Khalfin's Theorem and neutral mesons subsystem*

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1 Introduction

Lee–Oehme –Yang (LOY) approximation[1]:

$$\Downarrow$$

$$i\,\frac{\partial|\psi;t\rangle}{\partial t} = H\;|\psi;t\rangle, \qquad |\psi;t=0\rangle = |\psi_0\rangle, \tag{1}$$

- H is the total selfadjoint Hamiltonian for the system containing neutral kaons,
- units: $\hbar = c = 1$,
- $|\psi;t\rangle, |\psi_0\rangle \in \mathcal{H}$,
- $\mathcal{H}$ is the Hilbert space of states of the total system,

Within this problem:

$$H \;=\; H^{(0)} + H^{(1)}, \tag{2}$$

and $|K_0\rangle \equiv |\mathbf{1}\rangle$ and $|\overline{K}_0\rangle \equiv |\mathbf{2}\rangle$ are discrete eigenstates of $H^{(0)}$ for the 2–fold degenerate eigenvalue m_0,

$$\begin{aligned} H^{(0)}|\mathbf{j}\rangle &= m_0|\mathbf{j}\rangle, \quad (j=1,2); \\ H^{(0)}|\varepsilon,J\rangle &= \varepsilon\,|\varepsilon,J\rangle. \end{aligned}$$

[1] T. D. Lee, R. Oehme, C. N. Yang, *Phys. Rev.*, **106** (1957) 340. T. D. Lee, C. S. Wu, *Ann. Rev. Nucl. Sci.*, **16** (1966) 471. J. W. Cronin, *Acta Phys.Polon.*, **B 15** (1984) 419. V. V. Barmin *et al*, *Nucl.Phys.*, **B 247** (1984) 428. L. Lavoura, *Ann.Phys. (N.Y.)*, **207** (1991) 428. M. K. Gaillard, M. Nicolic (Eds.), *Weak Interaction*, INPN et de Physique des Particules, Paris, 1977, Ch. 5, Appendix A. T. D. Lee, *Particle Physics and Introduction to Field Theory*, Harwood academic publishers, London 1990.

We have:

$$\begin{aligned}\langle \mathbf{j}|\mathbf{k}\rangle &= \delta_{jk}, \quad (j,k=1,2),\\ \langle \varepsilon', L|\varepsilon, N\rangle &= \delta_{LN}\,\delta(\varepsilon-\varepsilon'),\\ \langle \varepsilon, J|\mathbf{k}\rangle &= 0,\end{aligned}$$

where J, L, N denotes such quantum numbers as charge, spin, etc.

Here $H^{(1)}$ induces the transitions from $|K_0\rangle \equiv |\mathbf{1}\rangle$ and $|\overline{K}_0\rangle \equiv |\mathbf{2}\rangle$ to other (unbound) eigenstates $|\varepsilon, J\rangle$ of $H^{(0)}$, and, consequently, also between $|K_0\rangle$ and $|\overline{K}_0\rangle$.

The problem = the time evolution of an initial state, which is a superposition of $|\mathbf{1}\rangle$ and $|\mathbf{2}\rangle$ states.

Solutions $|\psi;t\rangle$ of the the Schrödinger equation (1) for this problem (for $t \geq t_0 \equiv 0$)[2]:

$$|\psi;t\rangle = a_1(t)|\mathbf{1}\rangle + a_2(t)|\mathbf{2}\rangle + \sum_{J,\,\varepsilon} F_J(\varepsilon;t)|\varepsilon, J\rangle, \tag{3}$$

$$|a_1(t)|^2 + |a_2(t)|^2 + \sum_{J,\,\varepsilon} |F_J(\varepsilon,t)|^2 = 1,$$

$$F_J(\varepsilon; t=0) = 0.$$

Here $|F_J;t\rangle \equiv \sum_\varepsilon F_J(\varepsilon;t)|\varepsilon, J\rangle$ represents the decay products in the channel J.

[2] T. D. Lee, R. Oehme, C. N. Yang, *Phys. Rev.*, **106** (1957) 340. M. K. Gaillard, M. Nicolic (Eds.), *Weak Interaction*, INPN et de Physique des Particules, Paris, 1977, Ch. 5, Appendix A. T. D. Lee, *Particle Physics and Introduction to Field Theory*, Harwood academic publishers, London 1990.

Inserting $|\psi; t\rangle$, (3), into the Schrödinger equation (1)

$$\Downarrow$$

System of coupled equations for amplitudes $a_1(t)$, $a_2(t)$ and $F_J(\varepsilon; t)$.

$$\Downarrow$$

An adaptation of Weisskopf–Wigner approximation to this system of coupled equations for $a_1(t)$, $a_2(t)$ and $F_J(\varepsilon; t)$

$$\Downarrow$$

Lee–Oehme–Yang theory[3]

$$i\frac{\partial a_1(t)}{\partial t} = h_{11}^{LOY}\, a_1(t) \,+\, h_{12}^{LOY}\, a_2(t),$$

etc., where $t \gg t_0 = 0$, and

$$h_{jk}^{LOY} = m_0 \delta_{jk} - \Sigma_{jk}(m_0), \quad (j, k = 1, 2).$$

We have $H_{jJ}^{(1)}(\varepsilon) = \langle \mathbf{j}|H^{(1)}|\varepsilon, J\rangle$, and

$$\Sigma_{jk}(x) = \sum_{J,\,\varepsilon} H_{jJ}^{(1)}(\varepsilon) \frac{1}{\varepsilon - x - i0} H_{Jk}^{(1)}(\varepsilon).$$

[3]T. D. Lee, R. Oehme, C. N. Yang, *Phys. Rev.*, **106** (1957) 340. M. K. Gaillard, M. Nicolic (Eds.), *Weak Interaction*, INPN et de Physique des Particules, Paris, 1977, Ch. 5, Appendix A.

