ππ scattering lengths from NA48/2 data on Ke4 and three-pion decays of charged kaons

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on behalf of the **NA48/2** Collaboration:

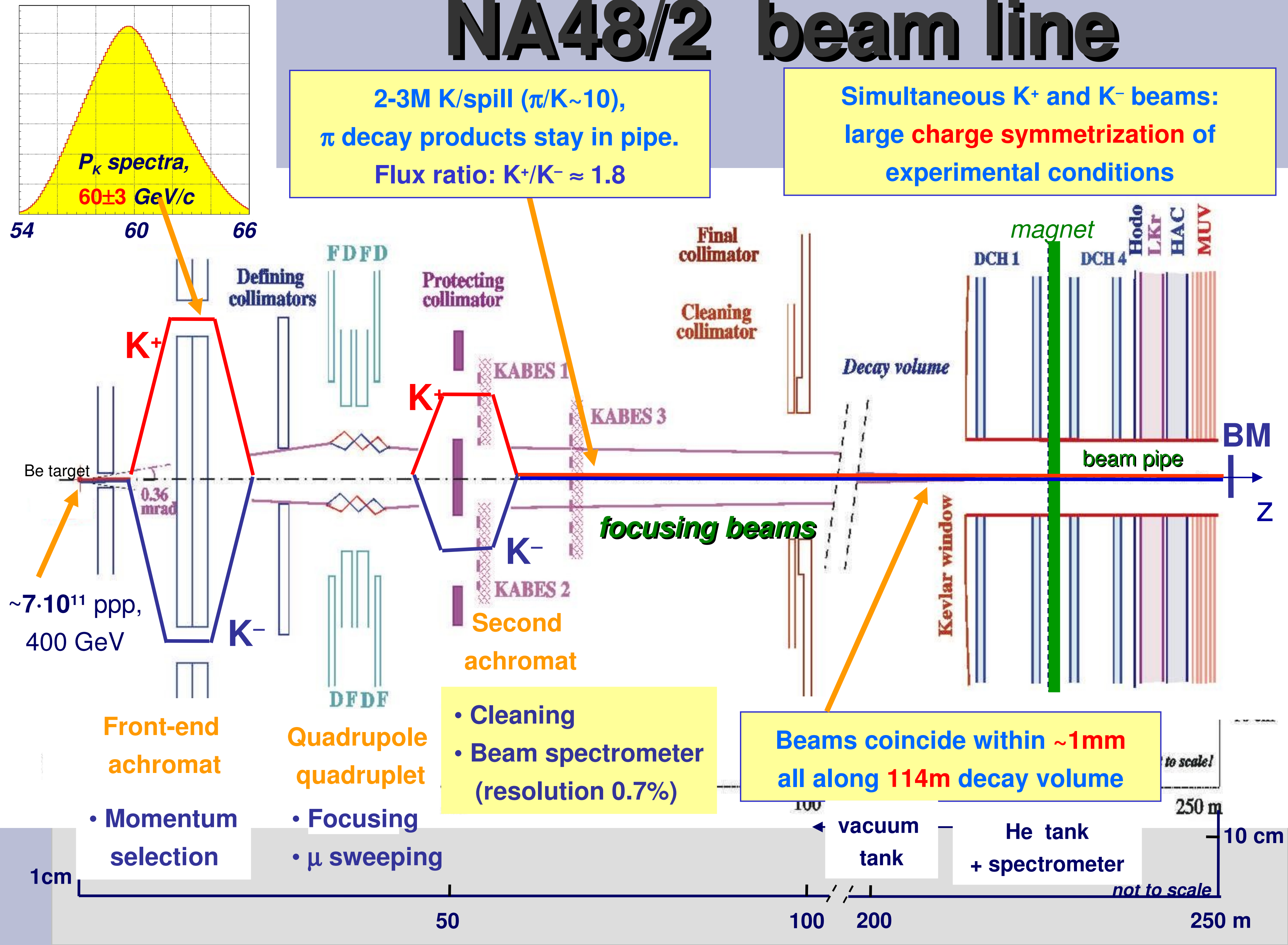
Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Vienna

CERN

Overview

- NA48/2 experiment at CERN: setup, data taking
- Study of $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$ Dalitz plot distribution: an updated measurement of $\pi\pi$ scattering lengths ($a_0 - a_2$, a_2) and the Dalitz plot slopes
- **Ke4 ($K^{\pm} \rightarrow \pi^{+}\pi^{-}e^{\pm}\nu$)**
  - Form factors and pion scattering lengths
  - Data 2003: Preliminary results

NA48/2 beam line



The NA48 detector

Main detector components:

- Magnetic spectrometer (4 DCHs):

  4 views/DCH: redundancy ⇒efficiency;

  used in trigger logic;

  p/p = 1.0% + 0.044%*p [GeV/c].
- Hodoscope

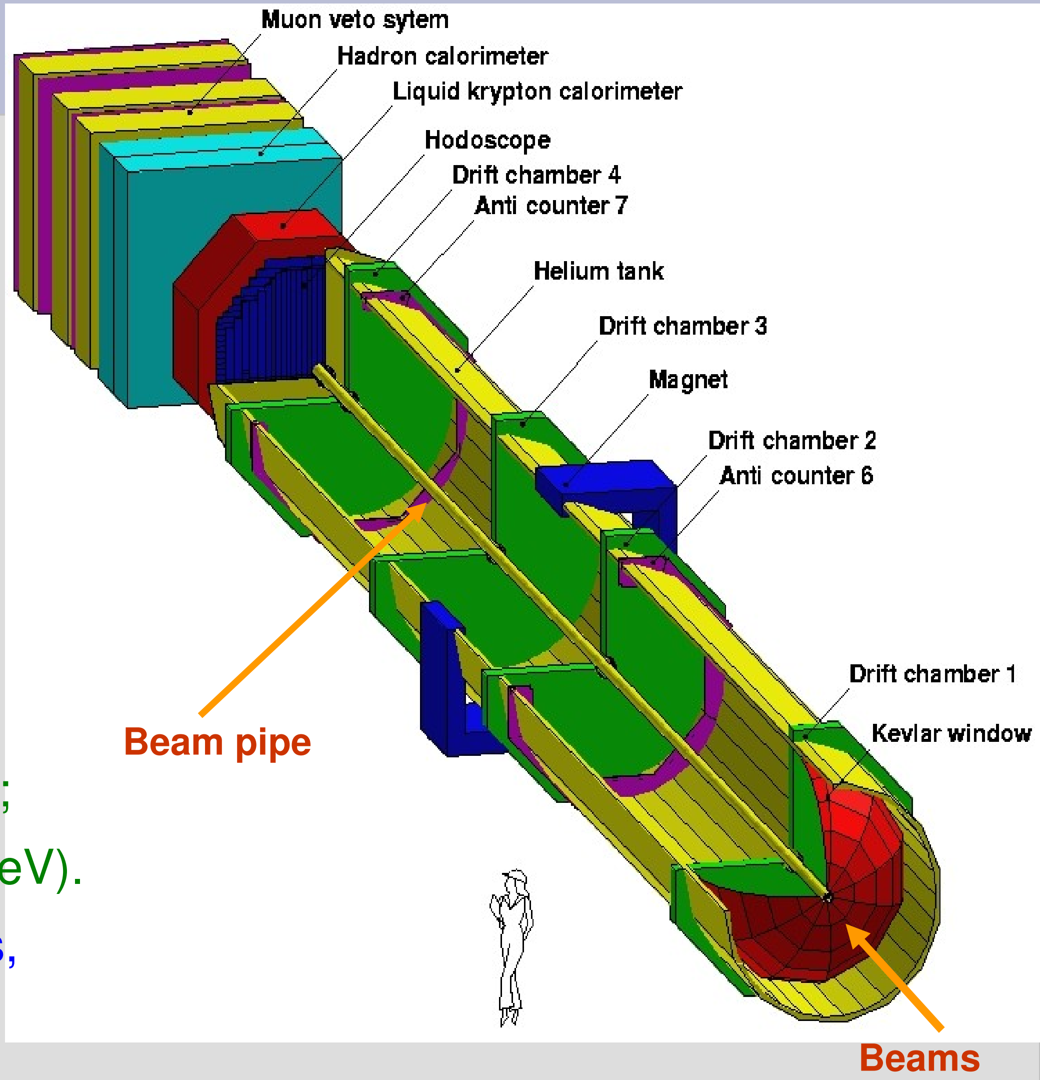
  fast trigger;

  precise time measurement (150ps).
- Liquid Krypton EM calorimeter (LKr)

  High granularity, quasi-homogenious;

  $\sigma_E/E = 3.2\%/E^{1/2} + 9\%/E + 0.42\%$ [GeV];

  $\sigma_x=\sigma_y=0.42/E^{1/2} + 0.6$mm (1.5mm@10GeV).
- Hadron calorimeter, muon veto counters, photon vetoes.



NA48/2 data:

2003 run: ~ 50 days

2004 run: ~ 60 days

A view of the NA48/2 beam line

Total statistics in 2 years:

$K^\pm \to \pi^-\pi^+\pi^\pm$: ~$4\cdot10^9$

$K^\pm \to \pi^0\pi^0\pi^\pm$: ~$1\cdot10^8$

Rare $K^\pm$ decays:

BR's down to 10^{-9}

can be measured

>200 TB of data recorded

$\pi\pi$ scattering lengths

The important free parameter of ChPT is the quark condensate <qq>, it determines the relative size of mass and momentum terms in the power expansion.

a_0 and a_2 are S-wave $\pi\pi$ scattering lengths in isospin states I=0 and I=2, correspondingly. They enter into all $\pi\pi$ scattering amplitudes.

The relation between <qq> and the scattering lengths a_0 and a_2 is known from this theory with a high precision, so the experimental measurement of a_0 and a_2 provides an important constrains for ChPT Lagrangian parameters.

