

Evidence for *D^o-D^o* Mixing at BaBar

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for the BABAR collaboration

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<u>Outline</u>

- $D^0 \overline{D^0}$ oscillations
- Search for Mixing / CP violation using $D^0 \rightarrow K^ \pi^+$ decays
- Other searches for mixing / CPV:
 - Lifetime Ratios: $\tau(D^0 \rightarrow K^+ K^-, \pi^+ \pi^-)$ vs $\tau(D^0 \rightarrow K^- \pi^+)$
 - CPV in time-integrated $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ rates.
 - Mixing study using $D^0 \rightarrow K^+ \pi^- \pi^0$ decays
- Comparison with other results, theory





D^0 - $\overline{D}{}^0$ Oscillations



Neutral Meson systems

- Two-level system (M⁰,M⁰)
 - Weak interactions remove degeneracy, make them unstable

Time evolution by Schrödinger eq.:
$$i \frac{\partial}{\partial t} \begin{pmatrix} |M^{0}(t)\rangle \\ |\overline{M}^{0}(t)\rangle \end{pmatrix} = \begin{pmatrix} \mathsf{M} - \frac{i}{2} \Gamma \\ \mathsf{M} - \frac{i}{2} \Gamma \end{pmatrix} \begin{pmatrix} |M^{0}(t)\rangle \\ |\overline{M}^{0}(t)\rangle \end{pmatrix}$$

2x2 hermitian matrices Mesons decay

Mass eigenstates:

$$|M_{1,2}\rangle = p|M^0\rangle \pm q|\overline{M}^0\rangle$$

Propagate with separate mass $m_{1,2}$ and width $\Gamma_{1,2}$: $|M_{1,2}(t)\rangle = e^{-i(m_{1,2}-i\Gamma_{1,2}/2)t}|M_{1,2}(t=0)\rangle$



Neutral meson oscillations

Time evolution for meson of *known flavor at t=0*

m

$$x = \frac{m_2 - m_1}{\Gamma}$$
$$y = \frac{\Gamma_2 - \Gamma_1}{2\Gamma}$$
$$\Gamma = \frac{\Gamma_2 + \Gamma_1}{2}$$

$$|M^{0}(t)\rangle = e^{-\bar{\gamma}t/2} \left(\cosh(\Delta\gamma t/2) |M^{0}\rangle - \frac{q}{p} \sinh(\Delta\gamma t/2) |\overline{M}^{0}\rangle \right)$$

Where $\Delta\gamma = (y + ix)\Gamma$ $\bar{\gamma} = (\Gamma_{1} + \Gamma_{2})/2 - i(m_{1} + m_{2})$

M⁰ "oscillates" into M⁰! (also dubbed "mixing")

An opposite flavor component appears after a while!



Short and Long distance

• Predictions for x and y:

$$\begin{pmatrix} \mathbf{M} - \frac{i}{2} \mathbf{\Gamma} \end{pmatrix}_{ij} = \frac{\langle D_i | H_{\text{eff}} | D_j \rangle}{2m_D} = m_D^{(0)} \delta_{ij} \\ + \frac{\langle D_i | H_w | D_j \rangle}{2m_D} + \frac{1}{2m_D} \sum_n \frac{\langle D_i | H_w | n \rangle \langle n | H_w | D_j \rangle}{m_D^{(0)} - E_n + i\epsilon}.$$

$$\mathbf{y} \quad \Gamma_{ij} = \frac{1}{2m_D} \sum_n \langle D_i | H_w | n \rangle \langle n | H_w | D_j \rangle \, \delta(E_n - m_D).$$

$$\mathbf{Sum of intermediate}_{\mathbf{REAL states}}$$





SM prediction for charm mixing

SM charm mixing box has down-type quarks in loop



 $x, y \sim \sin \theta_c^2 \times [SU(3) \text{ breaking}]. \longrightarrow$ Naively $x, y \sim \sin \theta_c^2 \times \left(\frac{m_s}{\Lambda_{1-1}}\right)^2 \lesssim O(10^{-3})$

Always hard to evaluate SU(3) breaking !!!

(HQET, propagation of common hadronic states,...)