A compact form of the evolution equations for $a_1(t),\ a_2(t)$:

$$i\frac{\partial}{\partial t}|\psi;t\rangle_{\|} = H_{LOY}|\psi;t\rangle_{\|}, \quad (t \geq t_0),$$

where

$$|\psi;t\rangle_{\|} = a_1(t)|\mathbf{1}\rangle + a_2(t)|\mathbf{2}\rangle \in \mathcal{H}_{\|} \subset \mathcal{H},$$

and $\mathcal{H}_{||}$ is spanned by vectors $|\mathbf{1}\rangle, |\mathbf{2}\rangle$.

$$H_{LOY} \equiv M_{LOY} - \frac{i}{2}\,\Gamma_{LOY},$$

where $M_{LOY} = M_{LOY}^{+},\ \Gamma_{LOY} = \Gamma_{LOY}^{+}$ are (2×2) matrices acting in two–dimensional subspace $\mathcal{H}_{||}$, and

$$h_{jk}^{LOY} = \langle \mathbf{j}|H_{LOY}|\mathbf{k}\rangle.$$

Solutions of the eigenvalue problem for H_{LOY}
— eigenvectors:

$$\begin{aligned} |K_S\rangle \; &= \frac{1}{(|p_S|^2 + |q_S|^2)^{\frac{1}{2}}}\; (p_S\; |K_0\rangle \; - q_S\; |\overline{K}_0\rangle), \\ |K_L\rangle \; &= \frac{1}{(|p_L|^2 + |q_L|^2)^{\frac{1}{2}}}\; (p_L\; |K_0\rangle \; + q_L\; |\overline{K}_0\rangle). \end{aligned}$$

— eigenvalues:

$$\mu_S = m_S - \frac{i}{2}\gamma_S, \quad \mu_L = m_L - \frac{i}{2}\gamma_L.$$

Assumption

The total system under considerations is CPT–invariant,

$$[\Theta, H] = 0. \tag{4}$$

Here

$$\Theta \stackrel{\rm def}{=} \mathcal{C}\mathcal{P}\mathcal{T},$$

is an antiunitary operator and $\mathcal{C}$ is the charge conjugation operator, $\mathcal{P}$ — space inversion, and the antiunitary operator $\mathcal{T}$ represents the time reversal operation.

Consequences of (4).

$$h_{11}^{LOY} = h_{22}^{LOY}, \tag{5}$$

$$p_S = p_L \equiv p, \quad q_S = q_L \equiv q, \tag{6}$$

$$(\frac{q}{p})^2 = \frac{h_{21}^{LOY}}{h_{12}^{LOY}} = \text{const.} \tag{7}$$

$$|K_S\rangle \equiv \frac{1}{(|p|^2 + |q|^2)^{\frac{1}{2}}} \, (p\, |K_0\rangle \, - q\, |\overline{K}_0\rangle), \tag{8}$$

$$|K_L\rangle \equiv \frac{1}{(|p|^2 + |q|^2)^{\frac{1}{2}}} \, (p\, |K_0\rangle \, + q\, |\overline{K}_0\rangle), \tag{9}$$

$$\langle K_S|K_L\rangle \equiv [\langle K_S|K_L\rangle]^* = \frac{|p|^2 - |q|^2}{|p|^2 + |q|^2}. \tag{10}$$

The standard results of the LOY approach:

Properties (5) – (10) and the property that $|\frac{q}{p}| \neq 1$ in CPT invariant system when CP is violated[4]

Remark

If one describes the properties of neutral mesons and the time evolution of their state vectors using the LOY method then, in fact, one assumes that Hamiltonians $H, H^{(0)}$ and $H^{(1)}$ acting in $\mathcal{H}$ exist and that the solutions of Schrödinger equation (1) describe the time evolution of states in $\mathcal{H}$.

The aims of the talk

- to confront the main predictions of the LOY theory with predictions following from the rigorous treatment of two state quantum mechanical subsystems and from the properties of the exact effective Hamiltonian for such subsystems.
- to show graphically how the Khalfin's Theorem "works" in a model of neutral kaon complex.

[4] S. Eidelman *et al*, Review of Particle Physics, *Phys. Lett.* **B 592**, No 1–4, (2004).

2 Khalfin's Theorem

Principles of quantum mechanics:

$$\begin{array}{ccc} t = 0 & & t > 0 \\ |\psi_1\rangle \in \mathcal{H} & \xrightarrow{t} & |\psi_2\rangle = U(t)|\psi_1\rangle \in \mathcal{H} \end{array}$$

where $U(t)$ acts in $\mathcal{H}$ and

$$U(t)\,U^{+}(t) = U^{+}(t)\,U(t) = \mathbb{I}, \tag{11}$$

$$U(t_1)\,U(t_2) = U(t_1 + t_2) = U(t_2)\,U(t_1). \tag{12}$$

$$\Downarrow$$

$$U(0) = \mathbb{I} \quad \text{and} \quad [U(t)]^{-1} \equiv [U(t)]^{+} = U(-t). \tag{13}$$

The transition amplitude

$$A_{jk}(t) = \langle\psi_j|U(t)|\psi_k\rangle, \tag{14}$$

where $(j, k = 1, 2)$, determines the probability to find the system in the state $|\psi_j\rangle$ at time $t > 0$ if it was earlier at instant $t = 0$ in the initial state $|\psi_k\rangle$.

