Parity violating thin shells in the framework of QED

Or better to say

QFT systems with 2d spatial defects

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Contents

- Physical motivation
- Primary Ideas
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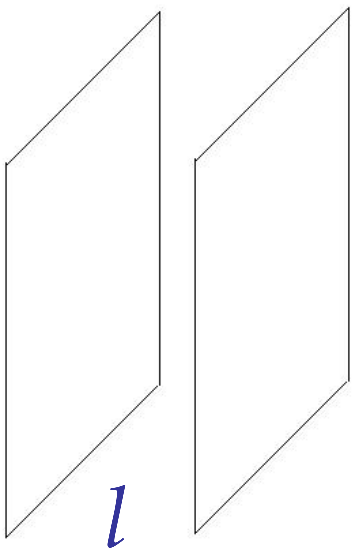
Fialkovsky I.V., Markov V.N., Pis'mak Yu.M.
Field of homogeneous plane in Quantum Electrodynamics
Int. J. Mod. Phys. A, Vol. 21, No. 12, pp. 2601-2616 (2006)
hep-th/0311236

Fialkovsky I.V., Markov V.N., Pis'mak Yu.M.
Renormalizable mean field calculation in QED with fermion background
J. Phys. A: Math. Gen. 39 (2006) 6357 – 6363

Markov V.N., Pis'mak Yu.M.
hep-th/0505218

Casimir motivation



$\Rightarrow$

in vacuum
neutral
∞-thin
conducting plates

$$F = -\frac{\pi^2}{240}\frac{1}{l^4}$$

Casimir, 1941

also known under the names

- vacuum polarization
- vacuum energy
- Casimir-Polder interaction

Essential: modification of the ground vacuum state and ***its energy*** in presence of boundaries (=spatial defects)

Usually: boundary conditions, simple (scalar fields') models + ζ-function regularization
Brings **only** the force, **no way** in principle for loop corrections.

Most recent Reviews

Mostepanenko, quant-ph/0702061
Klimchitskaya, Mostepanenko, quant-ph/0609145

Need for self consistent method for calculation of

- interaction of charge, wire current with the defect
- quantum corrections to classical fields of the defect
- **<u>all other observables in all loops</u>** (at least in principle)

$$\Downarrow$$

QFT

- **Locality**
- **Gauge invariance**
- **Lorenz invariance**
- **Renormalizability**

QFT model construction

2d objects:
singular (delta-function) potentials into Lagrangian

L_{def} - defect Lagrangian

- Lorenz & Gauge invariance - known from volume (ordinary) terms
- Locality: $\boldsymbol{L_{def}}$ must be polynomial in fields and derivatives taken in one point at the defect
- Renormalizability - **Simanzik, 1981**

- all possible terms with constants of non-negative dimension
- no terms with with constants of negative dimension

- **for QED we would like to reconstruct known physics**

Defect Lagrangian

Defect surface equation $\Phi(x)=0$

	L_0	L_{def}
Scalar field	$\phi(\partial^2+m^2)\phi$	$\lambda^2\delta(\Phi(x))\phi^2, \phi\partial_n\phi$
Fermion field	$\psi(i\hat{\partial}+m)\psi$	$\delta(\Phi(x))\bar{\psi}\hat{Q}\psi$ $\hat{Q}=\lambda+q_\mu\gamma^\mu+a_{\mu\nu}\sigma^{\mu\nu}$
EM field	$F^{\mu\nu}F_{\mu\nu}$	$a\delta(\Phi(x))\varepsilon_{\mu\nu\rho\sigma}\partial^\mu\Phi(x)A^\nu F^{\rho\sigma}$

$\lambda, q_\mu, a_{\mu\nu}, a$ describe physical properties of the defect

Path Integral Approach
$$Z[J]=\int D\varphi e^{i\int dx(L_0+L_{def})+J\varphi}$$

$$\varphi=\phi,\psi,A_\mu,\ldots$$

Calculations

Knowledge of ***Z[J]*** gives complete quantative description of all physical phenomena in the system

Vacuum energy $E = -\frac{1}{T} \ln Z[0]$ Propagator $D = \frac{\delta^2}{\delta J} Z[J]|_{J=0}$

To calculate ***Z[J]*** explicitly we introduce auxiliary fields living on the surface ***S*** of the defect

$$e^{-\lambda^2 \int \phi^2 dS} = N \int D\psi e^{\int (\frac{1}{4\lambda}\psi^2 + \phi\psi) dS}$$

$$\Downarrow$$

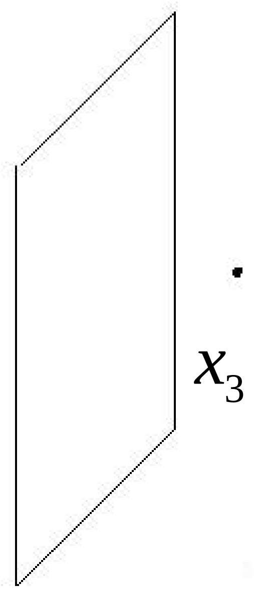
$$Z[J] = (DetQ)^{-1/2} e^{\frac{1}{2} JSJ}$$

$$S = D - 2\lambda(D\Omega)Q^{-1}(\Omega D) \qquad Q = 1 + 2\lambda(\Omega D \Omega)$$