G. Burdman and I. Shipsey, Ann. Rev. Nucl. and Part. Sci. 53, 431 (2003).

SU(3) breaking effect more important for y

$$x \lesssim 10^{-3}, \quad y \lesssim 10^{-2}.$$



New Physics in Charm?



 Δ : Standard model predictions for x

□: Standard model predictions for y

- •: New physics predictions for x
 - Hard to see a clear prediction
 - Pushing the limit down excludes models



Standard Model mixing predictions

olina





Charm Mixing in $D^{O} \rightarrow K\pi$ Decay at BaBar

(Phys. Rev. Lett. 98:211802, 2007)



Principle of Mixing Measurement

- Produce clean sample of D^0 and \overline{D}^0
- *****Identify flavor (D^0 or \overline{D}^0 ?) at decay time
- Measure rate of mixed decays as function of time (Distributions shown without time smearing)



Time-Evolution of $D^0 \rightarrow K\pi$ Decays



Time evolution:

$$\frac{\Gamma_{WS}(t)}{e^{-t/\tau}} \propto R_D + \sqrt{R_D} y'\left(\frac{t}{\tau}\right) + \left(\frac{x'^2 + y'^2}{4}\right) \left(\frac{t}{\tau}\right)^2$$

where $x' = x \cos \delta + y \sin \delta$ $y' = y \cos \delta - x \sin \delta$

and δ is the phase difference between DCS and CF decays.





m(K π)- Δ m Fit Results



RS Decay Time Fit RS decay time, signal region

D⁰ lifetime and resolution function fitted in RS sample

 $\tau = (410.3 \pm 0.6 (stat.))$ fs

Consistent with PDG (410.1±1.5 fs)

Systematics dominated by resolution function



WS Fit with no Mixing



WS Fit with Mixing



Signal Significance with Systematics

Including systematics decreases signal significance



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Validation: Alternative Fit Strategy

Rate of WS events clearly increases with time:





Allowing for CP Violation

CP violation could introduce different time dependence for D^0 (+) and D^0 (-):

$$\frac{T_{\rm WS}^{\pm}(t)}{e^{-\Gamma t}} = \sqrt{\frac{1 \pm A_{\rm D}}{1 \mp A_{\rm D}}} R_{\rm D} + \sqrt{R_{\rm D}} \sqrt[4]{\frac{(1 \pm A_{\rm D})(1 \pm A_{\rm M})}{(1 \mp A_{\rm D})(1 \mp A_{\rm M})}} (y' \cos \varphi \mp x' \sin \varphi) \Gamma t + \sqrt{\frac{1 \pm A_{\rm M}}{1 \mp A_{\rm M}}} \frac{{x'}^2 + {y'}^2}{4} (\Gamma t)^2$$

Three possible types of CP violation:

- Direct CP violation in DCS decay
- ♦ CP violation in mixing
- CP violation in interference between mixing and decay

Simpler to fit D^0 (+) and D^0 (-) separately:

$$\frac{\Gamma_{WS}^{\pm}(t)}{e^{-t/\tau}} \propto R_D^{\pm} + \sqrt{R_D^{\pm}} y'^{\pm} \left(\frac{t}{\tau}\right) + \left(\frac{x'^{\pm 2} + y'^{\pm 2}}{4}\right) \left(\frac{t}{\tau}\right)^2$$

CP violation if one or more "±" parameters are different



CPV Allowed Contours

<u>Results of fitting D^0 and \overline{D}^0 separately:</u>

 y'^+ : (9.8±6.4±4.5)x10⁻³

 x'^{+2} : (-0.24±0.43±0.30)x10⁻³ x'^{-2} : (-0.20±0.41±0.29)x10⁻³ y'-: (9.6±6.1±4.3)x10-3

 $A_{D} = (-2.1 \pm 5.2 \pm 1.5)\%$



No evidence for CP violation found





Other searches for D^{0} mixing and for CP violation in D^{0} decays



$D^0 - D^0$ Mixing in Lifetime Ratio of $D^0 \rightarrow K^+ K^-$, $\pi^+ \pi^- \text{vs } D^0 \rightarrow K^- \pi^+$

