Bilinear R-parity Violation in Rare Meson Decays

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$$M^+ \to M'^- \ell^+ \ell'^+$$

$$\Delta L = \pm 2$$

SM $\Rightarrow$ $M^+ \not\to M'^- \ell^+ \ell'^+$

Beyond the SM:
- Massive Majorana neutrinos
- *R*-parity-violating SUSY

The Bethe-Salpeter Formalism

Mesons as bound states of a quark and an antiquark:

$$\chi_p(x_1,x_2) = -\frac{i}{\sqrt{N_C}}\left\langle 0\right| T\left\{q_1^a(x_1)\bar{q}_{2a}(x_2)\right\}\left|M(p)\right\rangle$$

$a=1,2,3$ - color index

$N_C=3$ -the number of colors

$$\chi_p(q) = \int d^4x e^{iq\cdot x}\chi_p(x) = \gamma^5(1-\delta_M\hat{p})\varphi_p(q)$$

$\hat{p}=\gamma^\mu p_\mu$

m_M - the mass of the meson *M*

$\delta_M = (m_1+m_2)/m_M^2$

m_1, m_2 - q_1, q_2 quark masses

$q=(p_1-p_2)/2$ - the relative 4-momentum of the meson *M*

$p=p_1+p_2$ - the total 4-momentum of the meson *M*

$\varphi_p(q)$ - model dependent scalar function

The definition of the decay constant of the meson:

$$if_M p^\mu = \left\langle 0 \left| \bar{q}_{2a}(0)\gamma^\mu\gamma^5 q_1^a(0) \right| M(p) \right\rangle = -i\sqrt{N_C}\, Tr\{\gamma^\mu\gamma^5\chi_p(x=0)\}$$

$$f_M = 4\sqrt{N_C}\,\delta_M \int \frac{d^4q}{(2\pi)^4}\varphi_p(q)$$

Rare Meson Decays in R-parity-violating SUSY Theories

R-parity is a discrete, multiplicative symmetry defined as

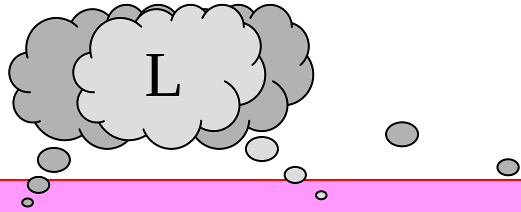
$R_p = (-1)^{3B+L+2S}$ S, B, L – spin, barion and lepton quantum numbers

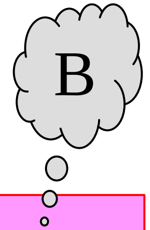
The SM fields, including additional Higgs boson fields appearing in the extended gauge models, have $R_p = +1$ while their superpartners have $R_p = -1$. This symmetry has been imposed on the minimal supersymmetric SM to ensure B and L number conservation. However, SUSY doesn't require R_p conservation.

MSSM → R_p →
- L and B number conservation
- the lightest superparticle stability

The most general gauge invariant form of the superpotential in minimal SUSY SM is

$$W = W_{R_p} + W_{RPV}$$





$$W_{RPV} = \frac{1}{2}\lambda_{ijk} L_i L_j \overline{E}_k + \lambda'_{ijk} L_i Q_j \overline{D}_k + \varepsilon_i L_i H_u + \lambda''_{ijk} \overline{U}_i \overline{Q}_j \overline{D}_k$$

i, j, k - generation indices; L, Q are lepton and quark doublet superfields

and $\overline{E}, \overline{U}, \overline{D}$ are lepton and up, down quark singlet superfields.

All λ are the coupling constants.

