Bound state problems and radiative effects in extended electrodynamics with Lorentz violation

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The Background [1]

- There exist some theories (but not the Standard Model itself!) that suggest a self-consistent description of quantum gravity. However, they are too complicated and contain many undetermined parameters
- In string and some other theories, a spontaneous breaking of Lorentz and CPT invariance can occur at 'low' energies $E \ll M_P \sim 10^{19}$GeV [Kostelecký, Jackiw, Coleman, Glashow, Colladay, et al.]
- For the study of Lorentz violation in these conditions, The Standard Model Extension (SME) was elaborated [Kostelecký, Colladay, et al.] that describes it in the most general way
- No evidence for the existence of Lorentz violation has been found to date; all SME-couplings are tightly constrained, except for the few ones, e.g. the zero component of the axial vector b_μ:

$$
\begin{aligned}
|b_0| &\lesssim 10^{-2}\text{eV}, \\
|\boldsymbol{b}| &\lesssim 10^{-19}\text{eV}.
\end{aligned}
$$

The Background [2]

- The study of atom within SME has yet concerned, primarily, its spectroscopic properties [Kostelecký, Bluhm, Russell, Lane, Ferreira et al.]. Solving the atomic eigenstate problem would also give the possibility to study its radiative properties
- Atomic parity can be effectively violated due to the weak interaction [Zeldovich, Khriplovich, Novikov, Bouchiat, Curtis-Michel] and directly by the interaction with the Lorentz-violating condensate b_0.

- Quantum theory of synchrotron radiation (SR) has been developed within the Standard Model [Sokolov, Ternov, Bagrov, Zhukovsky et al.] that even took the electron AMM into account.
- This theory was based on the method of exact solutions in the external magnetic field
- Although this method has been employed in SME [Lobanov, Zhukovsky, Murchikova], SR has been treated only classically [Altschul]

The Essence

Working within the context of Lorentz-violating extended electrodynamics (a part of the Standard Model Extension (SME)), we investigate the following physical systems:

- electron bound state in the Coulomb potential (hydrogen-like atom),
- electron in a constant homogeneous magnetic field and synchrotron radiation (SR).

Consideration of these problems is quite similar. It includes studying the following aspects:

- integrals of motion in the external field,
- one-particle eigenstates and spectrum in the external field,
- quantum transitions: interaction with photons within the Furry picture, radiation distribution.

Extended electrodynamics

We use the minimal CPT-odd form of the extended electrodynamics with electrons and photons:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\left(i\gamma^{\mu}D_{\mu} - m_{e} - b_{\mu}\gamma_{5}\gamma^{\mu}\right)\psi.$$

Electron charge $q_e = -e < 0$, $\alpha = e^2/4\pi$; $D_\mu = \partial_\mu - ieA_\mu$, $\gamma_5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$.

b^μ is a constant axial vector condensate that introduces a Lorentz-violating CPT-odd interaction

The Model

- One-particle approximation for e^- in an external field $A_\mu(\boldsymbol{r}, t)$ within extended electrodynamics:

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}_{\mathrm{D}}(t)\psi, \ \|\psi\|^2 = 1;$$

$$\hat{H}_{\mathrm{D}}(t) = c\boldsymbol{\alpha} \cdot \hat{\boldsymbol{P}} + \gamma^0 m_e c^2 - eA_0 - b_0\gamma_5 - \boldsymbol{b} \cdot \boldsymbol{\Sigma},$$

$$\alpha_i = \gamma^0\gamma^i, \ \Sigma_i = -\alpha_i\gamma_5, \ \hat{\boldsymbol{P}} = \hat{\boldsymbol{p}} + \frac{e}{c}\boldsymbol{A}.$$

- Expansion into a series with respect to b^μ.
- Of interest is a spherically-symmetric potential, e.g. the Coulomb potential: $A^\mu = \left\{\frac{Ze}{4\pi r}, \mathbf{0}\right\}$.
  In such a field, P-parity is unbroken unless $b^0 \neq 0$.
- We will show both the quasirelativistic (for $Z\alpha \ll 1$) and relativistic (for $Z\alpha < 1$) approaches.