Study of the cusp-effect in $K^{\pm}\to\pi^{\pm}\pi^{0}\pi^{0}$ decays

[2003 result: PLB 633 (2006) 173,
Cabibbo-Isidori theoretical framework]

[Now: a preliminary 2003+2004 result,
updated selection and analysis of systematic effects,
the same theoretical framework]

$K^{\pm}\rightarrow\pi^{\pm}\pi^{0}\pi^{0}$ selection

For each photon pair (i,k) a decay vertex reconstructed along beam axis under the assumption of $\pi^0\rightarrow\gamma\gamma$ decay

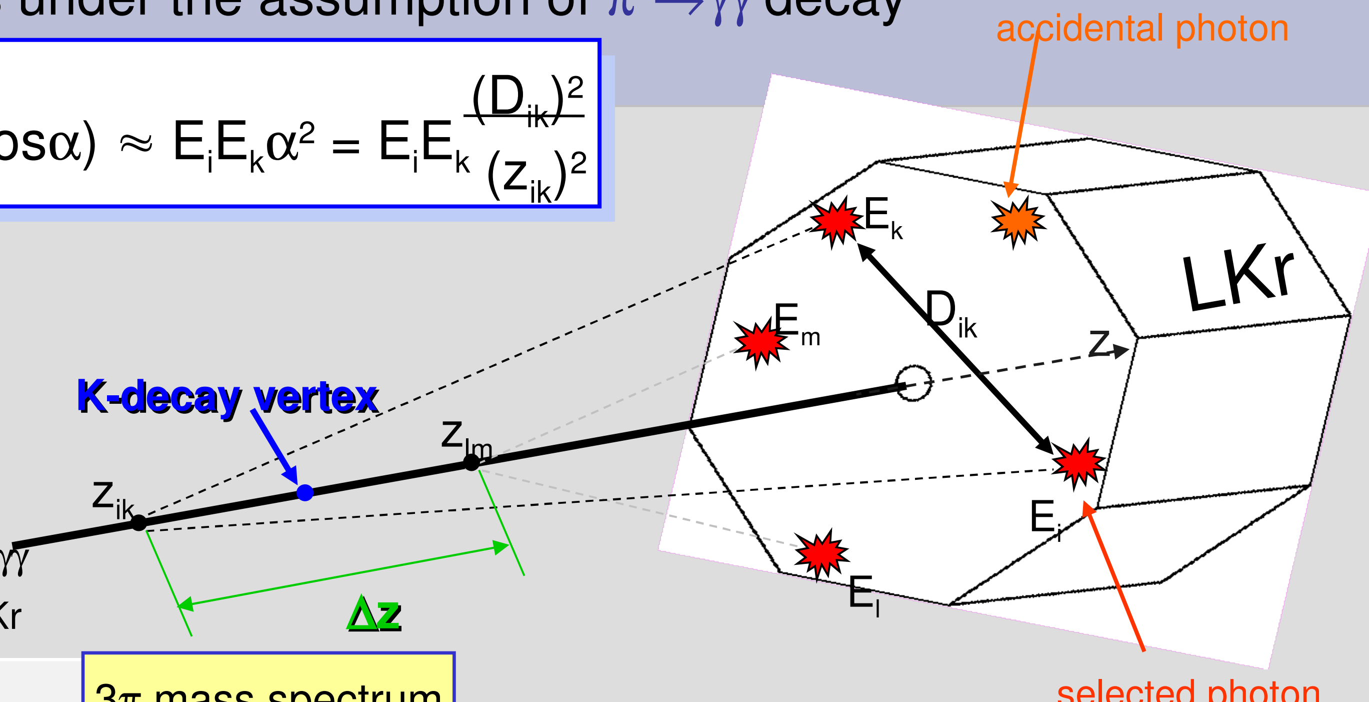
$$m_0^2 = 2E_iE_k(1-\cos\alpha) \approx E_iE_k\alpha^2 = E_iE_k\frac{(D_{ik})^2}{(z_{ik})^2}$$

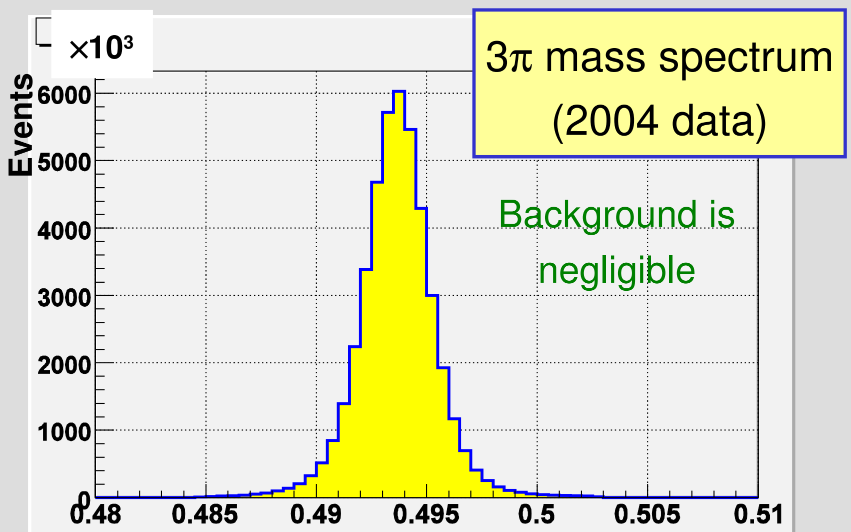
m_0^2 – mass of π^0

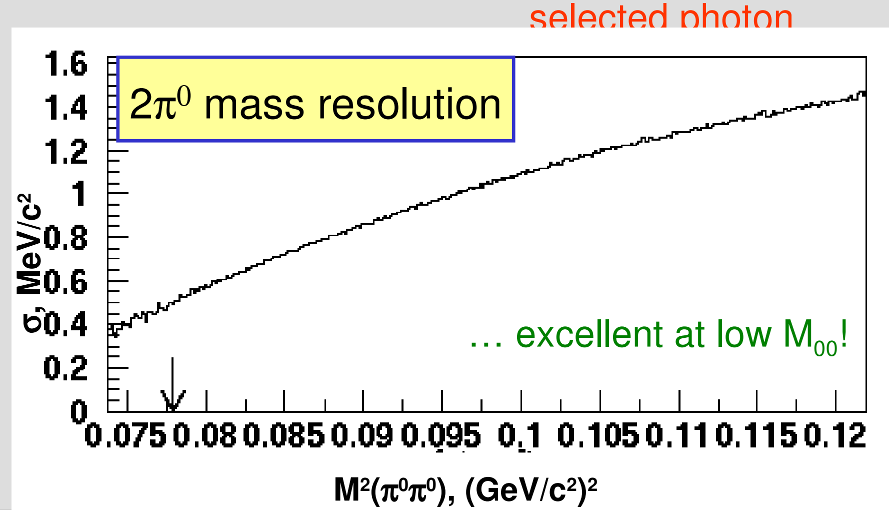
E_i, E_k – energy of γ_i, γ_k

D_{ik} – distance between γ_i and γ_k on LKr

z_{ik} – distance from $\pi^0\rightarrow\gamma\gamma$ decay vertex to LKr







Observation of the cusp

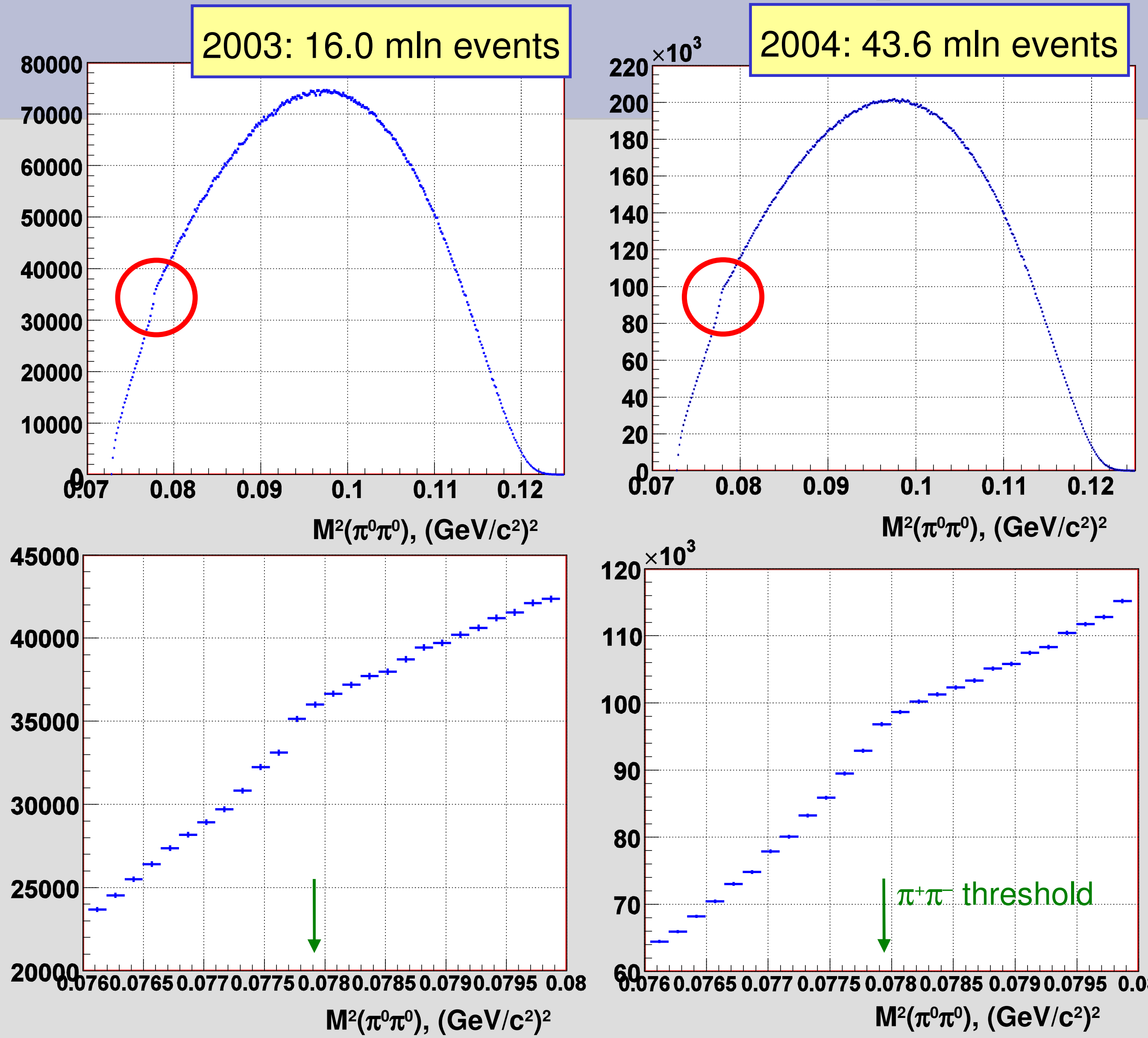
First observation of the cusp was made with the 2003 data

Inclusion of 2004 data: statistics increased by a factor of 3.7!