The trilinear part of lepton number violating Lagrangian:

$$\Im_{\lambda} = \frac{1}{2}\lambda_{ijk}\left[\tilde{\nu}_L^i \bar{\ell}_R^k \ell_L^j + \tilde{\ell}_L^j \bar{\ell}_R^k \nu_L^i + (\tilde{\ell}_R^k)^* (\bar{\nu}_L^i)^c \ell_L^j - (i \leftrightarrow j)\right] + h.c.,$$

$$\Im_{\lambda'} = \lambda'_{ijk}\left[\tilde{\nu}_L^i \bar{d}_R^k d_L^j + \tilde{d}_L^j \bar{d}_R^k \nu_L^i + (\tilde{d}_R^k)^* (\bar{\nu}_L^i)^c d_L^j - \tilde{\ell}_L^i \bar{d}_R^k u_L^j - \tilde{u}_L^j \bar{d}_R^k \ell_L^i - (\tilde{d}_R^k)^* (\bar{\ell}_L^i)^c u_L^j\right] + h.c.$$

The bilinear terms of the *R*-parity breaking superpotential induce mixing between the SM leptons and the MSSM chargino and neutralinos in the mass-eigenstate basis:

$$\Im_{LH} = -\frac{g}{\sqrt{2}} k_n W_\mu^- \bar{\ell} \gamma^\mu P_L \tilde{\chi}_n^0 + \sqrt{2} g (\beta_k^d \bar{\upsilon}_k P_R d \tilde{d}_R^* + \beta_k^u \bar{\upsilon}_k P_R u^c \tilde{u}_L +$$
$$+ \beta_{ki}^\ell \bar{\upsilon}_k P_R \ell^c \tilde{\ell}_{Li} + \beta^c \bar{u} P_R \ell^c \tilde{d}_L) + h.c.$$

The Lagrangian terms corresponding to gluino $\tilde{g}$ and neutralino $\tilde{\chi}^0$ interactions with fermions $\psi^i = \{u^i, d^i, \ell^i\}$, $q^i = \{u^i, d^i\}$ and their superpartners $\tilde{\psi}^i = \{\tilde{u}^i, \tilde{d}^i, \tilde{\ell}^i\}$, $\tilde{q}^i = \{\tilde{u}^i, \tilde{d}^i\}$:

$$\Im_{\tilde{g}} = -\sqrt{2} g_3 \sum_{a,b,i=1}^{3} \sum_{\alpha=1}^{8} \frac{\lambda_{ab}^{(\alpha)}}{2} (\overline{q}_{L\,i}^{a} \tilde{g}^{(\alpha)} \tilde{q}_{L\,i}^{b} - \overline{q}_{R\,i}^{a} \tilde{g}^{(\alpha)} \tilde{q}_{R\,i}^{b}) + h.c.,$$

Here $\lambda^{(\alpha)}$ are 3x3 Gell-Mann matrices.

$$\Im_{\chi} = -\sqrt{2} g_2 \sum_{\sigma=1}^{4} \sum_{i=1}^{3} (\varepsilon_{L\sigma}(\psi) \overline{\psi}_L^i \chi_\sigma \tilde{\psi}_L^i + \varepsilon_{R\sigma}(\psi) \overline{\psi}_R^i \chi_\sigma \tilde{\psi}_R^i) + h.c.$$

$$\varepsilon_{L\sigma}(\psi) = -T_3(\psi) N_{\sigma 2} + \tan\theta_W (T_3(\psi) - Q(\psi)) N_{\sigma 1},$$

$$\varepsilon_{R\sigma}(\psi) = Q(\psi) \tan\theta_W N_{\sigma 1},$$

Here $Q(\psi)$ and $T_3(\psi)$ are the electric charge and weak isospin of the field ψ,

$N_{\delta\sigma}$ - the 4x4 neutralino mixing matrix.

Bilinear R-parity breaking

$$\Im_{bil} = \Im_{SM} + \Im_{LH} + \Im_{\tilde{\chi}} + \Im_{\tilde{g}}$$

$$\Im^{bil}_{eff} = -\frac{g^4}{4m_W^4}(\bar{\ell}^c \gamma_\mu \gamma_\nu P_L \ell')\sum_{\delta=1}^{4}\frac{(k^*_\delta)^2}{m_{\tilde{\chi}_\delta}}[V^*_{12}V^*_{34}(\bar{q}^a_3 \gamma_\mu P_L q_{4a})(\bar{q}^b_2 \gamma_\nu P_L q_{1b}) +$$