Ω - Projector operator to the surface of the defect

Results

Fermion field with a single defect plane



$$L = \psi(i\hat{\partial} + m)\psi + \lambda\delta(x_3)\bar{\psi}\gamma_0\psi$$

Mean Electromagnetic Field

$$E \approx \frac{ekm^2}{8\pi^2\lambda^2}\left((1+\lambda^2)\,\mathrm{Arctr}(\lambda) - \lambda\right)\left(\frac{1}{m^2 {x_3}^2} - 2\right) \qquad x_3 \to 0$$

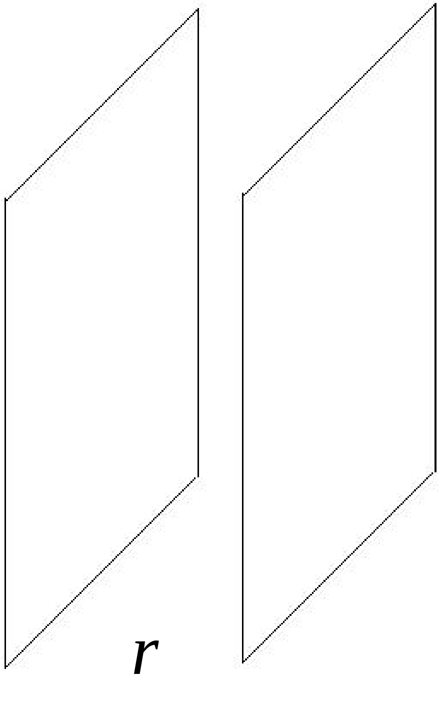
$$E \approx -\frac{em^2}{8\pi^2\lambda^2}\left(\mathrm{Arctr}(\lambda) - \lambda\right) + O\left(e^{-m|x_3|}\right) \qquad x_3 \to \infty$$

- Classical behavior at large distances

<u>Quantum corrections</u> at short distances

Results

EM field, two defect planes



$$L = F^{\mu\nu}F_{\mu\nu} + a\left(\delta(x_3) + \delta(x_3 - r)\right)\varepsilon_{3\nu\rho\sigma}A^{\nu}F^{\rho\sigma}$$

Casimir Force $$F = -\frac{\pi^2}{240 r^4} f(a)$$

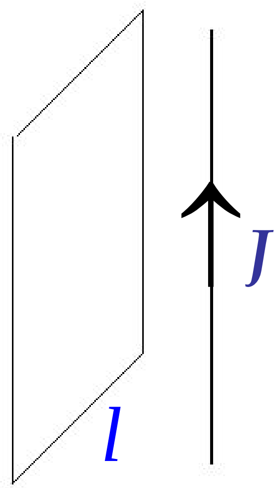
$a \to \infty$ Reproduces perfectly conducting case

- No parity violation manifest

sign of the force depending on ***a***

Results

EM defect plane, interaction with a current



Usual magnetic field H

Anomalous electric field E

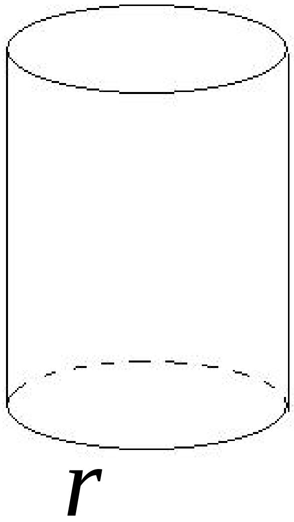
$$E_2 = \frac{2Ja}{a^2+1}\frac{x^2}{\tau^2} \qquad E_3 = \frac{2Ja}{a^2+1}\frac{|x_3|+l}{\tau^2}$$

$$\tau = \sqrt{{x_2}^2 + (|x_2|+l)^2}$$

- Parity violation manifest

Behavior similar to magnetoelecrtircs

Results



EM field: circular cylinder shell

$$L = F^{\mu\nu}F_{\mu\nu} + a\delta(\Phi(x))\varepsilon_{\mu\nu\rho\sigma}\partial^{\mu}\Phi(x)A^{\nu}F^{\rho\sigma}$$

$$\Phi(x) = x_1^2 + x_2^2 - R^2$$

Casimir energy (divergent)

$$E = \frac{1}{2(2\pi)^2 R^2}\int_0^{\infty} pdp \sum_{-\infty}^{\infty} \ln\left(1 + a^2 p^2 I_n(p) K_n(p) I_n'(p) K_n'(p)\right)$$

With Pauli-Villars regularization

$$E = E_{Cas} + RM^3 A_3 + M^2 A_2 + \frac{M}{R} A_1$$

$a \to \infty$ Reproduces perfectly conducting case

Results

EM field: circular cylinder shell

$$E_{Cas} = \frac{1}{4\pi^2 R^2}\int_0^\infty pdp\ \varepsilon_{fin} + \frac{c_1 + 2c_2}{4\pi R^2}$$

$$\varepsilon_{fin} = \ln\left(\frac{1+a^2 Y_0(p)}{1+a^2/4}\right) - F(1) + 2\sum_{n=1}^{\infty}\left[\ln\left(\frac{1+a^2 Y_n(p)}{1+a^2/4}\right) - F(N)\right]$$

$$c_1 = \frac{a^2}{4+a^2}\left(\frac{\ln 2}{2} + \frac{1}{16}\right) \qquad c_2 = \frac{a^2}{32(4+a^2)}\left(4\ln\pi - 4\ln 2 - 1\right)$$

$$Y_n(p) = -p^2 I_n(p) K_n(p) I'_n(p) K'_n(p)$$

$$F(n) - \text{first order term of assymptotic expansion of } Y_n(p, M)$$