frac{\omega^4}{1+\beta' y} S_l \sin\theta d\theta}\ , \quad \tilde{\beta}' = \frac{p' - \alpha m}{E' - \alpha m}\ , \quad \tilde{\beta} = \frac{p + \alpha m}{E - \alpha m}\ ,$$

$$y = \frac{\omega - p\cos\theta}{p'}, \quad K = \frac{E - \alpha m - p\cos\theta}{\alpha m}\ ,$$

$$\omega = \frac{2(E - \alpha m)(K\beta - 1)}{K^2 - 1}$$

where

$$\boxed{S_l = \frac{1}{2}(1 + l\beta')(1 + l\beta)(1 - l\cos\theta)(1 + ly)}$$

$l = \pm 1$ correspond to the photon **right** and **left circular polarizations** .

- In the limit of **low matter density** $\alpha \ll 1$: $E_0 = \sqrt{p^2 + m^2}$

$$\boxed{I^{(l)} \simeq \frac{64}{3}\mu^2 \alpha^4 p^4 \left(1 - l\frac{p}{2E_0}\right)}\ , \qquad I^{(+1)} > I^{(-1)}, \quad \text{however} \quad I^{(+1)} \sim I^{(-1)} .$$

- In **dense matter** $(\alpha \gg \frac{m}{p}$ for $p \gg m$, and $\alpha \gg 1$ for $p \ll m)$:

$$\boxed{\begin{array}{ccc} I^{(+1)} & \simeq & I \\ I^{(-1)} & \simeq & 0 \end{array}} \longleftarrow$$

In a dense matter $SL\nu$ is right-circular polarized.

Experimental identification of $SL\nu$ from astrophysical and cosmological sources

A.Grigoriev, A.Studenikin, A.Ternov, Phys.Lett.B 622 (2005) 199, **hep-ph/0507200**

- **Fireball model of GRBs** **B.Zhang, P.Meszaros,** Int.J.Mod.Phys. **A19** (2004) 2385; **T.Piran,** Rev.Mod.Phys. **76** (2004) 1143.
- **Gamma-rays** can be expected to be produced during **collapses** or **coalescence** processes of neutron stars, owing to $\boxed{SL\nu}$ in dense matter.
- Another favorable situation for effective $\boxed{SL\nu}$ production can be realized during a **neutron star** being **"eaten up"** by the **black hole** at the center of our Galaxy .

For estimation, consider a neutron star with mass $M_{NS} \sim 3M_{\odot}$, $M_{\odot} = 2 \cdot 10^{33} g$

⇒ $n \sim 8 \cdot 10^{38}\ cm^{-3}$, matter density parameter $\alpha \sim 23$,

if $m_{\nu} \sim 0.1\ eV$.

Then for **relativistic neutrinos** $(p \gg m)$

the $\boxed{SL\nu}$ **photon energy** $\langle \omega \rangle \sim \frac{1}{3} p$ ⇐ **totally polarized** **gamma-rays.**

It is possible to have $\tau = \frac{1}{\Gamma_{SL\nu}} << $ **age of the Universe ?**

For ultra-relativistic ν
with momentum $p \sim 10^{20} eV$
and magnetic moment $\mu \sim 10^{-10} \mu_B$
in very dense matter $n \sim 10^{40} cm^{-3}$
from

$$\Gamma_{SL\nu} = 4\mu^2 \alpha^2 m_\nu^2 p$$

$$p \gg m_{plasmon}$$

recently also discussed by A.Kuznetsov, N.Mikheev, 2006

A.Lobanov, A.S., PLB 2003; PLB 2004
A.Grigoriev, A.S., PLB 2005
A.Grigoriev, A.S., A.Ternov, PLB 2005

$$\alpha m_\nu = \frac{1}{2\sqrt{2}} G_F n \left(1 + sin^2\theta_W\right)$$

it follows that

$$\tau = \frac{1}{\Gamma_{SL\nu}} = 1.5 \times 10^{-8} s$$

Spin Light

of Electron in matter

SLe

… a method of studying charged particles interaction in matter…

A.S.,
J.Phys.A: Math. Gen. 39 (2006) 6769

Quantum theory of spin light of electron (I)

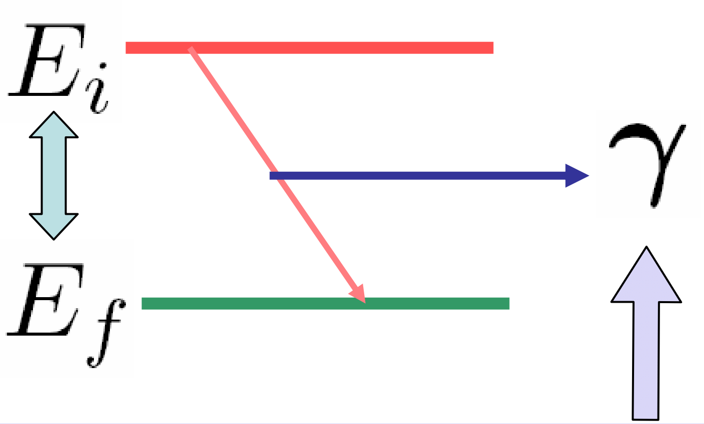
SLe

Spin light of electron in matter originates from the **two subdivided phenomena:**

