Simulation of the nuclear interaction

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- Problematic

$$L = L(t, \dot{r}) \quad \text{or} \quad L = L(t, \dot{r}, \ddot{r}, \dddot{r}, \ldots, \overset{\bullet(n)}{r})$$

???

Euler-Lagrange equation in this case

$$\delta S = \delta \int L(r, \dot{r}, \ddot{r}, \dddot{r}, \ldots, \dot{r}^{(n)}) dt = \int \sum_{n=0}^{N} (-1)^n \frac{d^n}{d\dot{r}^{(n)}} \delta \dot{r}^{(n)} dt = 0$$

or

$$\frac{\partial L}{\partial r} - \frac{d}{dt}\frac{\partial L}{\partial \overset{\bullet}{r}} + \frac{d^2}{dt^2}\frac{\partial L}{\partial \overset{\bullet\bullet}{r}} - \frac{d^3}{dt^3}\frac{\partial L}{\partial \overset{\bullet\bullet\bullet}{r}} + \ldots + (-1)^n \frac{d^{n-1}}{dt^{n-1}}\frac{\partial L}{\partial \overset{\bullet(n)}{r}} + \ldots = 0$$

Total Energy

$$E = kr^2 + k_1 \overset{\bullet}{r}^2 + k_2 \overset{\bullet\bullet}{r}^2 + \ldots k_n \overset{\bullet(n)}{r}^2 \quad \ldots$$

- Lagrange function in Ostrogradski's case

$$L = kr^2 - k_1\dot{r}^2 + k_2\ddot{r}^2 - k_3\dddot{r}^2 + \ldots + (-1)^{(\alpha)} k_\alpha \dot{r}^{(\alpha)2} + \ldots = \sum_{\alpha=0}^{\infty} (-1)^{(\alpha)} k_\alpha \dot{r}^{(\alpha)2}$$

- or

$$L = \sum_{\alpha=0}^{N} (-1)^{(\alpha)} g_{ik} \dot{r}^{(\alpha)i} \dot{r}^{(\alpha)k}$$

$$\frac{\partial L}{\partial r}-\frac{d}{dt}\frac{\partial L}{\partial \dot{r}}+\frac{d^2}{dt^2}\frac{\partial L}{\partial \ddot{r}}-\frac{d^3}{dt^3}\frac{\partial L}{\partial \dddot{r}}+\ldots+(-1)^n\frac{d^{n-1}}{dt^{n-1}}\frac{\partial L}{\partial \overset{\bullet(n)}{r}}+\ldots=0$$

- Momentums and forces

$$F=\frac{\partial L}{\partial r}=2kr,\ p=\frac{\partial L}{\partial \dot{r}}=-2k_1\dot{r}$$

$$F^{(2)}=\frac{d^2}{dt^2}\frac{\partial L}{\partial \ddot{r}}=2k_2\ddot{r},\ p^{(3)}=\frac{\partial L}{\partial \dddot{r}}=-2k_3\dddot{r}$$

$$F^{(4)}=\frac{d^4}{dt^4}\frac{\partial L}{\partial \overset{\bullet(4)}{r}}=2k_4r^{\bullet(4)},\ p^{(5)}=\frac{\partial L}{\partial \overset{\bullet(5)}{r}}=-2k_5r^{\bullet(5)}$$

....

$$F^{(2n)}=\frac{d^{2n}}{dt^{2n}}\frac{\partial L}{\partial \overset{\bullet 2n}{r}}=2k_nr^{\bullet(n)},\ p^{(2n+1)}=\frac{\partial L}{\partial \overset{\bullet(2n+1)}{r}}=-2k_{2n+1}r^{\bullet(2n+1)}$$

- Let’s compare

$$F + F^{(2)} + F^{(3)} + \ldots F^{(n)} = \frac{dp}{dt} + \frac{d^3 p^{(2)}}{dt^3} + \ldots + \frac{d^n p^{n-1}}{dt^n}$$

$$F(t) = F_0 + \dot{F}t + \frac{1}{2!}\ddot{F}t^2 + \frac{1}{3!}\dddot{F}t^3 + \ldots$$

$$F = ma + m_2\ddot{a} + m_4\dot{a}^{(4)} + \ldots. + m_n\dot{a}^{(2n)} + \ldots$$

$$F = ma$$

- In general case $L = \sum_{\alpha=0}^{N} (-1)^{(\alpha)} g_{ik} \dot{r}^{(\alpha)i} \dot{r}^{(\alpha)k}$

In our case we can represent Schwarzschild metric

$$ds^2 = (1 - \frac{r_g}{r}) c^2 dt^2 - (1 - \frac{r_g}{r}) dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2$$

as

$$ds^2 = \exp(-\frac{r_g}{r}) c^2 dt^2 - \exp(-\frac{r_g}{r}) dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2$$

here $r_g = \frac{2GM}{c^2}$ $g_{oo} = \exp(-\frac{r_g}{r})$

- In the particular case the gravitational field potential is

$$\varphi = G\frac{M}{r}\exp(-\frac{\lambda}{r})$$

- Or

$$\varphi = G\frac{M}{r}(1-\frac{\lambda}{r}+\frac{\lambda^2}{2r^2}-...)$$

- If constant λ is equal to the size of nuclear

λ =10^{-15}m than the gravitational force is equal to nuclei forces because at the this distant gravitational forces is change on exponential law and may be stronger than electromagnetic forces.

$$E = V + W + Q,$$

$$V = \alpha_0 r^2,$$

$$W = \alpha_1 \dot{r}^2,$$

$$Q = \alpha_2 \ddot{r}^2 + \ldots \alpha_n \overset{\bullet(n)}{r}{}^2 \ldots$$

$$-\frac{\partial S}{\partial t} = \frac{(\nabla S)^2}{2m} + V + Q$$

$$\psi = e^{\frac{i}{\hbar}S} \qquad Q \approx \alpha_3 \frac{\nabla^2 S}{m^2} \qquad \alpha_3 = \frac{i\hbar m}{2}$$

References

- Ostrogradskii M., Met. De l'Acad. De St.-Peterburg, v. 6. p. 385, 1850.

  (M. V. Ostrogradskii, "M´emoire sur les ´equations diff´erentielles relatives aux probl`e-mes des

  isop´erim`etres," M´emoires de l'Acad´emie Imp´eriale des Sciences de Saint-P´etersbourg, VIme s´erie,

  385, 1850.)
- T.F. Kamalov, Physics of non-Inertial Reference Frames, http://arxiv.org/abs/0708.1584v1

- Thanks!