2003 and 2004 data: slightly different selection conditions and different data/MC sample ratios.

⇓

Analysis is made independently for the two data sets, and then the results are averaged.



Theory: final state rescattering

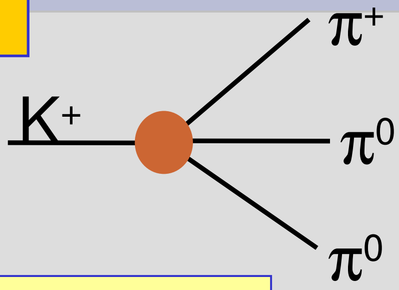
N. Cabibbo, PRL 93 (2004) 121801

$$\mathcal{M}(K^\pm \to \pi^\pm \pi^0 \pi^0) = \mathcal{M}_0 + \mathcal{M}_1$$

Kaon rest frame:

$u = 2m_K(m_K/3 - E_{odd})/m_\pi^2$

$v = 2m_K(E_1 - E_2)/m_\pi^2$



Direct emission:

$$\mathcal{M}_0 = A_0(1 + g_0 u/2 + h' u^2/2 + k' v^2/2)$$

Rescattering amplitude:

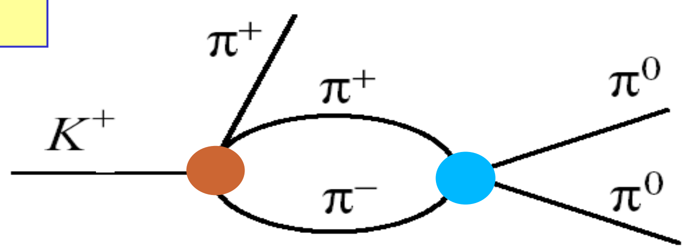
$$\mathcal{M}_1 = -2/3(a_0 - a_2)m_+ \mathcal{M}_+ \sqrt{1 - \left(\frac{M_{00}}{2m_+}\right)^2}$$

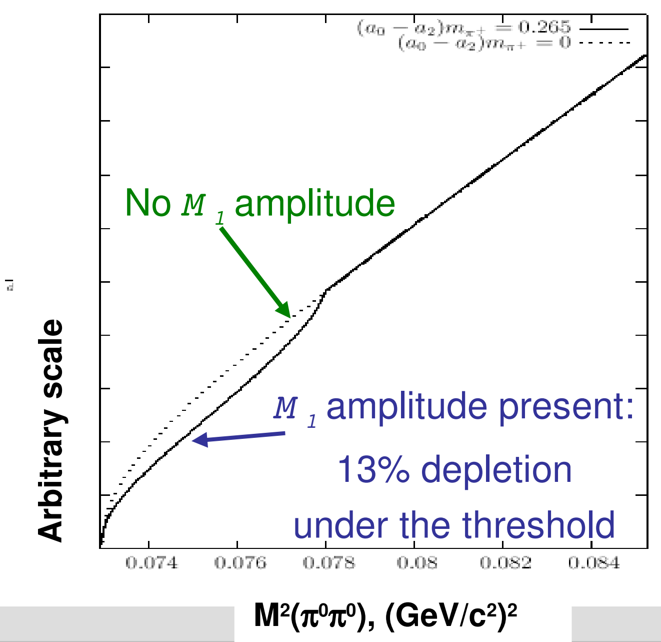
Combination of S-wave ππ scattering lenghts

(isospin symmetry assumed here)

$K^\pm \to 3\pi^\pm$ amplitude at threshold

Negative interference under threshold



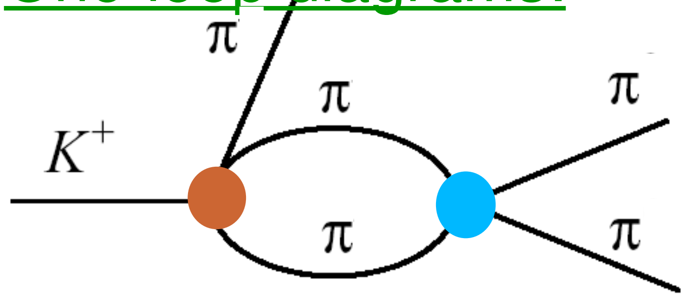


Theory: two-loop diagrams

N. Cabibbo and G. Isidori,
JHEP 503 (2005) 21

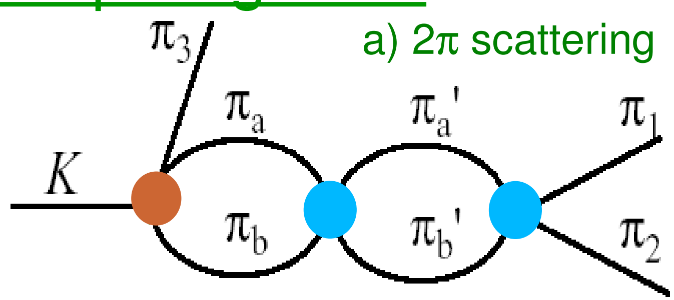
- Five S-wave scattering lengths (a_x, a_{++}, a_{+-}, a_{+0}, a_{00}) expressed as linear combinations of a_0 and a_2
- Isospin symmetry breaking accounted for following J. Gasser
- For example, $a_x = (1+\varepsilon/3)(a_0-a_2)/3$, where $\varepsilon=(m_+^2-m_0^2)/m_+^2=0.065$ is isospin breaking parameter
- All rescattering processes at one- & two-loop level
- Radiative corrections missing: (a_0-a_2) precision ~5%

One-loop diagrams:

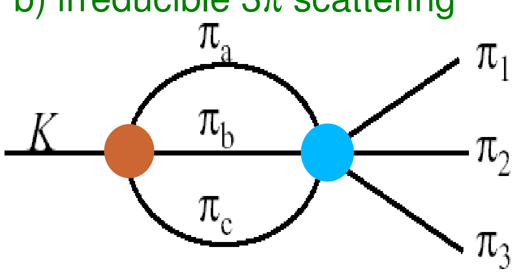


Two-loop diagrams:

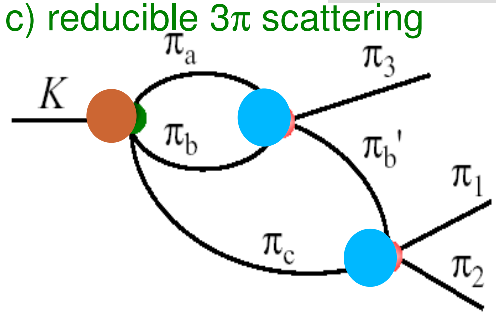
a) 2π scattering

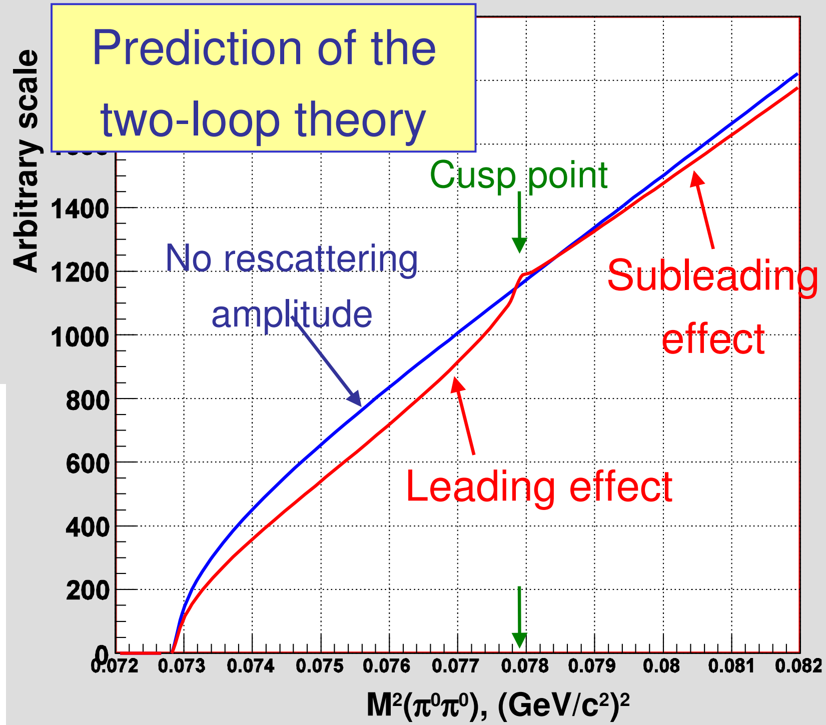


b) irreducible 3π scattering



c) reducible 3π scattering





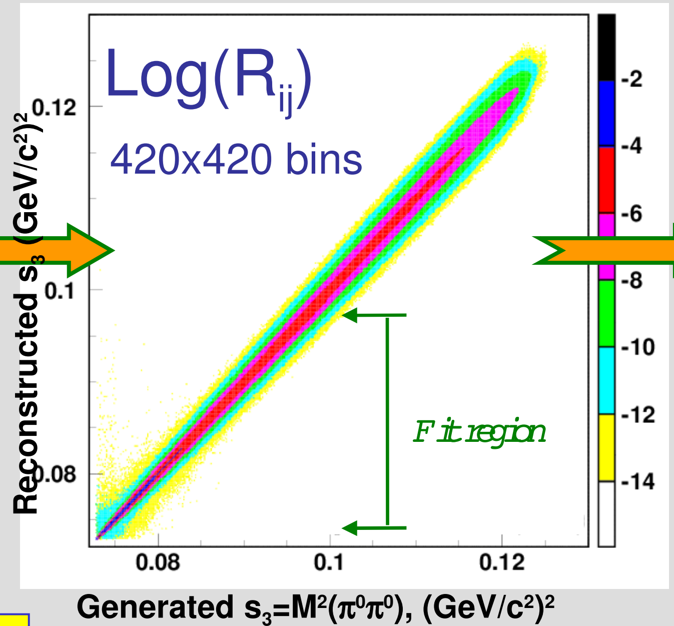
Fitting procedure

1-dimensional fit of the M_{00} projection

Detector response matrix R_{ij} obtained with a GEANT-based Monte-Carlo simulation