Relation (13)

$$\Downarrow$$

$$[A_{12}(-t)]^{*} = A_{21}(t). \tag{15}$$

P. K. Kabir and A. Pilaftsis[5]

$$\Downarrow$$

$$f_{21}(t) \stackrel{\text{def}}{=} \frac{A_{21}(t)}{A_{12}(t)}. \tag{16}$$

Relation (15)

$$[A_{12}(-t)]^* = A_{21}(t)$$

$$\Downarrow$$

$$[f_{21}(-t)]^* \, f_{21}(t) = 1. \tag{17}$$

Remark

The relation (17) as well as the property (15) are valid for any two states $|\psi_1\rangle$, $|\psi_2\rangle \in \mathcal{H}$.

[5]P. K. Kabir and A. Pilaftsis, *Phys. Rev.*, **A53**, (1996), 66.

Khalfin's Theorem[6]

If

$$f_{21}(t) \equiv \frac{A_{21}(t)}{A_{12}(t)} = \rho = \text{const.}$$

then there must be

$$R = |\rho| = 1.$$

Proof (Kabir and Pilaftsis)
From the property (17), i. e. that, $[f_{21}(-t)]^* f_{21}(t) = 1$, it follows that if $f_{21}(t) = \rho =$ const for every $t \geq 0$ then $[f_{21}(t')]^* = \zeta =$ const for all $t' \leq 0$. Now, if the functions $f_{21}(t)$ and $[f_{21}(t')]^*$ are continuous at $t = t' = 0$ then there must be

$$R = |\rho| = |\zeta| = 1,$$

which is the proof of the Khalfin's Theorem.

As it was pointed out by Kabir and Pilaftsis, the only problem in the above proof is to find conditions guaranteing the continuity of $f_{21}(t)$ at $t = 0$.

[6] L. A. Khalfin, Preprints of the CPT, The University of Texas at Austin: DOE-ER-40200-211, February 1990 and DOE-ER-40200-247, February 1991; (unpublished). L. A. Khalfin, *Foundations of Physics*, **27** (1997), 1549. C. B. Chiu and E. C. G. Sudarshan, *Phys. Rev.* **D42** (1990), 3712 P. K. Kabir and A. Pilaftsis, *Phys. Rev.*, **A53**, (1996), 66. P. K. Kabir, A. N. Mitra, *Phys. Rev.* **D52** (1995), 526. M. Nowakowski, *Int. J. Mod. Phys.* **A14**, (1999), 589. G. V. Dass and W. Grimus, *Phys. Rev.* **D67** (2003), 037901. G. V. Dass, *Phys. Rev.* **D60** (1999), 017501.

There are two possibilities:
1) Vectors $|\psi_1\rangle, |\psi_2\rangle$ are not orthogonal,

$$\langle \psi_1|\psi_2\rangle \neq 0.$$

2) Vectors $|\psi_1\rangle, |\psi_2\rangle$ are orthogonal,

$$\langle \psi_j|\psi_k\rangle = \delta_{jk}, \qquad (j, k = 1, 2).$$

The case **1)**: Functions $f_{21}(t)|_{t\geq 0}$ as well as $[f_{21}(t')]^*|_{t'\leq 0}$ are continuous at $t = t' = 0$.

The case **2)**: Here

$$A_{21}(0) = 0 \quad \text{and} \quad A_{12}(0) = 0,$$

so

$$f_{21}(t) \equiv \frac{A_{21}(t)}{A_{12}(t)}$$

need not be continuous at $t = 0$.

Quantum theory $\Rightarrow$

$$U(t) = e^{-itH},$$

or,

$$U_I(t) = \mathbb{T}\, e^{-i\int_0^t H_I(\tau)\, d\tau},$$

where $\mathbb{T}$ denotes the usual time ordering operator and $H_I(\tau)$ is the operator H in the interaction picture.

Remark

To assure the continuity of $f_{21}(t)$ at $t = 0$ it suffices that there exist such $n \geq 1$ that

$$\begin{aligned} \langle\psi_2|H^k|\psi_1\rangle &= 0, \qquad\qquad (0 \leq k < n), & (18)\\ \langle\psi_2|H^n|\psi_1\rangle &\neq 0 \quad \text{and} \quad |\langle\psi_2|H^n|\psi_1\rangle| < \infty. & (19) \end{aligned}$$

Proof

Assuming that (18), (19) hold and using the d'Hospital rule one finds that simply

$$\lim_{t\to 0+} f_{21}(t) = \frac{\langle\psi_2|H^n|\psi_1\rangle}{\langle\psi_1|H^n|\psi_2\rangle},$$

which means that $f_{21}(t)|_{t\geq 0}$ is continuous at $t = 0$. Similarly, the continuity of $[f_{21}(t')]^*|_{t'\leq 0}$ at $t' = 0$ is assured.

Remark

In the case of neutral mesons $\psi_1 = K_0,\, B_0,\, D_0 \ldots$ and $\psi_2 = \overline{K}_0,\, \overline{B}_0,\, \overline{D}_0 \ldots$. For neutral meson complexes according to the experimental results the particle–antiparticle transitions $|\psi_1\rangle \rightleftharpoons |\psi_2\rangle$ exist, which means that there must exist $n < \infty$ such that the relation (19) occurs. So, for the neutral meson complexes only the assumption of unitarity of the exact transition operator $U(t)$ assures the validity of the Khalfin's Theorem.

3 Properties of time evolution governed by a time–independent Hamiltonian acting in two state subspace

Assumption

The two–dimensional subspace $\mathcal{H}_{\|}$ of $\mathcal{H}$ is spanned by orthogonal vectors $|\psi_1\rangle$, $|\psi_2\rangle$.