the **shift** of the electron **energy levels** in the presence of the background matter, which is different for the two opposite electron helicity states,

$$E_\varepsilon^{(e)} = \varepsilon\sqrt{\mathbf{p}^2\left(1 - s\alpha_n \frac{m_e}{p}\right)^2 + m_e^2} + c\alpha_n m_e$$

$$s = \pm 1$$



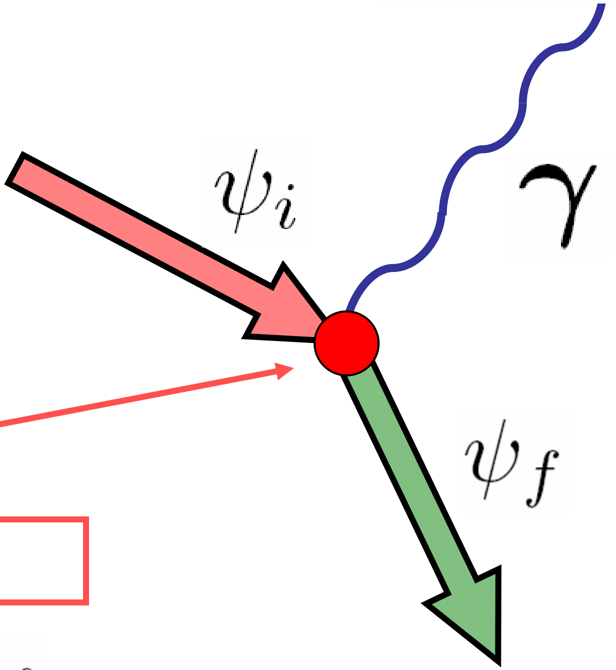
the radiation of the photon in the process of the electron transition from the **“excited” helicity state** to the **low-lying helicity state** in matter

electron-spin self-polarization effect in the matter

A.S., J.Phys.A: Math. Gen. 39 (2006) 6769

Theory of spin light of electron

SLe



The corresponding Feynman diagram is the one-photon emission diagram with the **initial** and **final** electron states described by the **“broad lines”** that account for the electron interaction with matter.

Electron interaction with quantized photon

the amplitude of the transition $\psi_i \longrightarrow \psi_f$

$$\boxed{S_{fi} = -ie\sqrt{4\pi}\int d^4x\bar{\psi}_f(x)\gamma^\mu e_\mu{}^*\frac{e^{ikx}}{\sqrt{2\omega L^3}}\psi_i(x)},$$

$k^\mu = (\omega, \mathbf{k})$, $\boldsymbol{\varkappa} = \mathbf{k}/\omega$ momentum

$\mathbf{e}^*$ polarization of photon

Order-of-magnitude estimation :

$$\boxed{R = \frac{\Gamma_{SLe}}{\Gamma_{SL\nu}} \sim \frac{e^2}{\omega^2\mu^2}} \; ,$$

then for $\mu \sim 10^{-10}\mu_0$ **and** $\omega \sim 5\ MeV$

$$\boxed{R \sim 10^{18}} \; ,$$

A.S.,
J.Phys.A: Math. Gen.
39 (2006) 6769

under these conditions $\boxed{SLe}$

is more effective than $\boxed{SL\nu}$

From exact calculations of $\boxed{SLe}$

$$n \sim 10^{37} \div 10^{40}\ \mathrm{cm}^{-3}$$
$$p \sim 1 \div 10^{3}\ \mathrm{MeV}$$
$$m_\nu = 1\ \mathrm{eV}$$
$$\mu = 10^{-10}\mu_0$$