Generated distribution

$G(M_{00}) = G(g_0, h', a_0, a_2, M_{00})$



Reconstructed distribution:

$F_j^{MC} = \Sigma R_{ij} G_i$

5 free parameters

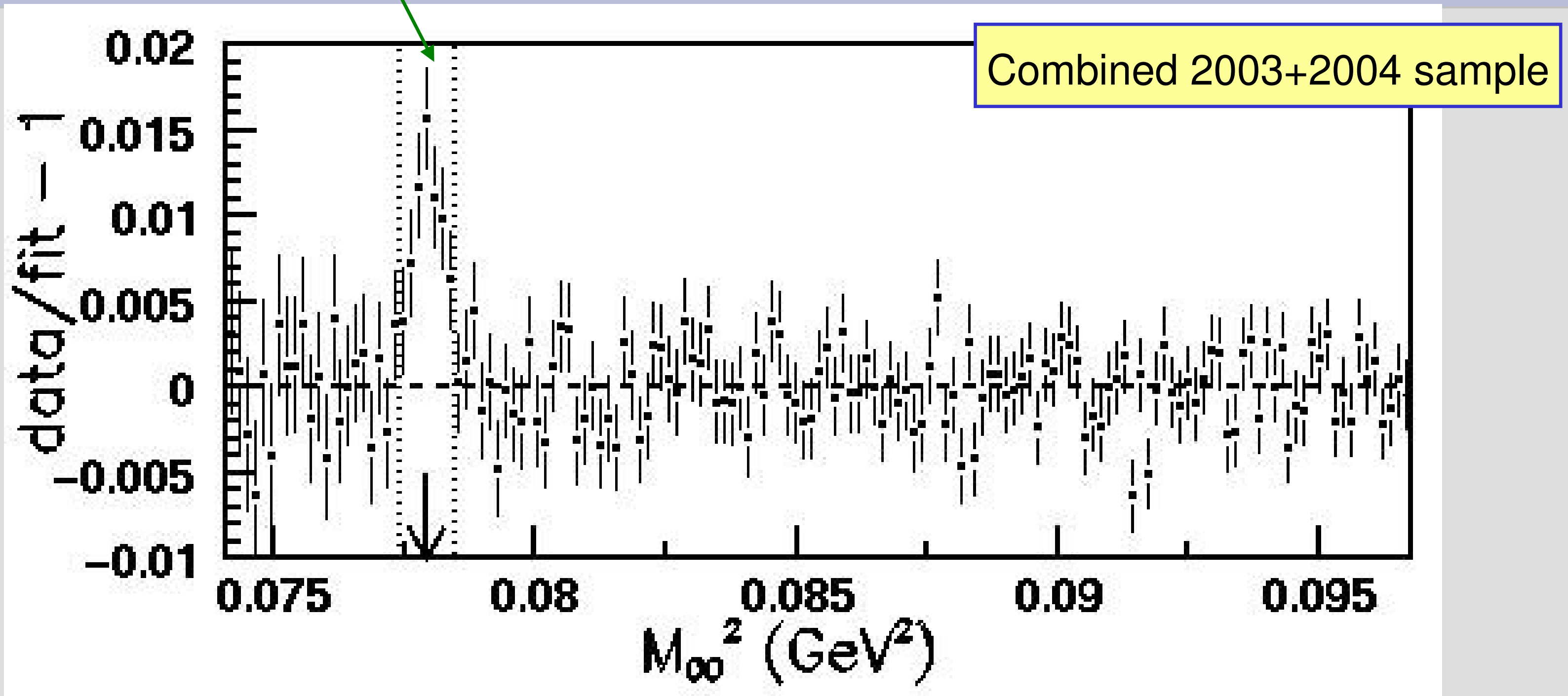
MINUIT minimization of χ^2 of data/MC spectra shapes

$$\chi^2(g, h', m_+(a_0 - a_2), m_+ a_2, N) = \sum_{s_3 \text{ bins}} \frac{(F_{DATA} - N F_{MC})^2}{\delta F_{DATA}^2 + N^2 \delta F_{MC}^2}$$

Fit quality & pionium signature

Points excluded from the fit due to absence of EM corrections in the used model

7 data bins skipped around the $M(\pi^+\pi^-)$ threshold



Excess of events in the excluded interval,

if interpreted as due to pionium decaying as $A_{2\pi}\to\pi^0\pi^0$,

gives $R=\Gamma(K^\pm\to\pi^+A_{2\pi})/\Gamma(K^\pm\to\pi^\pm\pi^+\pi^-) = (1.82\pm0.21)\times10^{-5}$.

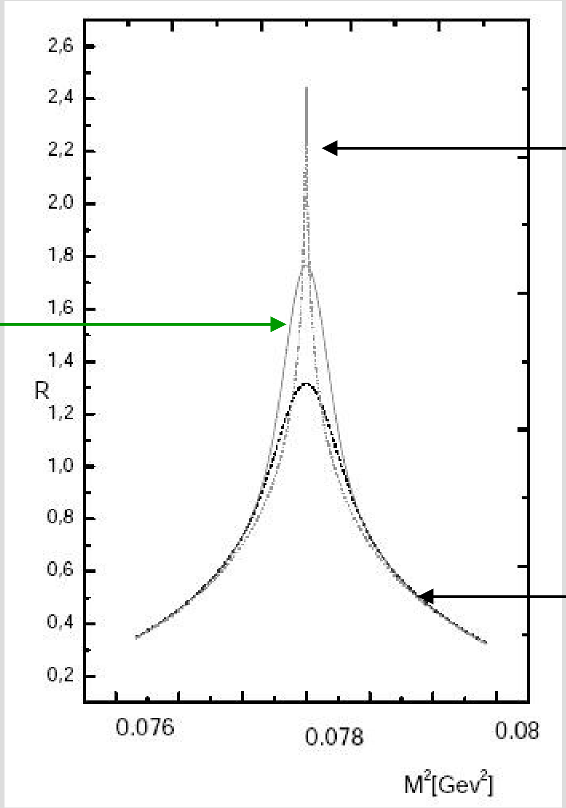
⇒ Prediction [Z.K. Silagadze, JETP Lett. 60 (1994) 689]: $R=0.8\times10^{-5}$.

Electromagnetic corrections to final state interactions in $K \rightarrow 3\pi$ decays

(Gevorkian, Tarasov, Voskresenskaya, hep-ph / 0612129)

Two contributions from $K^{\pm} \rightarrow \pi^{\pm}\pi^{+}\pi^{-}$ decay to the $K^{\pm} \rightarrow \pi^{\pm}\pi^{\circ}\pi^{\circ}$ cusp region:

- **Pionium formation : $\pi^{+}\pi^{-}$ atom $\rightarrow \pi^{\circ}\pi^{\circ}$ (negligible width)**
- **Additional $\pi^{+}\pi^{-}$ unbound states with resonance structure $\rightarrow \pi^{\circ}\pi^{\circ}$**



$\pi^{+}\pi^{-}$ resonant structure
(no experimental resolution)

$\pi^{+}\pi^{-}$ atoms and
$\pi^{+}\pi^{-}$ resonant structure
with experimental resolution

$\pi^{+}\pi^{-}$ resonant structure
with experimental resolution

Uncertainties & results (1)

(preliminary)

NA48/2 result with 2003+2004 statistics

$$(a_0 - a_2)m_+ = 0.261 \pm 0.006_{stat.} \pm 0.003_{syst.} \pm 0.0013_{ext.}$$

$$a_2 m_+ = -0.037 \pm 0.013_{stat.} \pm 0.009_{syst.} \pm 0.0018_{ext.}$$

- External uncertainty: due to $R = (A_{++-}/A_{+00})|_{threshold} = 1.975 \pm 0.015$;
- Theory precision (rad.corr. & higher order terms neglected): $\delta(a_0 - a_2)m_+ = 0.013$.

Systematic effect	$(a_0-a_2)\times 10^2$	$a_2 \times 10^2$
Analysis technique	±0.10	±0.20
Trigger inefficiency	negl.	±0.50
Description of resolution	±0.06	±0.11
LKr non-linearity	±0.06	±0.26
Geometric acceptance	±0.02	±0.01
MC sample	±0.03	±0.21
Simulation of LKr showers	±0.17	±0.50
V-dependence of amplitude	±0.17	±0.38
Total	±0.28	±0.90

Uncertainties & results (2)

Using a chiral symmetry constraint [Colangelo et al., PRL 86 (2001) 5008]:

$$a_2 = -0.0444 + 0.236(a_0-0.22) - 0.61(a_0-0.22)^2 - 9.9(a_0-0.22)^3$$

(preliminary)

$$(a_0-a_2)m_+ = \quad 0.263 \pm 0.003_{stat.} \pm 0.0014_{syst.} \pm 0.0013_{ext.}$$

Theory precision uncertainty is also applicable: $\delta(a_0-a_2)m_+ = 0.013$.

Systematic effect	$(a_0-a_2)\times10^2$
Analysis technique	±0.08
Trigger inefficiency	negl.
Description of resolution	±0.06
LKr non-linearity	±0.05
Geometric acceptance	±0.02
MC sample	±0.06
Simulation of LKr showers	±0.04
V-dependence of amplitude	±0.02
Total	±0.14

Results: Dalitz plot slopes

$$\mathcal{M}(K^\pm \to \pi^\pm\pi^0\pi^0) = \mathcal{M}_0 + \mathcal{M}_1$$

$\mathcal{M}_0 \sim (1+g_0u/2+h'u^2/2+k'v^2/2)$

NB: not the same parameterization as the PDG one:

$|\mathcal{M}_0|^2_{(PDG)} \sim (1+gu+hu^2+kv^2)$ $[g_0 \approx g,\ h' \approx h - g^2/4,\ k' \approx k]$

Technique: consecutive 1D-fit for rescattering and 2D-fit for the rest.