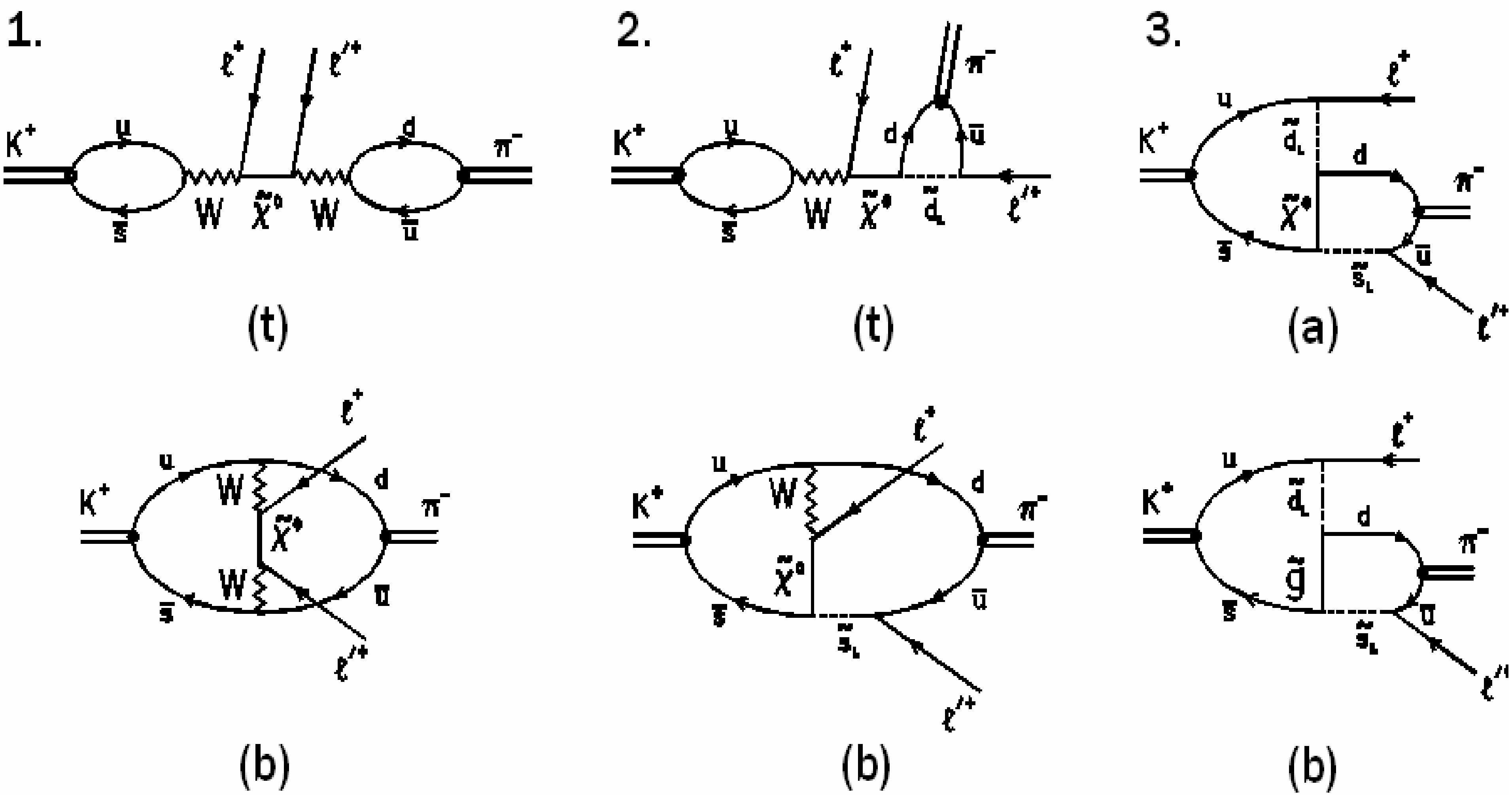
$$V^*_{13}V^*_{24}(\bar{q}^a_3 \gamma_\mu P_L q_{1a})(\bar{q}^b_2 \gamma_\nu P_L q_{4b})] + \frac{g^4 \beta^{c*}}{m_W^2}(\bar{\ell}'^c\, P_L q^b_4)\sum_{\delta=1}^{4}\frac{k^*_\delta}{m_{\tilde{\chi}_\delta}} \times$$