 $D^{0} \rightarrow K^{-} \pi^{+}: \text{CP-mixed} \quad D^{0}(t) \rightarrow K^{+} K^{-}, \ \pi^{+} \pi^{-}: \text{CP-even}$ $Determine \text{ the quantities} \qquad h = K \text{ or } \pi$ $\langle \tau_{hh} \rangle = (\tau_{hh}^{+} + \tau_{hh}^{-})/2$ $A_{\tau} = (\tau_{hh}^{+} - \tau_{hh}^{-})/(\tau_{hh}^{+} + \tau_{hh}^{-})$ $x \equiv 2 \frac{m_{1} - m_{2}}{\Gamma_{1} + \Gamma_{2}} \qquad y \equiv \frac{\Gamma_{1} - \Gamma_{2}}{\Gamma_{1} + \Gamma_{2}}$ $CPV \text{ in interference} \text{ of mixing and decay:} \qquad \varphi_{f} \equiv \arg\left(\frac{q}{p} \frac{\langle f | \mathcal{H}_{D} | \overline{D}^{0} \rangle}{\langle f | \mathcal{H}_{D} | D^{0} \rangle}\right) \neq 0$

If CP is conserved $y_{CP} = y, \Delta Y = 0$

y_{CP}	=	$y\cos arphi_f$
ΔY	=	$x\sin arphi_f$



Decay time fits to determine $(y_{CP}, \Delta Y)$



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BaBar (y_{CP} , ΔY) results

Tagged results from 384 fb⁻¹:



 $y_{CP} = (1.31 \pm 0.32 \pm 0.25)\%$

 $A_{\Gamma} = (0.01 \pm 0.30 \pm 0.15)\%$

M. Staric et al. (Belle Collab.), Phys. Rev. Lett. 98, 211803 (2007).



Search for direct CPV in time-integrated $D^0 \rightarrow K^+ K^-$, $\pi^+ \pi^-$ rates $A_{CP} = \frac{\Gamma(f) - \Gamma(\bar{f})}{\Gamma(f) + \Gamma(\bar{f})} = \frac{2 \operatorname{Im} A_1 A_2^* \sin(\delta_1 - \delta_2)}{|A_1|^2 + |A_2|^2 + 2 \operatorname{Re} A_1 A_2^* \cos(\delta_1 - \delta_2)}$ 2 weak amplitudes with phase difference strong phase difference

Two amplitudes with different strong & weak phases needed to observe CPV (in SM from tree and penguins)

Standard model predictions for direct CPV asymmetries in these modes: O(0.001% - 0.01%)

F. Bucella et al., Phys. Rev. **D51**, 3478 (1995).S. Bianco et al., Riv. Nuovo Cim. 26N7, 1(2003).

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e.g., $D^0 \rightarrow K^+K^-$:



Search for CPV in $D^0 \rightarrow K^+ K^-$, $\pi^+ \pi^-$



$$\begin{array}{lll} a_{CP}^{KK} = & (& 0.00 \pm 0.34 \; (\mathrm{stat.}) \pm 0.13 \; (\mathrm{syst.}))\% \\ a_{CP}^{\pi\pi} = & (-0.24 \pm 0.52 \; (\mathrm{stat.}) \pm 0.22 \; (\mathrm{syst.}))\% \end{array}$$

No evidence for CPV in either mode



Mixing in $D^0 \rightarrow K^+ \pi^- \pi^0$

Two types of WS Decays:

- Doubly Cabbibo-supressed (DCS)
- Mixing followed by Cabibbo-Favored (CF) decay $D^0 \rightarrow \overline{D}^0 \rightarrow K^+ \pi^- \pi^0$

 $D^{0} \to K^{+} \pi^{-} \pi^{0}$ $D^{0} \xrightarrow{}_{\text{mix}} \overline{D}^{0} \to K^{+} \pi^{-} \pi^{0}$

Two ways to reach same final state \Rightarrow interference!