Assumption

The evolution operator $U_{\|}(t)$ acting in this $\mathcal{H}_{\|}$ has the following form

$$U_{\|}(t) = e^{-itH_{\|}},$$

and that the operator $H_{\|}$ is a non–hermitian time–independent (2×2) matrix acting in $\mathcal{H}_{\|}$,

$$\frac{\partial h_{jk}}{\partial t} = 0,$$

where $h_{jk} = \langle \psi_j | H_{\|} | \psi_k \rangle$, $(j,k = 1,2)$.

$$\Downarrow$$

The operator $U_{\|}(t)$ is the (2×2) matrix and

$$U_{\|}(t_1)\, U_{\|}(t_2) \;=\; U_{\|}(t_2)\, U_{\|}(t_1) \;=\; U_{\|}(t_1 + t_2),$$

$$U_{\|}(0) \;=\; \mathbb{I}_{\|},$$

where $\mathbb{I}_{\|}$ is the unit matrix in $\mathcal{H}_{\|}$.

Remark
The operator $U_{\|}(t)$ is the solution of the Schrödinger–like evolution equation for the subspace $\mathcal{H}_{\|}$,

$$i\frac{\partial}{\partial t}U_{\|}(t)\,|\psi\rangle_{\|} \;=\; H_{\|}\,U_{\|}(t)|\psi\rangle_{\|}, \qquad U_{\|}(0) \;=\; \mathbb{I}_{\|},$$

where $|\psi\rangle_{\|} \in \mathcal{H}_{\|}$.

This equation is the equation of the same type as the evolution equation used within the Lee–Oehme–Yang theory to describe the time evolution in neutral mesons subspace of states.

Remark
There is

$$H_{\|} = h_0\,\mathbb{I}_{\|} + \vec{h}\,\cdot\,\vec{\sigma},$$

where σ_x, σ_y, σ_z are Pauli matrices,

$$\vec{h}\,\cdot\,\vec{\sigma} = h_x\,\sigma_x \,+\, h_y\,\sigma_y \,+\, h_z\,\sigma_z,$$

and $h_0 = \frac{1}{2}(h_{11} + h_{22})$, $h_z = \frac{1}{2}(h_{11} \,-\, h_{22})$, ect.

$$\Downarrow$$

$$\begin{aligned} U_{\|}(t) \;&=\; e^{-itH_{\|}} \\ &\equiv\; e^{-ith_0}\,[\mathbb{I}_{\|}\,\cos{(th)} \;-\; i\,\frac{\vec{h}\,\cdot\,\vec{\sigma}}{h}\,\sin{(th)}]. \end{aligned}$$

There is

$$h^2 = \vec{h} \cdot \vec{h} = h_x^2 + h_y^2 + h_z^2.$$

Matrix elements of $U_{||}(t)$,

$$\begin{aligned} u_{12}(t) &= -i\, e^{-ith_0}\, \frac{h_{12}}{h} \sin(th), && (20) \\ u_{21}(t) &= -i\, e^{-ith_0}\, \frac{h_{21}}{h} \sin(th), && (21) \\ u_{11}(t) &= e^{-ith_0} [\cos(th) - i\, \frac{h_z}{h} \sin(th)], && (22) \\ u_{22}(t) &= e^{-ith_0} [\cos(th) + i\, \frac{h_z}{h} \sin(th)]. && (23) \end{aligned}$$

Remark

From (20) and (21) it follows that

$$\frac{u_{21}(t)}{u_{12}(t)} \equiv \frac{h_{21}}{h_{12}} \stackrel{\text{def}}{=} r = \text{const}. \tag{24}$$

Remark

From (22) and (23) it follows that

$$u_{11}(t) - u_{22}(t) = -2i\, e^{-ith_0}\, \frac{h_z}{h} \sin(th), \tag{25}$$

so

$$u_{11}(t) = u_{22}(t) \Leftrightarrow h_{11} = h_{22}. \tag{26}$$

Remark

The above properties (24), (26), are true for every time–independent effective Hamiltonian $H_{||}$ acting in two–dimensional subspace $\mathcal{H}_{||}$.

In other words, they hold for the LOY effective Hamiltonian, H_{LOY}, as well as for every $H_{||} \neq H_{LOY}$.

Conclusion 1

If $|r| \neq 1$ and the time–independent effective Hamiltonian $H_{||}$ is the exact effective Hamiltonian for the subspace $\mathcal{H}_{||}$ of states of neutral mesons, that is if

$$u_{jk}(t) \equiv A_{jk}(t), \tag{27}$$

where $j \neq k$, $(j,k = 1,2)$, $r = \frac{u_{21}(t)}{u_{12}(t)} \equiv \frac{h_{21}}{h_{12}}$ (see (24)), $u_{jk}(t) \stackrel{\text{def}}{=} \langle\psi_j|U_{\|}(t)|\psi_k\rangle = \langle\psi_j|e^{-itH_{\|}}|\psi_k\rangle$, and $A_{jk}(t) = \langle\psi_j|U(t)|\psi_k\rangle$, (see (14)), then the evolution operator $U(t)$ for the total state space $\mathcal{H}$ can not be a unitary one.

Proof

Experimental results indicate that for the neutral kaon complex $|r| \neq 1$ So, this conclusion holds because from the Khalfin's Theorem it follows that if $|r| \neq 1$ and matrix elements $A_{jk}(t), \quad (j,k = 1,2)$ are the matrix elements of the exact evolution operator $U(t)$ then there must be $|r| \neq \text{const.}$ Thus if the relation, $u_{jk}(t) \equiv A_{jk}(t)$, is the true relation then there is only one possibility: The Khalfin's Theorem is not valid in this case.