$$[\frac{V^*_{12}\varepsilon_{L\delta}(q_3)}{m^2_{\tilde{q}_{3L}}}(\bar{q}^a_2 \gamma_\mu P_L q_{1a})(\bar{q}_{3b}\gamma_\mu P_L \ell) + \frac{V^*_{13}\varepsilon_{L\delta}(q_2)}{m^2_{\tilde{q}_{2L}}}(\bar{q}^a_3 \gamma_\mu P_L q_{1a})(\bar{q}_{2b}\gamma_\mu P_L \ell)] -$$

$$\frac{g^2(\beta^{c*})^2}{m^2_{\tilde{q}_{3L}} m^2_{\tilde{q}_{2L}}}(\bar{\ell}^c P_L q^a_1)(\bar{\ell}'^c\, P_L q^b_4)[\sum_{\delta=1}^{4}\frac{4g^2 \varepsilon_{L\delta}(q_3)\varepsilon_{L\delta}(q_2)}{m_{\tilde{\chi}_\delta}}(\bar{q}_{3a} P_R q^c_{2b}) +$$

$$\frac{g_3^2(\lambda_r)^e_a(\lambda_r)^d_b}{m_{\tilde{g}}}(\bar{q}_{3e} P_R q^c_{2d})]$$

Feynman diagrams for the rare meson decay $M^+ \to M'^- \ell^+ \ell'^+$ in SUSY with bilinear *R*-parity breaking



In this case the total decay width is *model independent*

$$\Gamma(M^+ \to M'^- \ell^+ \ell'^+) = (1 - \frac{1}{2}\delta_{\ell\ell'}) \frac{f_M^2 f_{M'}^2 m_M g^4}{2^6 \pi^3} \Phi_{\ell\ell'}^{bil} \left| -\frac{g^2 (k_n^*)^2}{8 m_W^4 m_{\tilde{\chi}_n}} (V_{12}^* V_{43}^* + \frac{V_{13}^* V_{42}^*}{N_c}) - \right.$$

$$\left. - \frac{g^2 k_n^* \beta^{c*}}{4 m_W^2 m_{\tilde{\chi}_n}^2} \left(\frac{V_{12}^* \varepsilon_{Ln}(q_3)}{m_{\tilde{q}_{3L}}^2} + \frac{V_{13}^* \varepsilon_{Ln}(q_2)}{N_c m_{\tilde{q}_{2L}}^2} \right) + \frac{g^2 \varepsilon_{Ln}(q_3) \varepsilon_{Ln}(q_2) (\beta^{c*})^2}{2 N_c m_{\tilde{q}_{3L}}^2 m_{\tilde{q}_{2L}}^2 m_{\tilde{\chi}_n}} + \frac{2 g_3^2}{N_c^2} \frac{(\beta^{c*})^2}{m_{\tilde{q}_{3L}}^2 m_{\tilde{q}_{2L}}^2 m_{\tilde{g}}} \right|^2,$$

where $\Phi_{\ell\ell'}^{bil} = \int_{l_+}^{h_-} dz z^2 \left(1 - \frac{h_+ + h_-}{2z}\right)^2 \left(1 - \frac{l_+ + l_-}{2z}\right) \sqrt{(h_+ - z)(h_- - z)(l_+ - z)(l_- - z)},$

$h_\pm = (1 \pm m_\pi / m_K)^2,$ $l_\pm = [(m_\ell \pm m_{\ell'})/m_K]^2,$ $z = (p - p')^2 / m_K^2$

p, p' - the 4-momentum of the initial and final mesons

Using the input parameters for mesons:

$$f_\pi = 131\,MeV\,,\quad f_K = 160\,MeV\,,\quad f_D = 228\,MeV\,.$$

And the following typical set of supersymmetric parameters :

a) *MSSM* – parameters: $m_0 = 70 GeV,\ \mu = 500 GeV,$

$$M_2 = 200 GeV,\quad tg\beta = 4;$$

b) *RPV* – parameters:

$$|\Lambda| = \sqrt{\sum_{i=1}^{3} |\Lambda_i|^2} = 0.1 GeV^2,\ 10\Lambda_1 = \Lambda_2 = \Lambda_3,\ |\varepsilon|^2 = \sum_{i=1}^{3} |\varepsilon_i|^2 = |\Lambda|,$$

$$\varepsilon_1 = \varepsilon_2 = \varepsilon_3.$$

$\Lambda_i = \mu \upsilon_i - \upsilon_d \varepsilon_i,$ υ_d - vacuum expectation values of down-type Higgs boson H_d ,

$\upsilon_i (<< \upsilon_d)$ - sneutrino vacuum expectation values.