 $\begin{array}{rcl} \text{Time dependent WS rate :} \\ & \Gamma_{\bar{f}}(s_{12}, s_{13}, t) &= e^{-\Gamma t} \{ |A_{\bar{f}}|^2 \leftarrow \text{DCS} \\ & \text{Interference} \rightarrow + & |A_{\bar{f}}| |\bar{A}_{\bar{f}}| \left[y'' \cos \delta_{\bar{f}} - x'' \sin \delta_{\bar{f}} \right] (\Gamma t) \\ & \text{Mixing} \rightarrow + & \frac{x''^2 + y''^2}{4} |\bar{A}_{\bar{f}}|^2 (\Gamma t)^2 \} \\ & \bar{f} = K^+ \pi^- \pi^0 \qquad A_{\bar{f}} = \langle \bar{f} |\mathcal{H}| D^0 \rangle, \ \bar{A}_{\bar{f}} = \langle \bar{f} |\mathcal{H}| \overline{D}^0 \rangle \end{array}$

 $egin{aligned} y'' &= y\cos\delta_{K\pi\pi^0} - x\sin\delta_{K\pi\pi^0} \ x'' &= x\cos\delta_{K\pi\pi^0} + y\sin\delta_{K\pi\pi^0} \end{aligned}$

 $\delta_{K\pi\pi^0}$: strong phase difference between CF and DCS decay amplitudes



$D^0 \rightarrow K^- \pi^+ \pi^0 \text{RS}$ Dalitz fit

Time-integrated analysis to determine CF amplitudes, $\bar{A}_{\bar{f}}$



$D^{0}(t) \rightarrow K^{+} \pi^{-} \pi^{0}$ WS Dalitz fit results

Through t-dependence, distinguish DCS amplitudes from the CF amplitudes arising from mixing.



Results are consistent with no mixing at 0.8%, including systematics

BaBar D^0 - \overline{D}^0 Mixing Summary

From $K^{\pm}\pi^{\mp}$ decays:

x'²: (-0.22±0.30±0.21) x 10⁻³, y': (9.7±4.4±3.1) x 10⁻³ Further evidence for $D^{0}-\overline{D}^{0}$ mixing from the *BaBar* experiment:

- $D^0 \rightarrow K^- \pi^+$ to $D^0 \rightarrow K^+ K^-$, $\pi^+ \pi^-$ lifetimes:

 $y_{CP} = (1.24 \pm 0.39 \text{ (stat.)} \pm 0.13 \text{ (syst.)})\%$

 $- D^{0} \rightarrow K^{+} \pi^{-} \pi^{0} \text{ time-dependent Dalitz analysis:}$ $x'' = (2.39 \pm 0.61 \text{ (stat.)} \pm 0.32 \text{ (syst.)})\%$ $y'' = (-0.14 \pm 0.60 \text{ (stat.)} \pm 0.40 \text{ (syst.)})\%$

In $D^0 \rightarrow K^+ K^-$, $\pi^+ \pi^-$ decays,

no evidence for direct CP violation

 $a_{CP}^{KK} = (0.00 \pm 0.34 \text{ (stat.)} \pm 0.13 \text{ (syst.)})\%$ $a_{CP}^{\pi\pi} = (-0.24 \pm 0.52 \text{ (stat.)} \pm 0.22 \text{ (syst.)})\%$

- no evidence for CP violation in mixing: $\Delta Y = (-0.26 \pm 0.36 \text{ (stat.)} \pm 0.08 \text{ (syst.)})\%$

Combining with other results, a comparison with Theory, and Conclusions

HFAG Results assuming no CPV

(Visit http://www.slac.stanford.edu/xorg/hfag/charm/index.html)

HFAG results allowing for CPV

(Visit <u>http://www.slac.stanford.edu/xorg/hfag/charm/index.html</u>)

Implications of Charm Mixing

BaBar and Belle mixing results first presented at Moriond electroweak conference on March 17

Several new hep-ph preprints on charm mixing since then, e.g.,

Five use D⁰ mixing results to evaluate limits on:
 ◆Certain SUSY models (flavor suppresion by "alignment") hep-ph/0703204 hep-ph/0703254, arXiv:0704.0601
 ◆Non-universal Z' model
 hep-ph/0703270

"Models are further constrained, "Light non-degenerate but constraints are limited by lack of precise SM value" "Light non-degenerate squarks unlikely to be observed at LHC"

Currently, only an observation of CP violation in mixing would be a clear sign of New Physics

BaBar studied $D^0 \rightarrow K\pi$ and other D^0 decays for mixing, CPV Evidence for mixing in $K\pi$ decays (3.9 σ)

 \bullet Evidence for mixing in lifetime differences (3.0 σ)