From the proof of this Theorem given in the previous Section and analysis of the case of neutral mesons performed there it follows that this Theorem holds if the evolution operator $U(t)$ for the total state space $\mathcal{H}$ of the system containing two state subsystem under considerations is a unitary operator. For the neutral mesons subsystem Khalfin's Theorem need not hold only if the total evolution operator $U(t)$ is not a unitary operator.

Remark

The description on neutral meson subsystem within the use of any time–independent effective Hamiltonian $H_{||}$ is not the exact description. It is only an approximation.

4 Symmetries CP, CPT and the exact evolution operator and effective Hamiltonian for neutral mesons subsystem

Properties of the exact effective Hamiltonian for the subspace $\mathcal{H}_{||}$ result from the properties of the exact (transition) evolution operator, $U_{||}(t) = PU(t)P$, for $\mathcal{H}_{||}$:

$$U_{||}(t) \equiv PU(t)P = \begin{pmatrix} \mathbf{A}(t) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix},$$

where

$$P = |\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2|,$$

$U(t)$ — the exact evolution operator for the total state space $\mathcal{H}$,
$\mathcal{H}_{||} = P\mathcal{H}$,
$|\psi_1\rangle$, $|\psi_2\rangle$ are orthonormal vectors,
$\mathbf{0}$ — zero–submatrices,

$$\mathbf{A}(t) = \begin{pmatrix} A_{11}(t) & A_{12}(t) \\ A_{21}(t) & A_{22}(t) \end{pmatrix},$$

$$A_{jk}(t) = \langle\psi_j|U(t)|\psi_k\rangle, \qquad (j,k = 1,2),$$

$\mathbf{A}(0) = \mathbb{I}_{||}$ is the unit operator for $\mathcal{H}_{||}$.

The exact effective Hamiltonian $H_{\|}$ governing the time evolution in $\mathcal{H}_{\|}$[7]:

$$H_{\|} = H_{\|}(t) \equiv i\frac{\partial \mathbf{A}(t)}{\partial t}[\mathbf{A}(t)]^{-1}. \tag{28}$$

The exact Schrödinger–like evolution equation for the subspace $\mathcal{H}_{\|}$:

$$i\,\frac{\partial}{\partial t}|\psi, t\rangle_{\|} = H_{\|}(t)\,|\psi, t\rangle_{\|}.$$

Here:
$|\psi, t\rangle_{\|} = a_1(t)\,|\psi_1\rangle + a_2(t)\,|\psi_2\rangle = \mathbf{A}(t)\,|\psi\rangle_{\|} \in \mathcal{H}_{\|}$,
$|\psi\rangle_{\|} = a_1|\psi_1 + a_2|\psi_2\rangle \in \mathcal{H}_{\|}$ is the initial state of the system,
$\|\,|\psi\rangle_{\|}\,\| = 1$.

From (28):

$$h_{11}(t) = \frac{i}{\det \mathbf{A}(t)}(\frac{\partial A_{11}(t)}{\partial t}A_{22}(t) - \frac{\partial A_{12}(t)}{\partial t}A_{21}(t)),$$
$$h_{22}(t) = \frac{i}{\det \mathbf{A}(t)}(- \frac{\partial A_{21}(t)}{\partial t}A_{12}(t) + \frac{\partial A_{22}(t)}{\partial t}A_{11}(t)).$$

[7] K. Urbanowski, *Bull. de L'Acad. Polon. Sci.: Ser. sci. phys. astron.*, **27**, (1979), 155. L. P. Horwitz, J. P. Marchand, *Helv. Phys. Acta*, **42**, (1969), 801. K. Urbanowski, *Acta Phys. Polon.* **B 14** (1983) 485. K. Urbanowski, *Phys. Rev.* **A 50**, (1994) 2847. K. Urbanowski, *Phys. Lett.*, **B 540**, (2002), 89; hep-ph/0201272. K. Urbanowski, *Acta Phys. Polon.* **B 37** (2006) 1727.

So,

$$h_{11}(t) - h_{22}(t) \;=\; \frac{i}{\det \mathbf{A}(t)} \{A_{11}(t)\, A_{22}(t)\, \frac{\partial}{\partial t} \ln (\frac{A_{11}(t)}{A_{22}(t)}) + A_{12}(t)\, A_{21}(t)\, \frac{\partial}{\partial t} \ln (\frac{A_{21}(t)}{A_{12}(t)})\}. \tag{29}$$

Assumptions

$$[\Theta, H] = 0,$$

$$|\psi_1\rangle \equiv |\mathbf{1}\rangle, \quad |\psi_2\rangle \equiv |\mathbf{2}\rangle.$$

$$\Downarrow$$

$$A_{11}(t) = A_{22}(t). \tag{30}$$

Assumptions

$$[\mathcal{CP}, H] = 0,$$

$$\mathcal{CP}|\mathbf{1}\rangle = e^{-i\alpha}|\mathbf{2}\rangle, \quad \mathcal{CP}|\mathbf{2}\rangle = e^{+i\alpha}|\mathbf{1}\rangle.$$

$$\Downarrow$$

$$A_{12}(t) = e^{2i\alpha} A_{21}(t), \;\text{ and }\; A_{11}(t) = A_{22}(t). \tag{31}$$

$$\Downarrow$$

$$|\,\frac{A_{21}(t)}{A_{12}(t)}\,| = 1 \;\equiv\; \text{const.}$$

Assumption

$$[\mathcal{CP}, H] \neq 0.$$

$$\Downarrow$$

$$\frac{A_{21}(t)}{A_{12}(t)}| \neq 1, \quad (\forall t > 0).$$

Conclusion 2

If $(h_{11}(t) - h_{22}(t)) = 0$ for $t > 0$ then there must be

a)

$$\frac{A_{11}(t)}{A_{22}(t)} = \text{const.}, \quad \text{and} \quad \frac{A_{21}(t)}{A_{12}(t)} = \text{const.}, \quad (\text{for} \;\; t > 0),$$

or,

b)

$$\frac{A_{11}(t)}{A_{22}(t)} \neq \text{const.}, \quad \text{and} \quad \frac{A_{21}(t)}{A_{12}(t)} \neq \text{const.}, \quad (\text{for} \;\; t > 0).$$

Remark

Case **a)** means that CP–symmetry is conserved and there is no information about CPT invariance.
Case **b)** denotes that the system under considerations is neither CP–invariant nor CPT–invariant.