Masses of superpartners:

$$m_{\tilde{s}_L}^2 \approx m_{\tilde{d}_L}^2 = m_0^2 + 0.83 m_{\tilde{g}}^2 - \frac{1}{2}\cos(2\beta) M_Z^2 (1 - \frac{2}{3}\sin^2\theta_W),$$

$$m_{\tilde{g}} = \frac{g_s^2}{g^2} M_2.$$

Masses of neutralino $m_{\tilde{\chi}}$ and the elements of neutralino mixing matrix N_{mn} were calculated numerically for the above MSSM input parameters.

Non-zero mass of neutrino:

$$m_{\nu_3} = \frac{M_1 g^2 + M_2 g'^2}{4\det(M_{\tilde{\chi}^0})} \left|\vec{\Lambda}\right|^2,$$

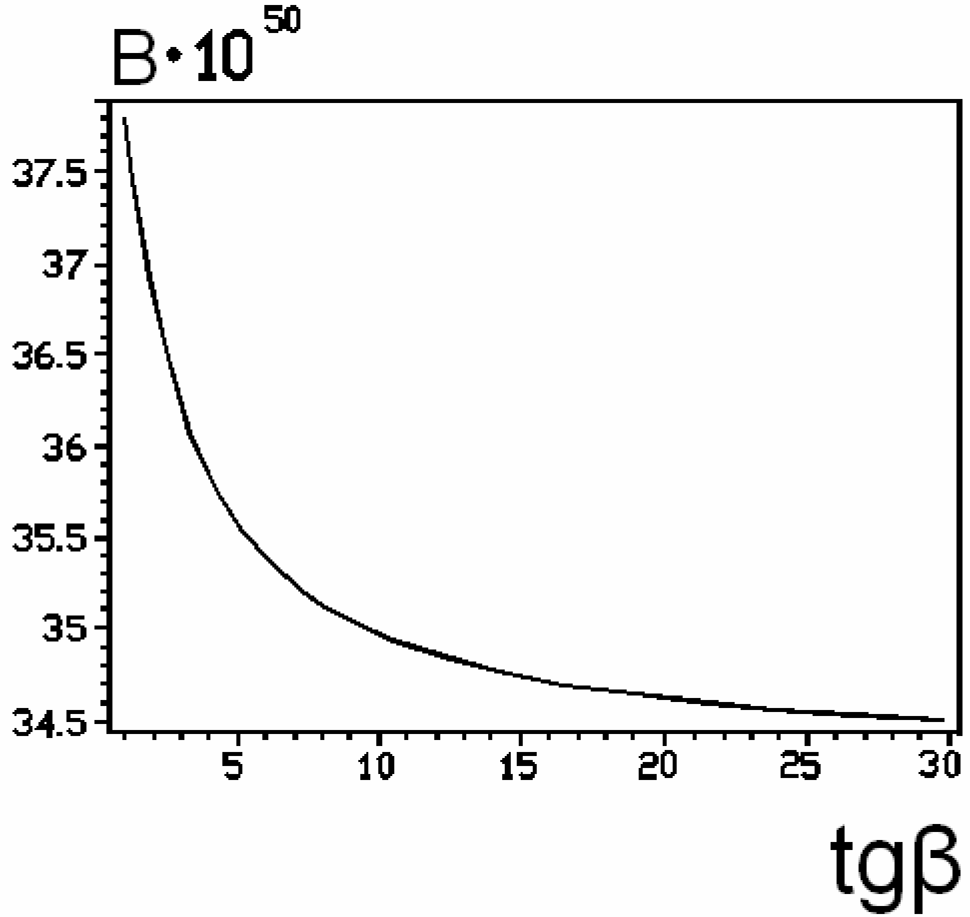
$$\det(M_{\tilde{\chi}^0}) = m_W^2 \mu (M_1 + M_2 tg^2\theta_W) \sin 2\beta - M_1 M_2 \mu^2,$$