✤No sign of CP violation at the ~½% level

Consistent with other measurements and SM

More BaBar data and analyses coming up

Backup Slides

PEP-II, a B-Factory (and Charm)

High-luminosity asymmetric energy e^+e^- collider at Υ (4S) resonance

B-Factory built for study of CP-violation and other CKMphysics in *B* meson decays

~10 Hz of $B\overline{B}$

The BaBar Experiment

BaBar is a large acceptance experiment with excellent particle reconstruction and identification capability

Signal Significance

Best fit is in unphysical region (x²<0)

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Systematic Uncertainties

Two types of systematic uncertainties considered:

Fit model variations:

Change signal and background models used in fit, to test assumptions made

Selection criteria:

✤Mainly decay time (error) ranges used in fit

Interpreting the results

And CP violation?

In the standard model, $\phi \sim 2\,A^2\lambda^4\eta \lesssim 10^{-3}$

Ciuchini et al. hep-ph/0703294

In general NP weakly constrained if SM not known Nevertheless SUSY coupling can be constrained hints on squark and gluino masses!

Neutral meson mixing always a window into unknown (virtual) states!

Double tag at v(3770) [CLEO-c]

- Reconstruct Double Tags: CP vs Kπ
- Asymmetry in CP+ vs CP- related to cosδ

D_{CP±} neutral D CP eigenstate

$$A = \frac{B(D_{CP+} \rightarrow K^{-}\pi^{+}) - B(D_{CP-} \rightarrow K^{-}\pi^{+})}{B(D_{CP+} \rightarrow K^{-}\pi^{+}) + B(D_{CP-} \rightarrow K^{-}\pi^{+})}$$

R_D is ratio of DCS to Cabibbo favored rates

$$\cos \delta = \frac{A}{2\sqrt{R_D}}$$

• Input $R_D = (3.60 \pm 0.08)\%$ from PDG2006+CDF ~±2%,

 $\psi(3770)$ decay conserves CP

Need to run On threshold

- Updated results with 281 pb⁻¹ at Winter Conferences
 - Expect $\sigma(y) \sim \pm 1.5\%$ and $\sigma(\cos \delta_{K\pi}) \sim \pm 0.3$
 - Including systematic uncertainties
- Full CLEO-c dataset ~750 pb⁻¹
 - Expect $\sigma(y)$ ~ ±1.0% and $\sigma(\cos \delta_{K\pi})$ ~ ±0.1-0.2

BaBar (y_{CP} , ΔY) systematics

Systematic uncertainties (%):

Systematic	Δy_{CP}^{KK}	$\Delta y_{CP}^{\pi\pi}$	Δy_{CP}	$\Delta(\Delta Y^{KK})$	$\Delta(\Delta Y^{\pi\pi})$	$\Delta(\Delta Y)$
Signal Model	0.130	0.059	0.085	0.072	0.265	0.062
Charm Bkgd	0.062	0.037	0.043	0.001	0.002	0.001
Combinatorial Bkgd	0.019	0.142	0.045	0.001	0.005	0.002
Selection	0.068	0.178	0.046	0.083	0.172	0.011
Detector Model	0.064	0.080	0.064	0.054	0.040	0.054
Quadrature sum	0.172	0.251	0.132	0.122	0.318	0.083

Variations:

- Signal: PDF shape, polar angle dependent resolution offset, signal interval
- Charm backgrounds: yields and charm lifetime
- Combinatorial backgrounds: yields, shape and sideband region
- Selection: σ_t criterion, treatment of multiple candidates
- Detector: Alignment and energy loss

Search for CPV in $D^0 \rightarrow K^+ K^-$, $\pi^+ \pi^-$

Measure the time integrated CP asymmetries

$$a_{CP}^{KK} = rac{\Gamma(D^0
ightarrow K^- K^+) - \Gamma(\overline{D}{}^0
ightarrow K^+ K^-)}{\Gamma(D^0
ightarrow K^- K^+) + \Gamma(\overline{D}{}^0
ightarrow K^+ K^-)} \ a_{CP}^{\pi\pi} = rac{\Gamma(D^0
ightarrow \pi^- \pi^+) - \Gamma(\overline{D}{}^0
ightarrow \pi^+ \pi^-)}{\Gamma(D^0
ightarrow \pi^- \pi^+) + \Gamma(\overline{D}{}^0
ightarrow \pi^+ \pi^-)}.$$