1. (a_0,a_2,g_0,h') measurement (fixed $k'=0$) [2003 data] →
2. k' measurement (fixed a_0,a_2,g_0,h') [2003 data] →
3. (a_0,a_2,g_0,h') second iteration (fixed k') [2004 data]

$$g = (\ 64.9 \pm 0.3_{stat.} \pm 0.4_{syst.})\%$$
$$h' = (\ -4.7 \pm 0.7_{stat.} \pm 0.5_{syst.})\%$$
$$k' = (\ -0.97 \pm 0.03_{stat.} \pm 0.08_{syst.})\%$$

NA48/2 result:

(preliminary)

(a_0,a_2 are not significantly affected by neglecting the $k'v^2$ term,

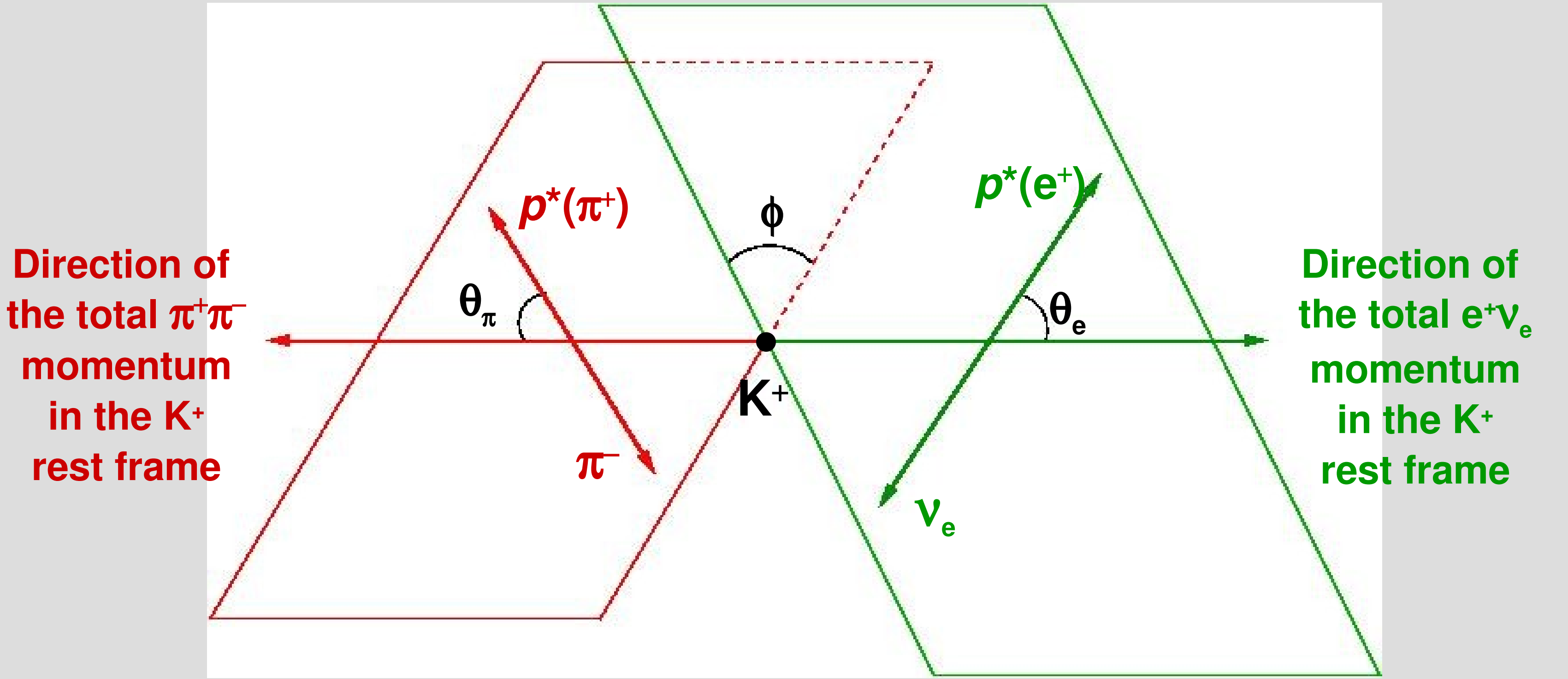
but g_0,h' are biased by $\delta g_0=-1.5\%$, $\delta h=-1.2\%$)

ππ scattering

in $K^{\pm} \rightarrow \pi^{+}\pi^{-}e^{\pm} \nu$ decay

K_{e4}: formalism

A rare decay [B.R. = (4.09 ± 0.09) x 10^{-5}] described by five independent variables



Cabibbo – Maksymowicz variables : $s_\pi \equiv M_{\pi\pi}^2$

$s_e \equiv M_{e\nu}^2$

θ_e

θ_π

ϕ

For $K^+ \Rightarrow K^-$

$\phi \Rightarrow \pi + \phi$

$\theta_e \Rightarrow \pi - \theta_e$

K_{e4}: formalism

In the partial wave expansion (only S and P waves) the amplitude can be written using form factors:

$F = F_s e^{i\delta s} + F_p e^{i\delta p} \cos\theta_\pi$

$G = G_p e^{i\delta p}$

$H = H_p e^{i\delta p}$

F (F_p,F_s), G, H and $\delta = \delta_p - \delta_s$ will be used as fit parameters

The form factors can be expanded as a function of $M_{\pi\pi}^2$ and $M_{e\nu}^2$:

$F_s = f_s + f_s' q^2 + f_s'' q^4 + f_e' (M_{e\nu}^2/4m_\pi^2) + \ldots$

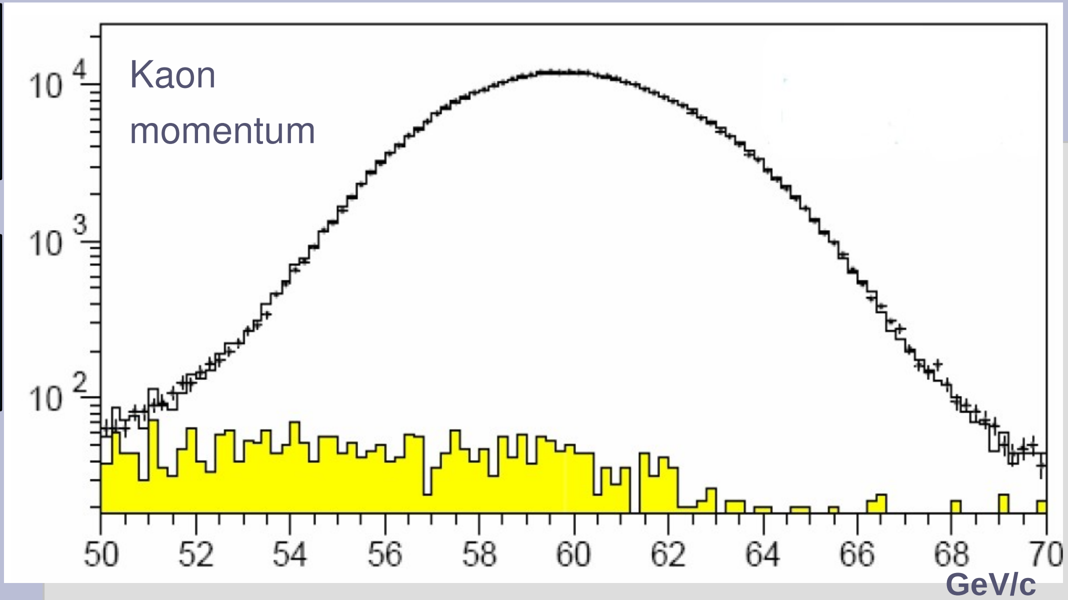
$F_p = f_p + f_p' q^2 + \ldots$

$q^2 = (M_{\pi\pi}^2/4m_\pi^2) - 1$

$G_p = g_p + g_p' q^2 + \ldots$

$H_p = h_p + h_p' q^2 + \ldots$

K_{e4}: selection & background



Selection:

- 3 tracks
- Missing energy and missing Pt
- LKr/DCH energy to electron PID

677500 decays

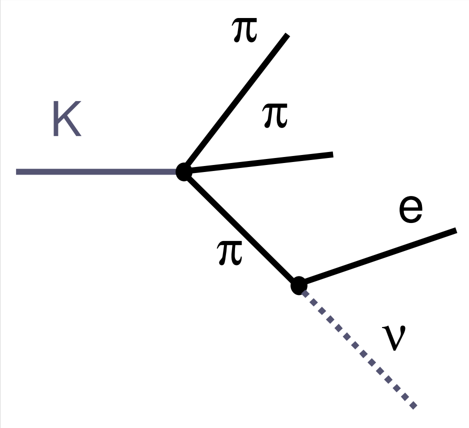
The background is studied using the electron *"wrong"* sign events (we assume $\Delta Q = \Delta S$ and total charge ±1) and cross check with MC. The total bkg is at level of 0.5%.