$$M_1 = \frac{5g'^2}{3g^2} M_2.$$

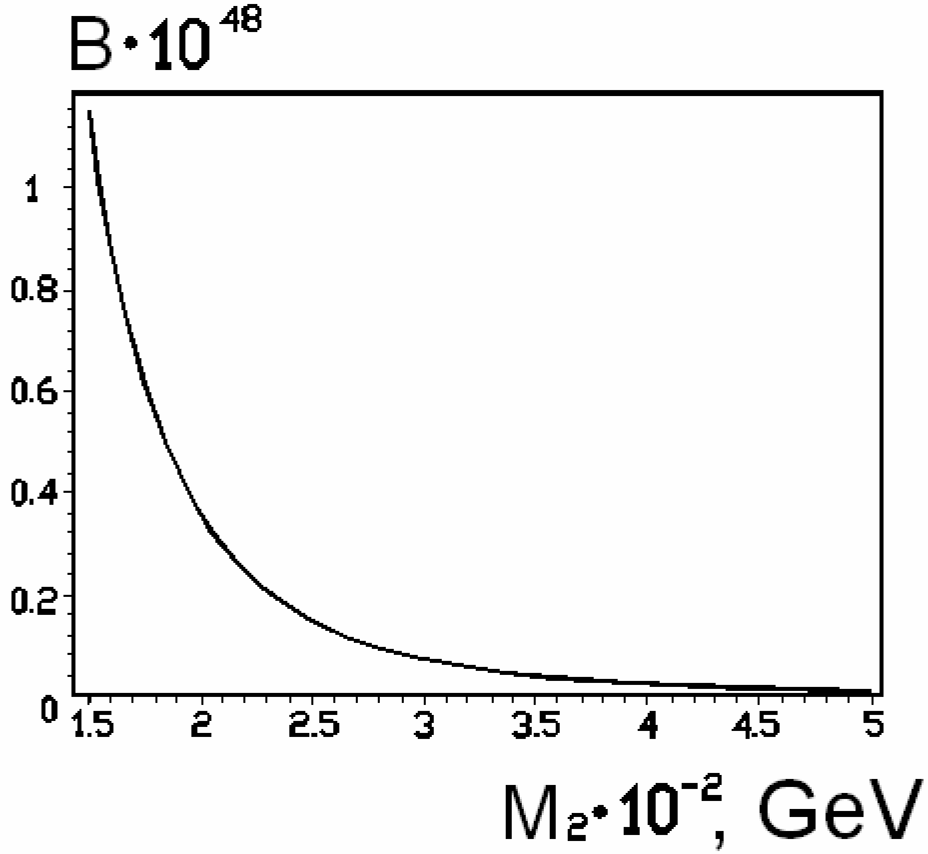
Experimental and indirect bounds on the branching ratios for the rare meson decays in bilinear R-parity breaking sypersymmetry

Decay	$B_{\ell\ell'}$ (bil $\not{R}$ MSSM)	Exp. upper bounds on $B_{\ell\ell'}$
$K^+ \to \pi^- e^+ e^+$	$3.6 \cdot 10^{-49}$	$6.4 \cdot 10^{-10}$
$K^+ \to \pi^- \mu^+ \mu^+$	$1.0 \cdot 10^{-49}$	$3.0 \cdot 10^{-9}$
$K^+ \to \pi^- e^+ \mu^+$	$4.3 \cdot 10^{-49}$	$5.0 \cdot 10^{-10}$
$D^+ \to K^- e^+ e^+$	$1.6 \cdot 10^{-48}$	$1.2 \cdot 10^{-4}$
$D^+ \to K^- \mu^+ \mu^+$	$1.5 \cdot 10^{-48}$	$1.3 \cdot 10^{-5}$
$D^+ \to K^- e^+ \mu^+$	$3.2 \cdot 10^{-48}$	$1.3 \cdot 10^{-4}$