Experimental procedure:

- fit $m, \Delta m$ distributions to determine raw signal weights
- Determine relative D^0/\overline{D}^0 soft pion tagging efficiency using $\overline{D}^0 \rightarrow K\pi$ data

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\Rightarrow greatly reduces	systematic	uncertainties	2 Dia
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	Category	$\Delta a_{CP}^{\kappa\kappa}$	$\Delta a_{CP}^{\pi\pi}$
2	-Dim. PDF shapes	$\pm 0.04\%$	$\pm 0.05\%$
	π_s correction	$\pm 0.08\%$	$\pm 0.08\%$
	a_{CP} extraction	$\pm 0.09\%$	$\pm 0.20\%$
	Quadrature sum	$\pm 0.13\%$	$\pm 0.22\%$

RS and WS ($m_{K\pi\pi}$, Δm) fits

Model	Approximate Constraint	
Fourth Generation (Fig. 2)	$ V_{ub'}V_{cb'} \cdot m_{b'} < 0.5 \; (GeV)$	
Q = -1/3 Singlet Quark (Fig. 4)	$s_2 \cdot m_S < 0.27 \; (\text{GeV})$	
Q = +2/3 Singlet Quark (Fig. 6)	$ \lambda_{uc} < 2.4 \cdot 10^{-4}$	
Little Higgs	Tree: See entry for $Q = -1/3$ Singlet Quark	
	Box: Region of parameter space can reach observed $x_{\rm D}$	
Generic Z' (Fig. 7)	$M_{Z^\prime}/C>2.2\cdot 10^3~{ m TeV}$	
Family Symmetries (Fig. 8)	$m_1/f > 1.2 \cdot 10^3 \text{ TeV}$	
	(with $m_1/m_2 = 0.5$)	
Left-Right Symmetric (Fig. 9)	No constraint	
Alternate Left-Right Symmetric (Fig. 10)	$M_R > 1.2 { m TeV} (m_{D_1} = 0.5 { m TeV})$	
	$(\Delta m/m_{D_1})/M_R > 0.4 \text{ TeV}^{-1}$	
Vector Leptoquark Bosons (Fig. 11)	$M_{VLQ} > 55(\lambda_{PP}/0.1) \text{ TeV}$	
Flavor Conserving Two-Higgs-Doublet (Fig. 13)	No constraint	
Flavor Changing Neutral Higgs (Fig. 15)	$m_H/C > 2.4 \cdot 10^3 \text{ TeV}$	
FC Neutral Higgs (Cheng-Sher ansatz) (Fig. 16)	$m_H/ \Delta_{uc} > 600 \text{ GeV}$	
Scalar Leptoquark Bosons	See entry for RPV SUSY	
Higgsless (Fig. 17)	$M > 100 { m ~TeV}$	
Universal Extra Dimensions	No constraint	
Split Fermion (Fig. 19)	$M/ \Delta y > (6\cdot 10^2 { m ~GeV})$	
Warped Geometries (Fig. 21)	$M_1 > 3.5 \text{ TeV}$	
Minimal Supersymmetric Standard (Fig. 23)	$ (\delta^u_{12})_{\rm LR,RL} < 3.5 \cdot 10^{-2} \mbox{ for } \tilde{m} \sim 1 \mbox{ TeV}$	
	$ (\delta^u_{12})_{\rm LL,RR} < .25$ for $\tilde{m} \sim 1~{\rm TeV}$	
Supersymmetric Alignment	$\tilde{m} > 2$ TeV	
Supersymmetry with RPV (Fig. 27)	$\lambda_{12k}'\lambda_{11k}'/m_{\tilde{d}_{R,k}} < 1.8\cdot 10^{-3}/100~{\rm GeV}$	
Split Supersymmetry 75	No constraint	

Table from Golowich, Hewett, Pakvasa and Petrov: arXiv:0705.3650 [hep-ph]

"... for some models (Split Fermions, Flavor Changing Neutral Higgs) the constraints can be strong."

"Such a list is by nature approximate, and we refer the reader to the body of the paper for a more precise presentation of our results."