Assumption

$$[\Theta, H] = 0 \quad \Rightarrow \quad A_{11}(t) = A_{22}(t).$$

The above property and relation (29):

$$\Downarrow$$

$$h_{11}(t) - h_{22}(t) = \frac{i}{\det \mathbf{A}(t)} \{A_{12}(t)\, A_{21}(t)\, \frac{\partial}{\partial t} \ln (\frac{A_{21}(t)}{A_{12}(t)})\}$$

$$\Downarrow$$

$$h_{11}(t) - h_{22}(t) = 0 \quad \Leftrightarrow \quad \frac{A_{21}(t)}{A_{12}(t)} \quad = \quad \text{const.}, \quad (t > 0).$$

Consequences of the Khalfin's Theorem:

$$\Downarrow$$

Conclusion 3

If $[\Theta, H] = 0$ and $[\mathcal{CP}, H] \neq 0$, that is if for $t > 0$ $A_{11}(t) = A_{22}(t)$ and $\left|\frac{A_{21}(t)}{A_{12}(t)}\right| \neq 1$, then there must be

$$(h_{11}(t) - h_{22}(t)) \neq 0 \text{ for } t > 0.$$

Remark

Within the exact theory (i.e. for real systems), the property

$$h_{11} = h_{22}$$

can not occur if CPT symmetry holds and CP is violated. In this case such a relation is only an approximation.

5 Model calculations

The model considered by Khalfin[8] and then by Nowakowski[9]:

Hamiltonian, H, for the system containing neutral kaon complex is a hermitian operator with a continuous spectrum of decay products labeled by α, β, etc.,

$$\begin{aligned} H|\phi_\alpha(m)\rangle &= m\,|\phi_\alpha(m)\rangle, \\ \langle\phi_\beta(m')|\phi_\alpha(m)\rangle &= \delta_{\alpha\beta}\delta(m'-m). \end{aligned}$$

$$\Downarrow$$

$$|K_S\rangle = \int_{\text{Spec }(H)} dm \sum_\alpha c_{S,\alpha}(m)|\phi_\alpha(m)\rangle,$$

$$|K_L\rangle = \int_{\text{Spec }(H)} dm \sum_\beta c_{S,\alpha}(m)|\phi_\beta(m)\rangle,$$

$$|\mathbf{j}\rangle = \int_{\text{Spec }(H)} dm \sum_\alpha c_{j,\alpha}(m)|\phi_\alpha(m)\rangle,$$

where $j = 1, 2$.

[8] L. A. Khalfin, Preprints of the CPT, The University of Texas at Austin: DOE-ER-40200-211, February 1990 and DOE-ER-40200-247, February 1991; L. A. Khalfin, *Foundations of Physics*, **27** (1997), 1549.

[9] M. Nowakowski, *Int. J. Mod. Phys.* **A14**, (1999), 589.

The exact $A_{jk}(t)$ is the Fourier transform of the density $\omega_{jk}(m)$, $(j,k=1,2)$:

$$A_{jk}(t) = \int_{-\infty}^{+\infty} dm\; e^{-imt} \omega_{jk}(m),$$

where

$$\omega_{jk}(m) = \sum_{\alpha} c_{j,\alpha}^{*}(m)\, c_{k,\alpha}(m).$$

Requirements for $\omega_{jk}(m)$:

$$\int_{-\infty}^{+\infty} dm\, |\omega_{jk}(m)| \;<\; \infty,$$

$$\omega_{jk}(m) = 0 \;\text{ if }\; m < m_g, \quad (m_g > -\infty).$$

$$\mathrm{Spec}(H) = [m_g,\, \infty).$$

Physical states:

$$|K_S\rangle \equiv \frac{1}{(|p|^2 + |q|^2)^{\frac{1}{2}}}\, (p\, |K_0\rangle \; - q\, |\overline{K}_0\rangle),$$

$$|K_L\rangle \equiv \frac{1}{(|p|^2 + |q|^2)^{\frac{1}{2}}}\, (p\, |K_0\rangle \; + q\, |\overline{K}_0\rangle).$$

$$\Downarrow$$

$$\omega_{jk}(m) = f(c_{S,\alpha}(m), c_{S,\alpha}(m)) \;\Rightarrow\; A_{jk}(t),\; (j,k=1,2).$$

Nowakowski:

$$c_{S,\beta}(m) = \Theta(m - m_g)\,\sqrt{\frac{\gamma_S}{2\pi}}\;\frac{a_{S,\beta}(K_S \to \beta)}{m - m_S + i\frac{\gamma_S}{2}},$$

$$c_{L,\beta}(m) = \Theta(m - m_g)\,\sqrt{\frac{\gamma_L}{2\pi}}\;\frac{a_{L,\beta}(K_L \to \beta)}{m - m_L + i\frac{\gamma_L}{2}},$$

where:

$a_{S,\beta}$ and $a_{L,\beta}$ are the decay (transition) amplitudes,

$$\Theta(m - m_g) \stackrel{\text{def}}{=} \begin{cases} 1 & \text{if } m \geq m_g, \\ 0 & \text{if } m < m_g, \end{cases} .$$

$$\Downarrow$$

$$\mathcal{A}_{SS}(t) \stackrel{\text{def}}{=} \langle K_S | e^{-itH} | K_S \rangle = \int_{-\infty}^{+\infty} dm\, \omega_{SS}(m)\, e^{-itm},$$

and so on.

$$\omega_{SS}(m) \;=\; \Theta(m - m_g)\,\frac{\gamma_S}{(m - m_S)^2 + \frac{\gamma_S^2}{4}}\,\frac{S}{2\pi},$$

$$S = \sum_{\alpha} |a_{S,\alpha}(K_S \to \alpha)|^2.$$

Assumptions

$$m_g = 0,$$

$$A_{11}(t) = A_{22}(t),$$

i.e., CPT symmetry holds.

Assumptions[10]

$m_S \simeq m_L \simeq m_{average} = 497.648 MeV,$
$\Delta m = 3.489 \times 10^{-12} MeV,$
$\tau_S = 0.8935 \times 10^{-10} s, \ \ (\gamma_S = 7.4 \times 10^{-12} MeV),$
$\tau_L = 5.17 \times 10^{-8} s, \ \ (\gamma_L = 1.3 \times 10^{-14} MeV).$

$$\Downarrow$$

$$Mathematica$$

$$\Downarrow$$

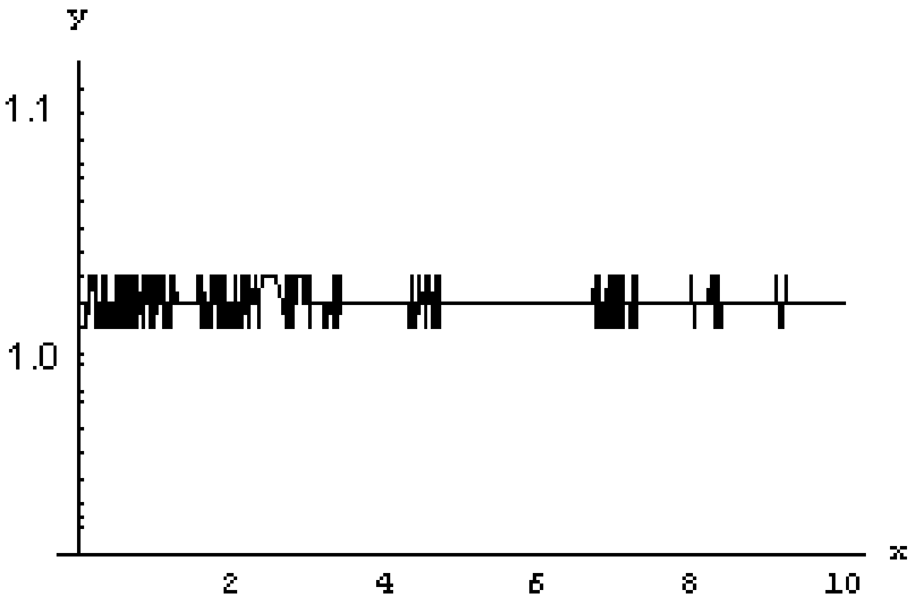

Figure 1: Numerical examination of the Khalfin's Theorem.

Here $y(x) = |r(t)| \equiv |\frac{A_{21}(t)}{A_{12}(t)}|$, $x = \frac{\gamma_L}{\hbar} \cdot t$, and $x \in (0.01, 10)$.

[10]S. Eidelman *et al*, Review of Particle Physics, *Phys. Lett.* **B 592**, No 1–4, (2004).

Remark

$$y_{max}(x) - y_{min}(x) \simeq 3.3 \times 10^{-16},$$

where

$$\begin{aligned} y_{max}(x) &= |r(t)|_{max}, \\ y_{min}(x) &= |r(t)|_{min}. \end{aligned}$$

Remark

For times t considered

$$\frac{A_{21}(t)}{A_{12}(t)} = R + \Delta r(t),$$

where, $R = \text{const}$ and $|\Delta r(t)| \leq 10^{-16}$.

Remark

If one is able to measure the modulus of the ratio $\frac{A_{12}(t)}{A_{21}(t)}$ only up to the accuracy 10^{-15} then one sees this quantity as a constant function of time.

The variations in time of $|\frac{A_{12}(t)}{A_{21}(t)}|$ become detectable for the experimenter only if the accuracy of his measurements is of order 10^{-16} or better.

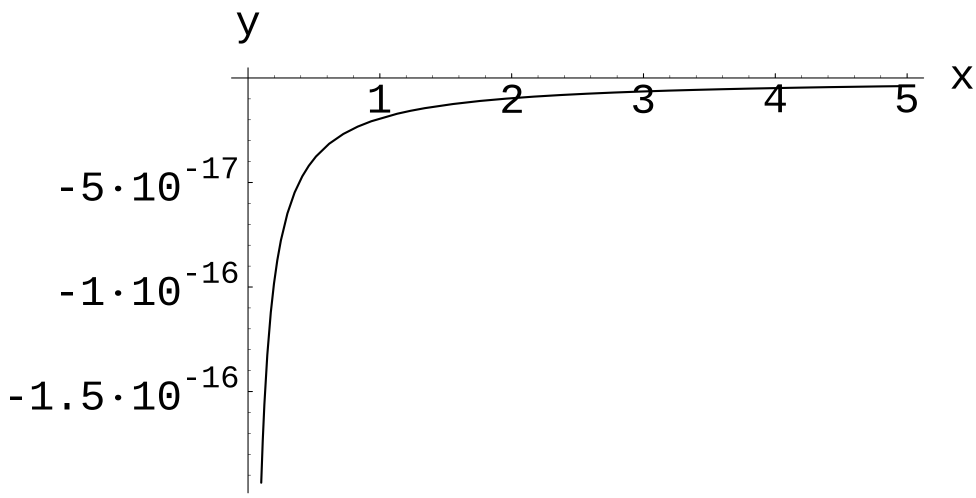

Figure 2: Real part of $(h_{11}(t) - h_{22}(t))$

There is $y(x) = \Re\left(h_{11}(t) - h_{22}(t)\right.$ in Fig 2.