Main background sources:

$\pi\pi\pi + \pi \rightarrow e\nu$

$\pi\pi\pi$ with π misidentified

$\pi\pi^0\pi^0$ or $\pi^0 + \pi^0$(Dalitz) +e misidentified and γs outside the LKr



K_{e4}: Fitting procedure

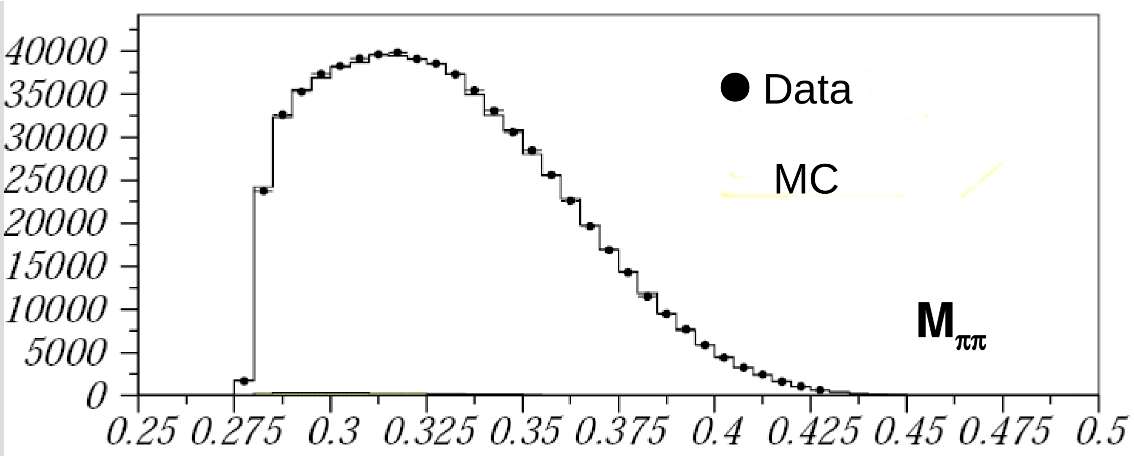
- The form factors (F,G,H and δ) are extracted minimizing a log-likehood estimator in each of **10(Mππ)x5(Meν)x5(cosθe)x5(cosθπ)x12(ϕ)=15000** equi-populated bins. In each bin the correlation between the 4+1 parameters is taken into account.

- The form factors structure is studied in 10 bins of Mππ, assuming constant form factors in each bins

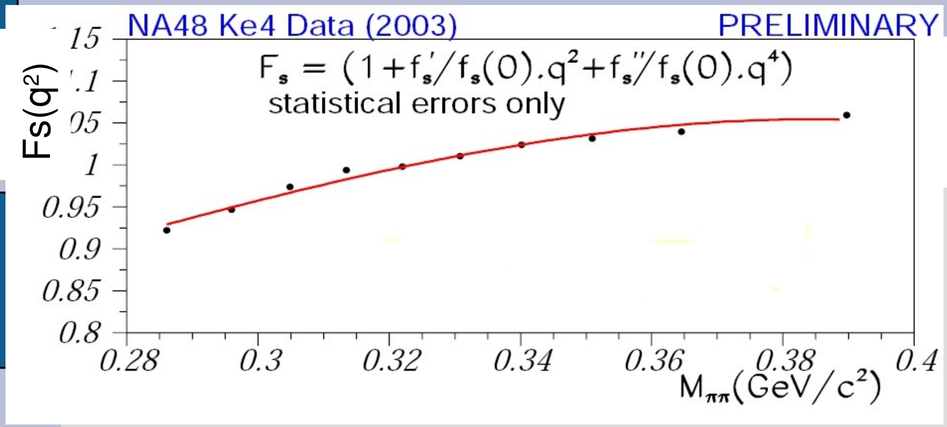
- A 2D fit (Mππ, Meν) is used to study the Fs expansion

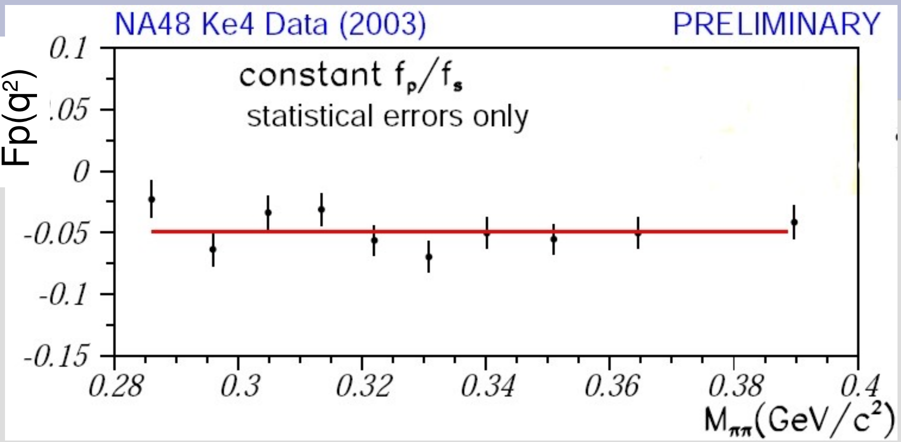
- All the results are given wrt to F_s(q=0) constant term, due to the unspecified overall normalization (BR is not measured)

		Data	MC
K+	evts	435654	10.0 M
	Evts/bin	29	667
K-	evts	241856	5.6 M
	Evts/bin	16	373

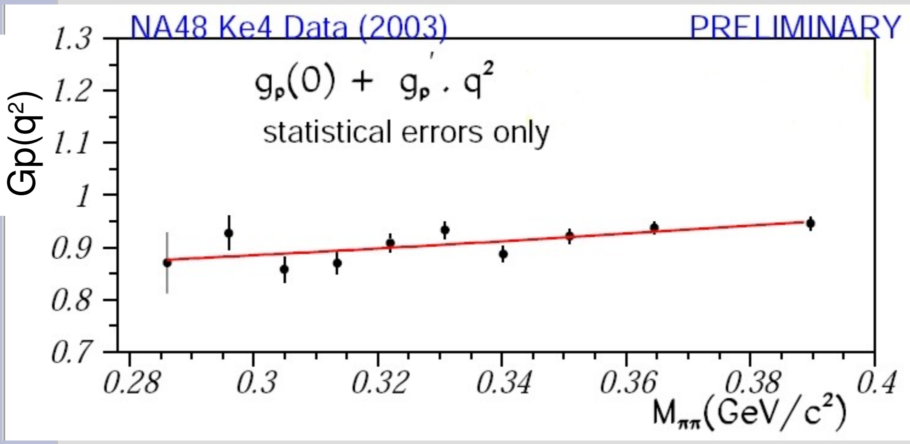


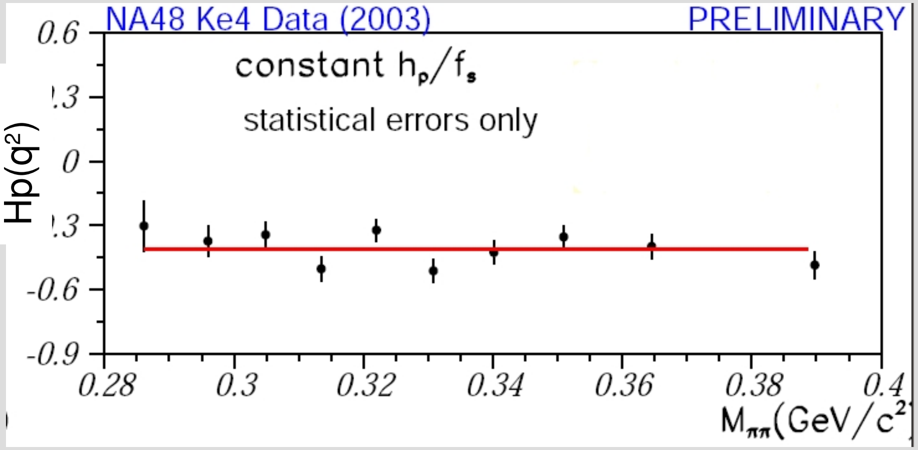
K_{e4}: Fitting results





- Fs is quadratic in q^2
- First measurement of Fp≠0





- Linear in q^2
- No linear term (h_p')

K_{e4}: form factors result

$f'_s/f_s = 0.165 \pm 0.011 \pm 0.006$

$f''_s/f_s = -0.092 \pm 0.011 \pm 0.007$

$f'_e/f_s = 0.081 \pm 0.011 \pm 0.008$

$f_p/f_s = -0.048 \pm 0.004 \pm 0.004$

$g_p/f_s = 0.873 \pm 0.013 \pm 0.012$

$g'_p/f_s = 0.081 \pm 0.022 \pm 0.014$

$h_p/f_s = -0.411 \pm 0.019 \pm 0.007$

Preliminay
(2003 data)

- Separately measured on K+ and K- and then combined (different statistical error)

Systematics checks:

- Acceptance
- Background
- PID
- Radiative corrections
- Evaluation of the sensitivity of the form factors on the Meν dependence of the normalization

- All the Form factors are measured relatively to fs
- first evidence of fp≠0 and fe'≠0
- The f.f. are measured at level of **<5%** of precision while the slopes at **~15%** (factor 2 or 3 improvement wrt previous measurements)

The Ke4 decay amplitude depends on two complex phases identified with

- δ_0 : the $\pi\pi$ scattering phase shift in the $I = 0$, $l = 0$ state (s – wave)
- δ_1 : the $\pi\pi$ scattering phase shift in the $I = 1$, $l = 1$ state (p – wave)

The Ke4 decay rate depends on the phase shift difference $\delta = \delta_0 - \delta_1$

δ is an increasing function of $M_{\pi\pi}$; $\delta \to 0$ for $M_{\pi\pi} \to 2m_+$