Quasirelativistic Hamiltonian in the External Field [1]

To obtain it, we will use the expansion into a series with respect to $1/c$, assuming $\hbar, c \neq 1$, $b^\mu = \{cb_t, \boldsymbol{b}\}$ (the Landau method).

0. Consider an electron with a wavefunction ψ in the positive energy continuum:

$$i\hbar\,\partial\psi/\partial t = \hat{H}_{\mathrm{D}}(t)\psi, \quad \|\psi\|^2 = 1.$$

Assume $\hat{\boldsymbol{P}}, \boldsymbol{E}, \boldsymbol{H} = O(c^0)$, when acting upon ψ, and $b_t, \boldsymbol{b} = O(c^0)$.

1.Perform an energy shift: $\psi(\boldsymbol{r}, t) = \exp\left\{-i\frac{m_e c^2}{\hbar}t\right\}\begin{pmatrix} u \\ v \end{pmatrix}$

2. In the standard representation of the Dirac matrices,

$$\begin{pmatrix} \hat{\lambda} & -c\hat{\Lambda} \\ -c\hat{\Lambda} & \hat{\lambda} + 2m_e c^2 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = 0, \qquad \hat{\Lambda} \equiv \boldsymbol{\sigma P} + b_t,\ \hat{\lambda} \equiv eA_0 + \boldsymbol{\sigma}\cdot\boldsymbol{b} + i\hbar\tfrac{\partial}{\partial t}.$$

Then $\quad v = \frac{1}{2m_e c}\left(1 - \frac{\hat{\lambda}}{2m_e c^2}\right)\hat{\Lambda}u + O(1/c^4).$

3. Quasirelativistic wavefunction: $\Phi(x) \equiv \left(1 + \frac{\hat{\Lambda}^2}{8m_e^2 c^2}\right)u.$

Quasirelativistic Hamiltonian in the External Field [2]

4. The equations of motion in the $1/c^2$ approximation (general form):

$$\left\{\hat{\lambda} - \frac{\hat{\Lambda}^2}{2m_e}\left(1 - \frac{\hat{\Lambda}^2}{4m_e^2c^2}\right) + \frac{1}{8m_e^2c^2}\left[\left[\hat{\lambda}, \hat{\Lambda}\right], \hat{\Lambda}\right]\right\}\Phi = O(1/c^3).$$

5. The quasirelativistic Hamiltonian (gives unitary evolution and $U(1)$ gauge invariance):

$$\begin{aligned} \hat{h} &= \frac{\hat{P}_b'^2}{2m_e}\left(1 - \frac{\hat{P}_b'^2}{4m_e^2c^2}\right) + \frac{e\hbar}{2m_ec}\boldsymbol{\sigma}\boldsymbol{H} - \boldsymbol{\sigma}\boldsymbol{b} - eA_0 + \\ &+ \frac{e\hbar}{4m_e^2c^2}\boldsymbol{\sigma}[\boldsymbol{E}\boldsymbol{P}] + \frac{e\hbar^2}{8m_e^2c^2}\operatorname{div}\boldsymbol{E} + \frac{\boldsymbol{\sigma}[\boldsymbol{P}[\boldsymbol{b}\boldsymbol{P}]]}{2m_e^2c^2}; \\ \hat{P}_b'^2 &\equiv \hat{\boldsymbol{P}}_b^2 - 2b_t^2, \quad \hat{\boldsymbol{P}}_b = \hat{\boldsymbol{P}} + b_t\boldsymbol{\sigma}. \end{aligned}$$

6. The results agree with [Kostelecký, Lane, 1999] and [Ferreira Jr., Moucherek, 2006].

The $1/c^2$-approximation in the Coulomb field

$eA_0 = \frac{Z\alpha\hbar c}{r}$, $\boldsymbol{A} = 0$, linear order in $b^\mu = \{cb_t, \boldsymbol{0}\}$