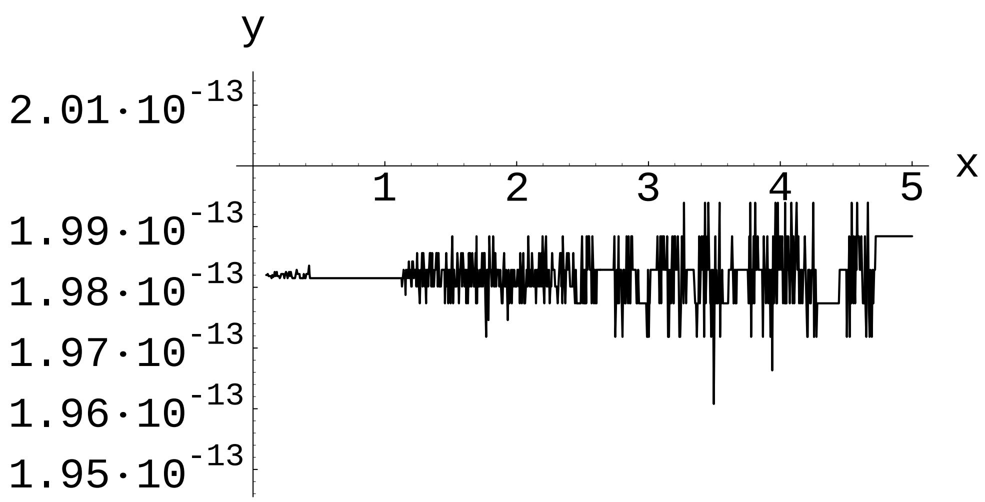

Figure 3: Imaginary part of $(h_{11}(t) - h_{22}(t))$

There is $y(x) = \Im\,(h_{11}(t) - h_{22}(t)$ in Fig 3.

In the above Figures $x = \frac{\gamma_L}{\hbar} \cdot t$, $x \in (0.01, 5.0)$.
Units on the y–axis are in [MeV].

Remark

An expansion of scale in Fig. 2 shows that continuous fluctuations, similar to those in Fig. 3, appear.

For more graphics and results see:
J. Jankiewicz, *Acta. Phys. Polon.* **B 36**, (2005), 1901.
J. Jankiewicz, hep-ph/0612178.

6 Final remarks

From *Conclusion 3*,

"If $[\Theta, H] = 0$ and $[\mathcal{CP}, H] \neq 0$, that is if $A_{11}(t) = A_{22}(t)$ and $\left|\frac{A_{21}(t)}{A_{12}(t)}\right| \neq 1$ for $t > 0$, then there must be $(h_{11}(t) - h_{22}(t)) \neq 0$ for $t > 0$."

drawn up earlier, it follows that, contrary to the standard interpretation following from the LOY theory,

$$\delta \neq 0, \quad \text{or,} \quad \varepsilon_s \neq \varepsilon_l,$$

when CPT symmetry holds and CP symmetry is violated.

Here

$$\delta \stackrel{\text{def}}{=} \frac{1}{2}(\varepsilon_s - \varepsilon_l) \equiv \frac{h_{11} - h_{22}}{D} \equiv \frac{2h_z}{D},$$

$$D \stackrel{\text{def}}{=} h_{12} + h_{21} + \Delta\mu,$$

and $\Delta\mu = \mu_S - \mu_L$.

$$|K_{L(S)}\rangle \equiv \frac{1}{\sqrt{1 + |\varepsilon_{l(s)}|^2}} \left(|K_{2(1)}\rangle + \varepsilon_{l(s)}|K_{1(2)}\rangle \right),$$

$$|K_{1(2)}\rangle \stackrel{\text{def}}{=} \frac{1}{\sqrt{2}}(|\mathbf{1}\rangle - (+)|\mathbf{2}\rangle),$$

$$\mathcal{CP}|K_{1(2)}\rangle = +(-1)|K_{1(2)}\rangle,$$

Here,

$$\mathcal{CP}|\mathbf{1}\rangle = (-1)|\mathbf{2}\rangle, \qquad \mathcal{CP}|\mathbf{2}\rangle = (-1)|\mathbf{1}\rangle.$$

Remark

The result that there must be

$$\varepsilon_s \neq \varepsilon_l,$$

if CPT symmetry holds CP is violated was also obtained by B. Machet, V. A. Novikov and M. I. Vysotsky, (see: *Int. J. Mod. Phys.* **A 20**, (2005), 5399, and, hep–ph/0407268 by V. A. Novikov, where within the quantum field theory binary systems such as the neutral meson complexes were analyzed).

For more details see:

K. Urbanowski, *Phys. Lett.*, **B 540**, (2002), 89; hep-ph/0201272,

K. Urbanowski, *Acta Phys. Polon.* **B 37** (2006) 1727.

K. Urbanowski, J. Jankiewicz, hep–ph/0707.3219.

Remark

All conclusions and results presented in this talk follow from mathematically rigorous treatment of the basic assumptions of the quantum theory.

Thank you