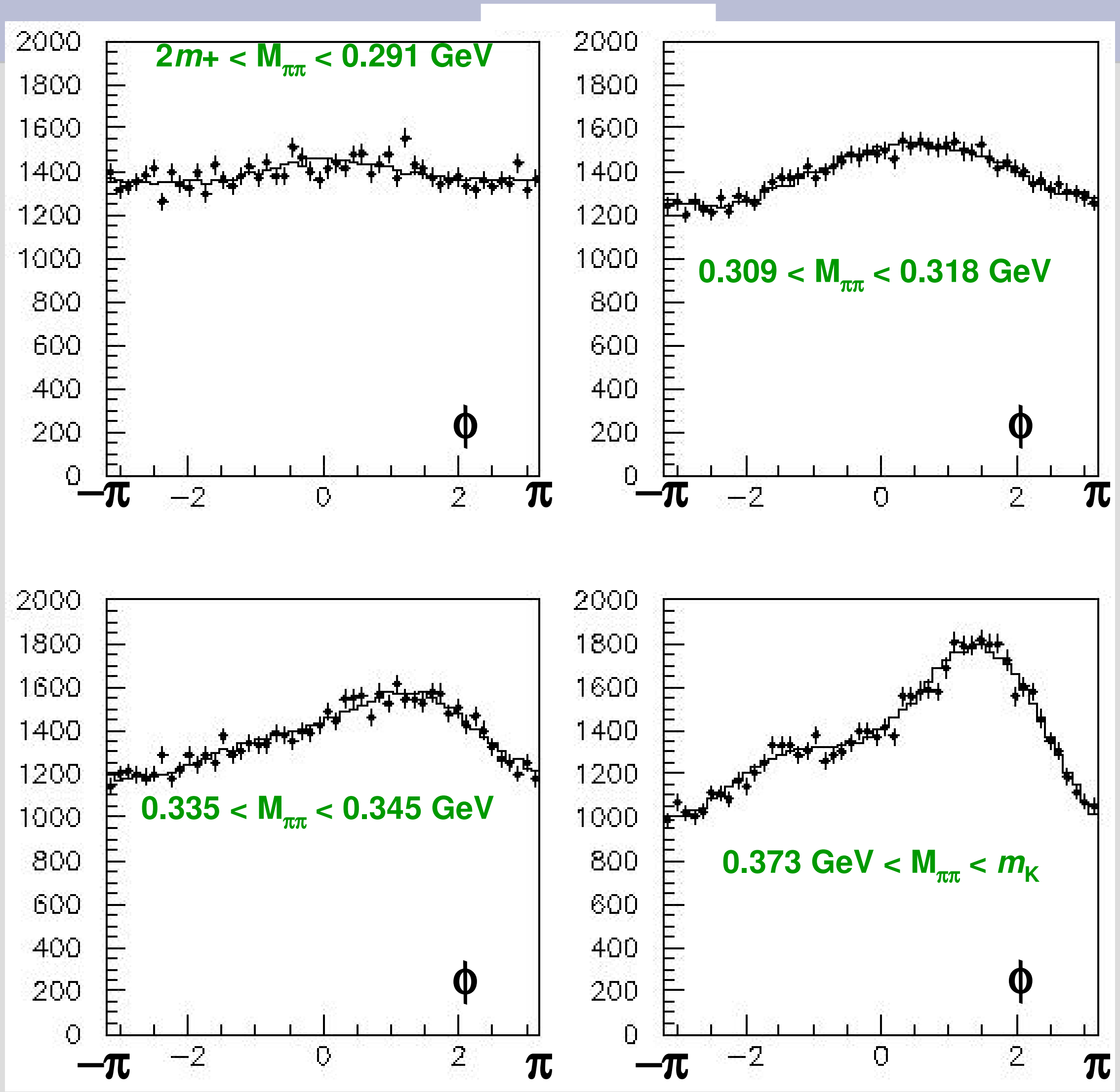
(from scattering theory : $\delta(k) \approx ak$ at very low centre-of-mass momentum k ;
a is the scattering length ; $a \neq 0$ for s – waves only)

$\delta \neq 0$ in the Ke4 decay amplitude ⟹ asymmetric distribution of the charged lepton direction with respect to the plane defined by the two pions (ϕ distribution)
(Shabalin 1963)

The asymmetry of the ϕ distribution increases with $M_{\pi\pi}$

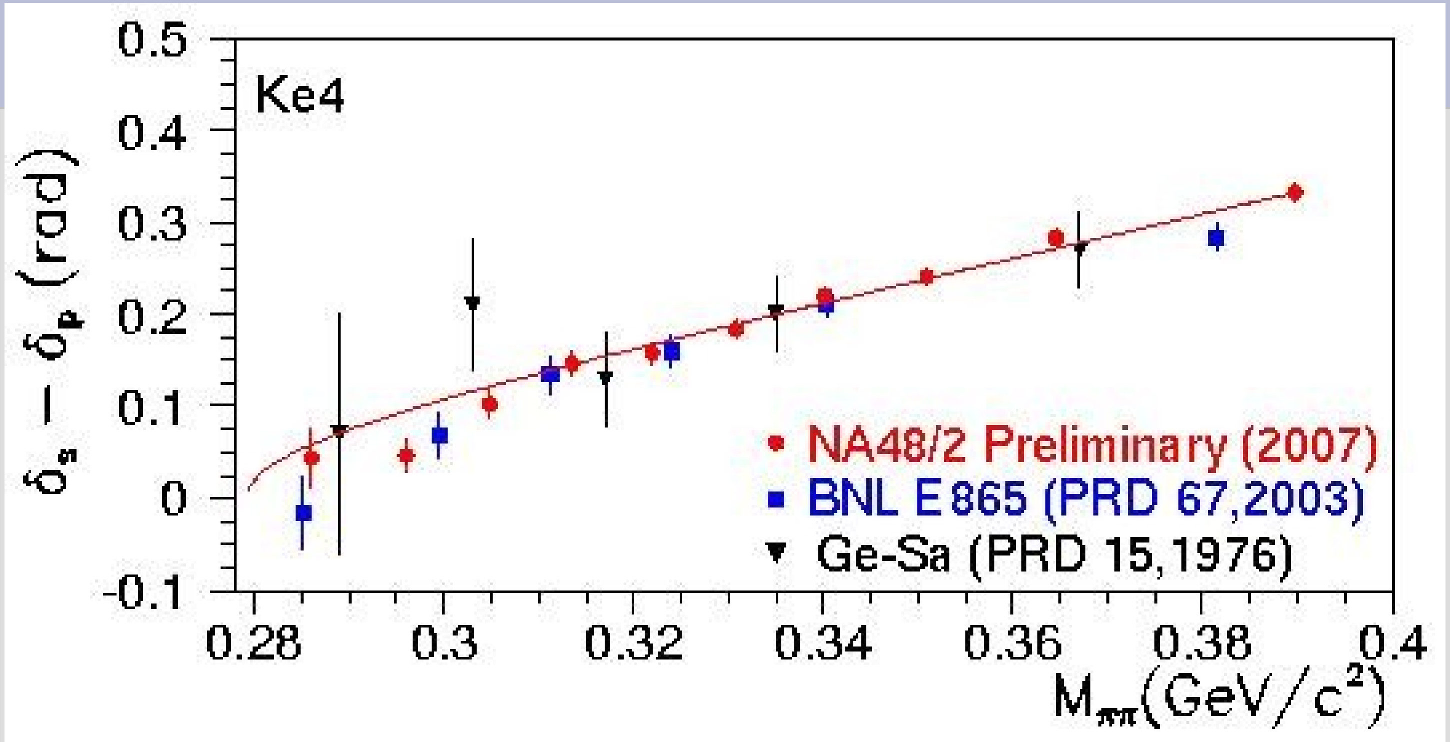
NA48 / 2 (Preliminary)

ϕ distributions for four M_{π} bins



δ versus $M_{\pi\pi}$

(no isospin – breaking corrections)



Geneva – Saclay : ~ 30,000 events , p_{K^+} = 2.8 GeV/c

BNL E865 : 406,103 events (with ~ 4.4% background), p_{K^+} = 6 GeV/c

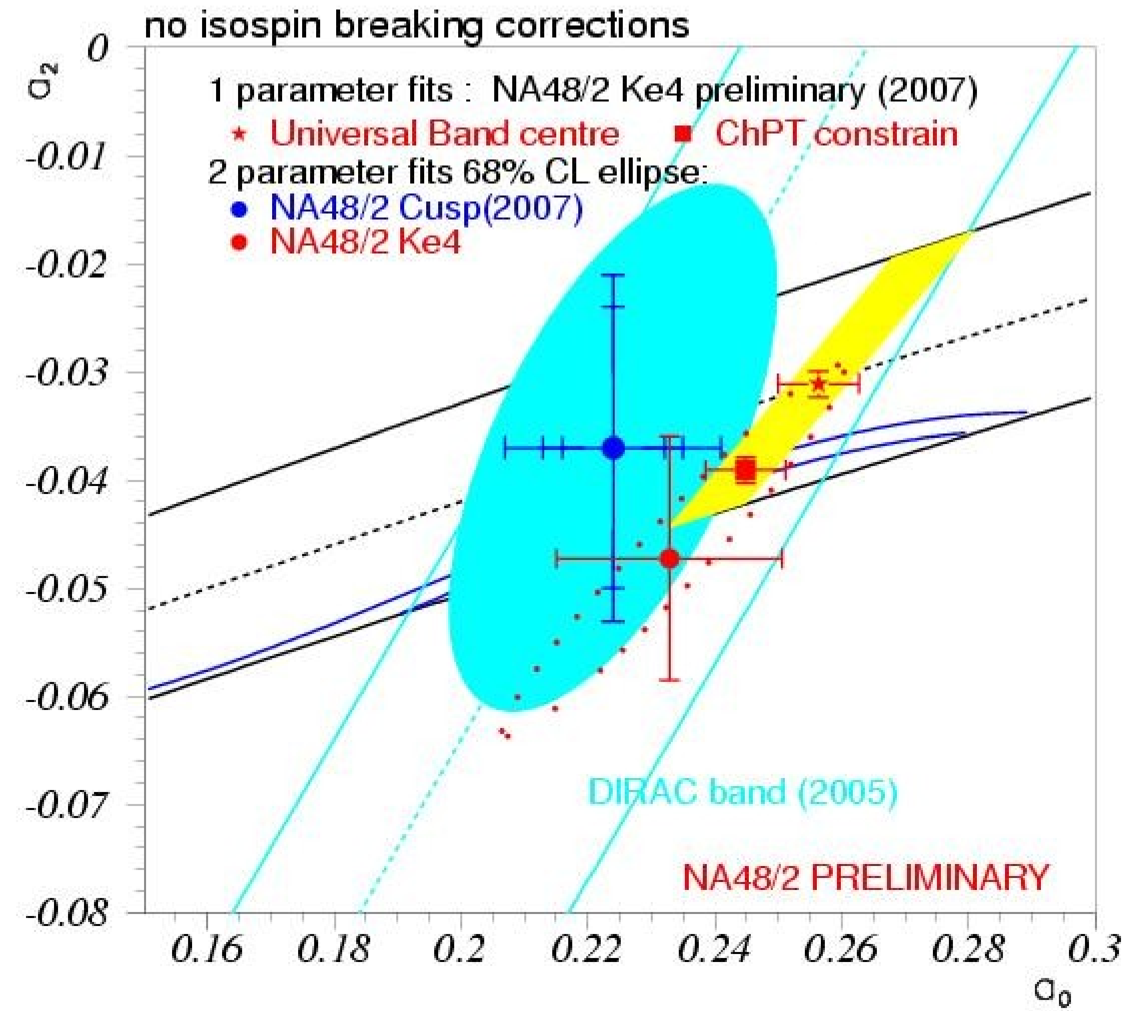
NA48/2 : 677,510 events (with ~0.5% background), $p_{K^\pm}$ = 60 GeV/c