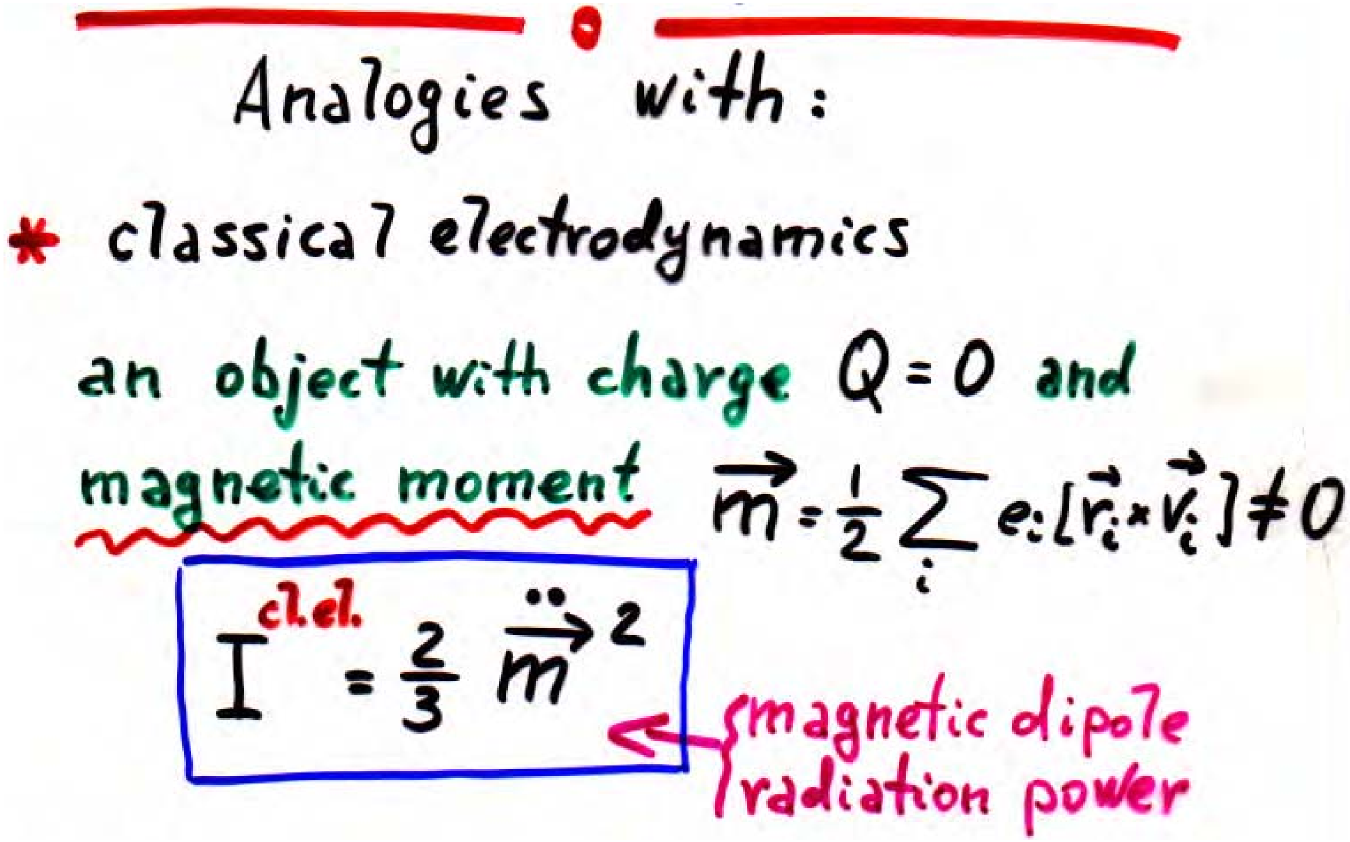
$$R_\Gamma = \frac{\Gamma_{SLe}}{\Gamma_{SL\nu}} \sim \boxed{10^{16} \div 10^{19}}$$

$$R_\mathrm{I} = \frac{\mathrm{I}_{SLe}}{\mathrm{I}_{SL\nu}} \sim \boxed{10^{15} \div 10^{19}}$$

Grigoriev, Shinkevich, Studenikin, Ternov, Trofimov, hep-ph/0611128, Izv.Vuz.Fiz. # 6 (2007) 66.

New mechanism of electromagnetic radiation

? **Why** **Spin Light** of neutrino $SL\nu$ / of electron SLe **in matter.**



Analogies with:

* classical electrodynamics

an object with charge $Q=0$ and

magnetic moment $\vec{m}=\frac{1}{2}\sum_i e_i[\vec{r}_i\times\vec{v}_i]\neq 0$

$$\boxed{I^{cl.el.}=\frac{2}{3}\,\ddot{\vec{m}}^{\,2}}$$

magnetic dipole radiation power

The developed approach to ν and e :

- **Modified Dirac equations** for neutrino and electron in **matter** (**background environment)**
- Exact solutions of **modified Dirac equations** in matter
  - **wave functions** and **energy spectra** in matter
- ***Spin light of neutrino*** in matter $\boxed{SL\nu}$
  - transition **rate,** radiation **power,** photon **energy**
  - spatial angular **distribution** and **polarization**
- ***Spin light of electron*** in matter $\boxed{SLe}$

. . . Applications to astrophysics and cosmology ?

Conclusion

ν **exhibits unexpected properties**

W. Pauli, 1930 :

> ***"… I have done a terrible thing –***
> ***I have introduced a particle***
> ***that can't be observed …"***

- neutron — **now we know that it is** **neutrino** ***E.Fermi, 1933***
- neutral — **now we know that** $\boxed{q_\nu \neq 0}$
  in **matter** **and** **external fields**

  and probably
- massless particle — **now we know that** $\boxed{\Delta m^2_{12} \neq 0}$

$$\boxed{\nu_e = \nu_1 \cos\theta + \nu_2 \sin\theta}$$

- ν **very important player** **(astrophysics, cosmology etc. . .)**