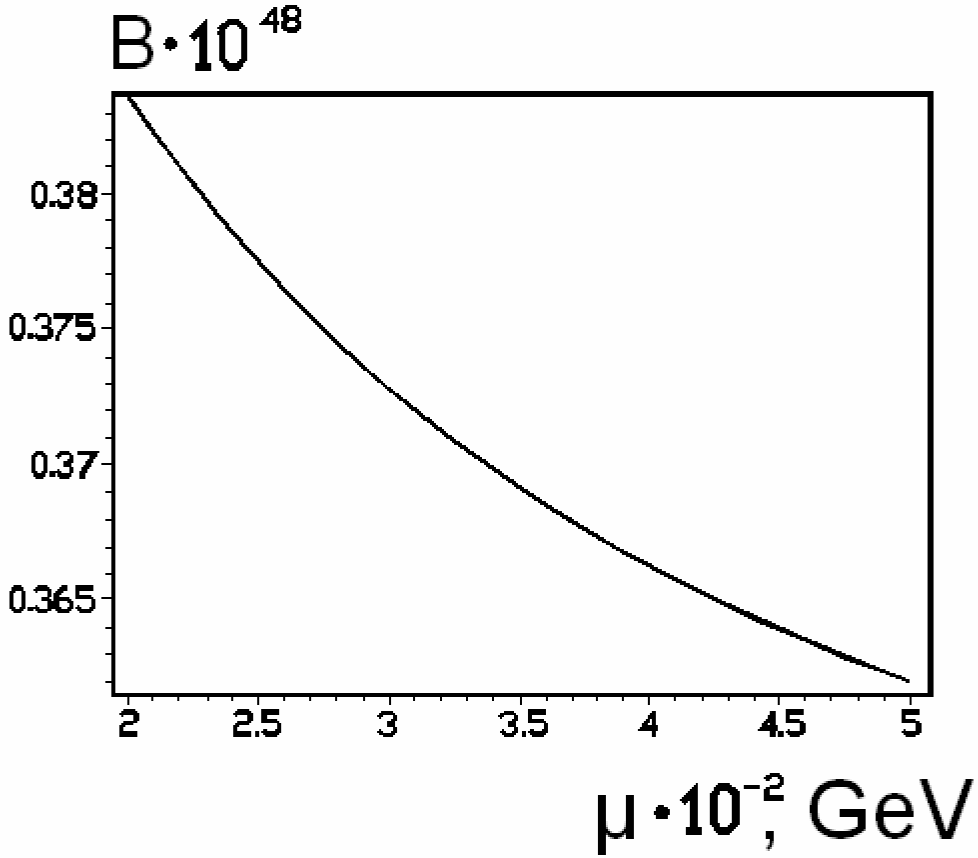
$B(K^+ \to \pi^- e^+ e^+)$ as a function of $tg\beta$ for $\mu = 500 GeV$ and $M_2 = 200 GeV$ in bilinear *R*-parity breaking supersymmetric theory



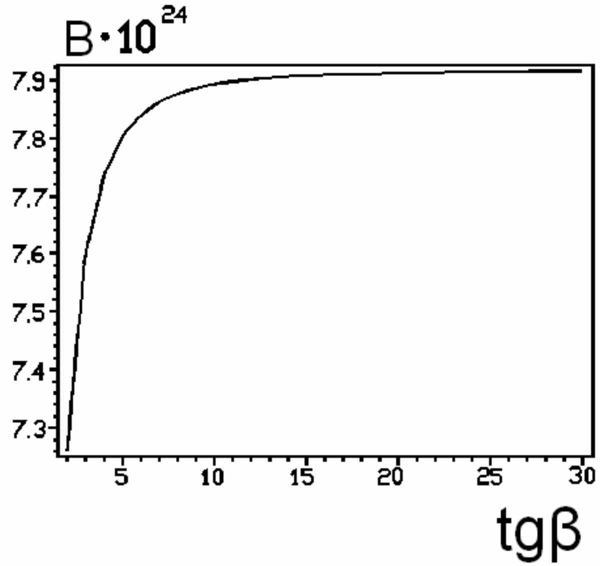
B($K^+ \to \pi^- e^+ e^+$) as a function of M_2 for $\mu = 500 GeV$ and $tg\beta = 4$ in bilinear *R*-parity breaking supersymmetric theory



$B(K^+ \to \pi^- e^+ e^+)$ as a function of μ for $M_2 = 200 GeV$ and $tg\beta = 4$ in bilinear R-parity breaking supersymmetric theory