Unitary transformation $\Phi = \exp\left\{\frac{ib_t}{\hbar}\left(1 + \frac{Zr_e}{2r}\right)\boldsymbol{\sigma}\cdot\boldsymbol{r}\right\}\tilde{\Phi}$, $\quad r_e = \frac{\alpha\hbar}{m_e c}$, reduces the problem to that without Lorentz-violating terms. After the inverse transformation, we obtain:

$$\Phi_{nljm_j}(\boldsymbol{r}) = R_{nlj}(r)\left\{Y^l_{jm_j}(\boldsymbol{r}/r) + \frac{\varkappa b_t r}{\hbar}\left(1 + \frac{Zr_e}{2r}\right)Y^{l'}_{jm_j}(\boldsymbol{r}/r)\right\},$$

$$E = E_{nj}|_{b^0=0} = -\frac{Z\hbar R}{n^2}\left[1 + \frac{Z^2\alpha^2}{n}\left(\frac{1}{j+1/2} - \frac{3}{4n}\right)\right],$$

$$\left(\hat{\boldsymbol{l}}^2 + \frac{b_t}{\hbar}\left(1 + \frac{Zr_e}{2r}\right)\boldsymbol{\sigma}([\hat{\boldsymbol{r}}\hat{\boldsymbol{l}}] - [\hat{\boldsymbol{l}}\hat{\boldsymbol{r}}])\right)\Phi_{nljm_j} = l(l+1)\Phi_{nljm_j};$$

$$R = \alpha^2 m_e c^2/2, \quad l' \equiv 2j - l, \quad \varkappa \equiv (-1)^{\frac{l-l'+1}{2}} = \mp 1 \text{ for } l = j \pm 1/2.$$

$R_{nlj}(r)$ remain the same radial functions that recover in the $b_0 = 0$ case.

We will consider this problem in more detail within the relativistic approach.

Gauge-invariant unitary transformation [1]

$\hbar = c = 1$, $\alpha = e^2/4\pi$, quadratic approximation in $b^\mu = \{b_0, \mathbf{0}\}$.

Consider an electron in a spherically-symmetric potential $\phi(r)$, and in a weak 'external' field $A_\mu^{(\mathrm{e})}(x)$, so that $A^\mu(x) = \{\phi(r) + A_0^{(\mathrm{e})}(x), \boldsymbol{A}^{(\mathrm{e})}(x)\}$.

The transformation:

$$\psi = e^{-ib_0\hat{\Delta}}\tilde{\psi}, \quad \hat{\tilde{H}}_{\mathrm{D}} - i\frac{\partial}{\partial t} = e^{ib_0\hat{\Delta}} \cdot \left(\hat{H}_{\mathrm{D}} - i\frac{\partial}{\partial t}\right) \cdot e^{-ib_0\hat{\Delta}};$$

$$\hat{\Delta} = \boldsymbol{\Sigma}\boldsymbol{r} - \frac{i}{m_e}(\boldsymbol{\Sigma}\hat{\boldsymbol{L}} + 1)\gamma^0\gamma_5, \qquad \hat{\boldsymbol{L}} = [\boldsymbol{r}\hat{\boldsymbol{P}}] = -[\hat{\boldsymbol{P}}\boldsymbol{r}].$$

The transformed Hamiltonian:

$$\hat{\tilde{H}}_{\mathrm{D}} \approx \hat{H}_{\mathrm{D}}\,|_{b_0=0} - \frac{b_0^2}{m_e}\hat{f}\gamma^0 - \hat{\boldsymbol{d}}_A\boldsymbol{E}^{(\mathrm{e})} - \hat{\boldsymbol{\mu}}_A\boldsymbol{H}^{(\mathrm{e})} + \hat{H}_{\mathrm{int}}^{(2)}[A^{(\mathrm{e})}],$$

where $\hat{H}_{\mathrm{int}}^{(2)}[A^{(\mathrm{e})}]$ contains b_0^2-corrections to the interaction with the external field, and $\hat{f} \equiv \boldsymbol{\Sigma}\hat{\boldsymbol{l}} + 1$.

Gauge-invariant unitary transformation [2]

Corrections to the magnetic and electric dipole moment operators:

$$\hat{\boldsymbol{\mu}}_A = \frac{eb_0}{m_e}\gamma^0[\boldsymbol{\Sigma r}], \qquad \hat{\boldsymbol{d}}_A = i\gamma_5\hat{\boldsymbol{\mu}}_A.$$

These two operators have zero expectation values in any eigenstate of the nonperturbed Hamiltonian. Moreover, since $[\hat{\Delta}, \phi(r)] = 0$, $\hat{\tilde{H}}_{\mathrm{D}}$ does not include additional terms containing $\phi(r)$, and that holds exactly, i.e. in every order in b_0.