NOTE: the isospin – breaking corrections reduce δ by 0.01 – 0.012 (J. Gasser)

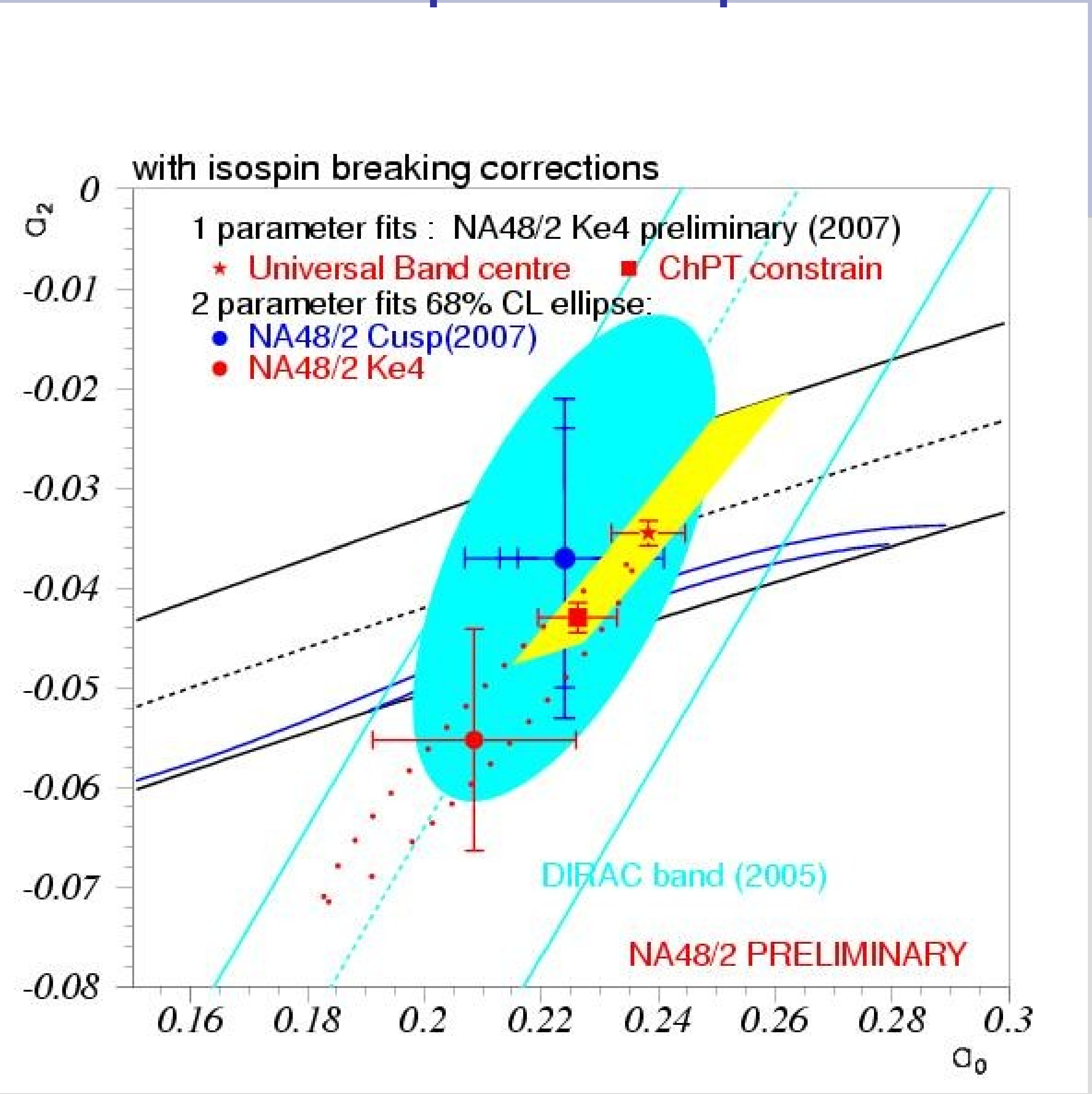
To extract scattering lengths from $\delta = \delta_0 - \delta_1$ some external data (I=2 $\pi\pi$ scatteringg at higher energies) and theoretical input are needed, for example:

- Solution of Roy equations - ACGL, Phys.Rep.353(2001), DFGS, EPJ C24 (2002) that relates δ and (a_0,a_2) .
- The Universal Band centre line parametrisation corresponds to 1-parameter fit with $a_2=f(a_0)$
- ChPT constrain gives another relation between (a_0,a_2) inside Universal Band.

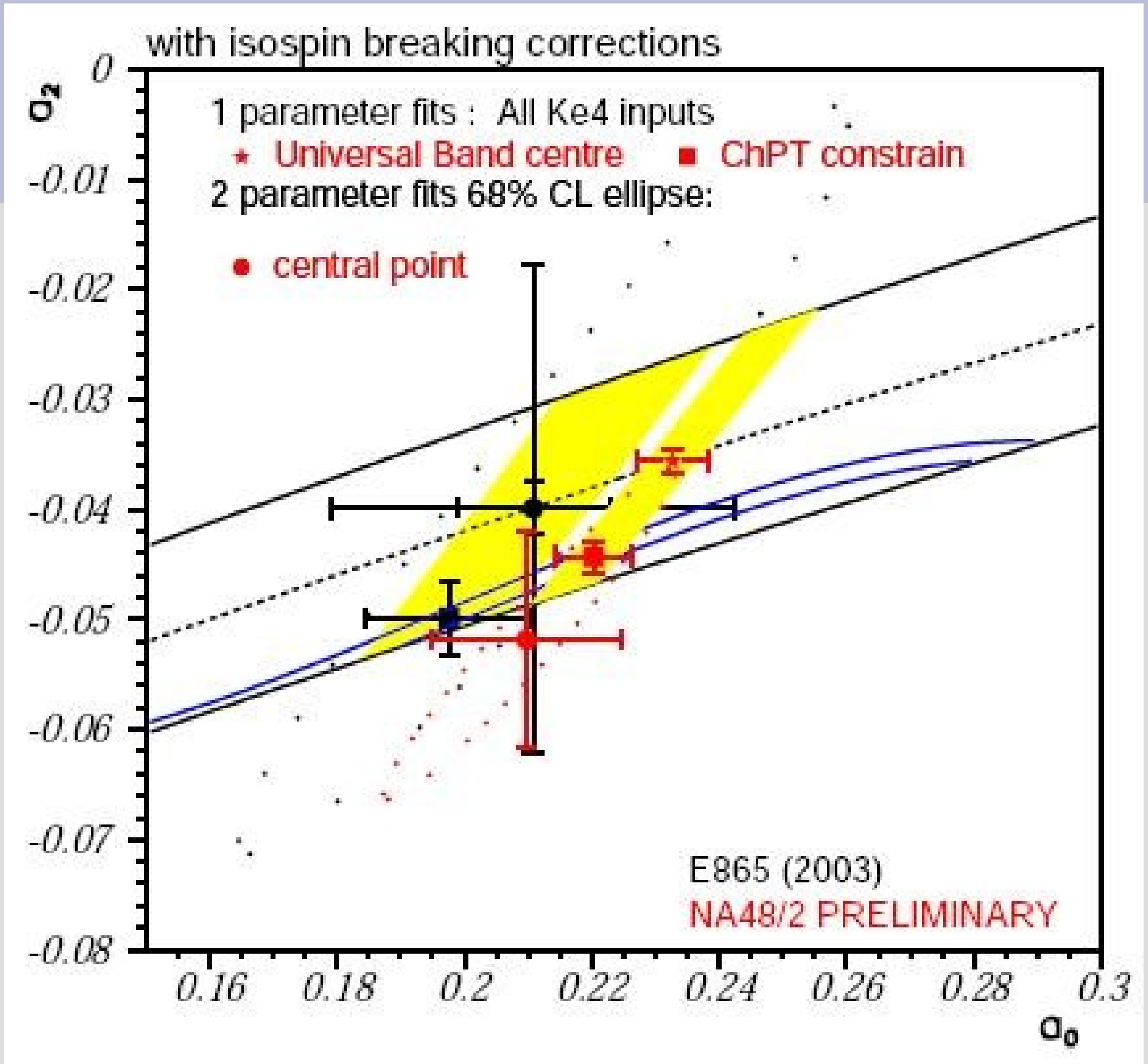
NA48/2 Cusp – Ke4 comparison



NA48/2 Cusp – Ke4 comparison



Ke4: NA48/2 – BNL E865 comparison



Marginal agreement for constrained fits

Conclusions

- A new step made in $(a_0 - a_2)$ measurement by analysis of $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$ decay with the Cabibbo-Isidori framework:
  - Data sample size increased by almost a factor of 4;
  - A significant improvement in experimental statistical and systematic precision on scattering lengths: in particular,

    $(a_0 - a_2) = (0.268 \pm 0.011_{exper.})$ [2003] → $(0.261 \pm 0.007_{exper.})$ [2003+2004]
  - However the largest uncertainty comes from the theory precision: $\delta(a_0 - a_2) = 0.013$ → a challenge to the theory!
- **Ke4**
  - Expect further improvement from NA48/2 by the addition of the 2004 data ($\Rightarrow$ total number of events > 10^6).
  - A new development: isospin – breaking corrections. Better agreement between NA48/2 Ke4 and cusp.
  - Agreement between NA48/2 and E865 is marginal and should be understood