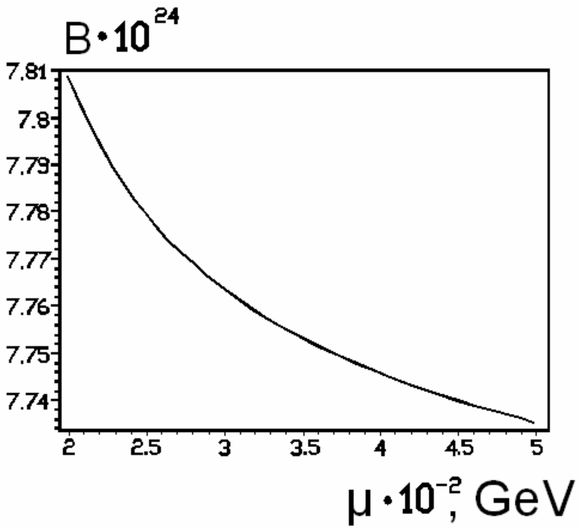


B($K^+ \to \pi^- e^+ e^+$) as a function of $tg\beta$, μ and M_2 in trilinear *R*-parity breaking supersymmetric theory



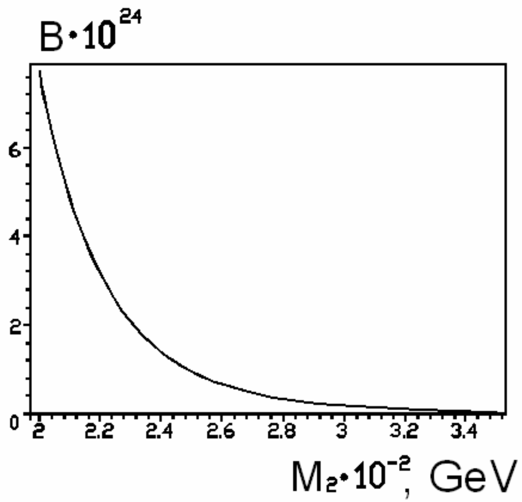
$M_2 = 200 GeV$

$\mu = 500 GeV$



$tg\beta = 4$

$M_2 = 200 GeV$



$tg\beta = 4$

$\mu = 500 GeV$

Conclusion

Experimental and indirect bounds on the branching ratios for the rare meson decays mediated by heavy Majorana neutrinos and in trilinear and bilinear R-parity breaking sypersymmetry

Decay	$B_{\ell\ell'}$ $(\nu_M\ SM)$	$B_{\ell\ell'}$ (tril $\not{R}$ MSSM)	$B_{\ell\ell'}$ (bil $\not{R}$ MSSM)	Exp. upper bounds on $B_{\ell\ell'}$
$K^+ \to \pi^- e^+ e^+$	$5.9 \cdot 10^{-32}$	$7.7 \cdot 10^{-24}$	$3.6 \cdot 10^{-49}$	$6.4 \cdot 10^{-10}$
$K^+ \to \pi^- \mu^+ \mu^+$	$1.1 \cdot 10^{-24}$	$2.7 \cdot 10^{-24}$	$1.0 \cdot 10^{-49}$	$3.0 \cdot 10^{-9}$
$K^+ \to \pi^- e^+ \mu^+$	$5.1 \cdot 10^{-24}$	$1.0 \cdot 10^{-23}$	$4.3 \cdot 10^{-49}$	$5.0 \cdot 10^{-10}$
$D^+ \to K^- e^+ e^+$	$1.5 \cdot 10^{-31}$	$2.9 \cdot 10^{-25}$	$1.6 \cdot 10^{-48}$	$1.2 \cdot 10^{-4}$
$D^+ \to K^- \mu^+ \mu^+$	$8.9 \cdot 10^{-24}$	$2.7 \cdot 10^{-25}$	$1.5 \cdot 10^{-48}$	$1.3 \cdot 10^{-5}$
$D^+ \to K^- e^+ \mu^+$	$2.1 \cdot 10^{-23}$	$5.6 \cdot 10^{-25}$	$3.2 \cdot 10^{-48}$	$1.3 \cdot 10^{-4}$