The correction $\hat{\boldsymbol{\mu}}_A$ generates a nonzero anapole magnetic moment $\boldsymbol{T}$ of the electron orbital, which interacts as $-\boldsymbol{T}\cdot\mathrm{rot}\,\boldsymbol{H}^{(\mathrm{e})}$ with the external magnetic field. In the ground state of hydrogen ($Z = 1$),

$$\boldsymbol{T} = 2er_{\mathrm{B}}^2\left(\frac{b_0}{m_e c^2}\right)m_s\boldsymbol{e}_z, \quad r_{\mathrm{B}} \text{ is the Bohr radius; } m_s = \pm 1/2.$$

Anapole moment is specific for systems with broken parity [Zeldovich,1957; Borisov, Zhukovsky, Ternov, 1989].

Removing b_0 in a spherically-symmetric potential [1]

$A^{(\mathrm{e})} = 0 \;\Rightarrow\; \hat{\Delta} = \boldsymbol{\Sigma}\cdot\boldsymbol{r} - \frac{i}{m_e}\hat{f}\gamma^0\gamma_5$; $\phi(r)$ is arbitrary!

Transformed Hamiltonian: $\hat{\tilde{H}}_{\mathrm{D}} \approx \boldsymbol{\alpha}\hat{\boldsymbol{p}} + m_e\gamma^0 - e\phi(r) - \frac{b_0^2}{m_e}\hat{f}\gamma^0$.

This Hamiltonian is P-even, then $\tilde{\psi}$ can be taken as follows:

$$\tilde{\psi}_{nljm_j}(\boldsymbol{r},t) = \begin{pmatrix} R^{(1)}_{nlj}(r)Y^{l}_{jm_j}(\boldsymbol{r}/r) \\ \varkappa R^{(2)}_{nlj}(r)Y^{l'}_{jm_j}(\boldsymbol{r}/r) \end{pmatrix},$$

$j = \frac{1}{2}, \frac{3}{2}, \ldots,\quad m_j = \overline{-j, j};\quad l = j \pm 1/2$ determines the parity $P \equiv (-1)^l$.

Moreover, $\hat{f}\gamma^0\tilde{\psi} = f\tilde{\psi}$, $f \equiv \varkappa(j+1/2) = \mp 1$ for $l = j \pm 1/2$. $\hat{\tilde{H}}_{\mathrm{D}}$ therefore formally describes an electron in the potential $\phi(r)$ without Lorentz violation, but with a splitting correction $(-b_0^2 f/m_e)$ to the energy:

$$E = \tilde{E} = E^{(0)}_{nlj} \pm (j+1/2)\frac{b_0^2}{m_e} \qquad \text{for } l = j \pm 1/2,$$

where $\{E^{(0)}_{nlj}\}$ is the spectrum in the $b_0 = 0$ case.

Removing b_0 in a spherically-symmetric potential [2]

In the initial representation: modified parity: $\hat{P}_b = e^{-2ib_0\hat{\Delta}}\hat{P}$,

eigenfunction: $\psi_{nljm_j}(\boldsymbol{r},t) = e^{-iE_{nlj}t}\exp\left\{-\frac{b_0^2}{2}\left(r^2+\frac{(j+1/2)^2}{m_e^2}\right)\right\}\times$

$$\times\begin{pmatrix} R^{(1)}_{nlj}Y^{l}_{jm_j} - b_0\varkappa\left(\frac{f}{m_e}R^{(2)}_{nlj}(r) - rR^{(1)}_{nlj}(r)\right)Y^{l'}_{jm_j} \\ \varkappa R^{(2)}_{nlj}Y^{l'}_{jm_j} - b_0\left(\frac{f}{m_e}R^{(1)}_{nlj}(r) + rR^{(2)}_{nlj}(r)\right)Y^{l}_{jm_j} \end{pmatrix},$$

$$E_{nlj} = E^{(0)}_{nlj} \pm (j+1/2)\frac{b_0^2}{m_e} \qquad \text{for } l = j\pm 1/2.$$

In the Coulomb potential ($e\phi(r) = Z\alpha/r$), the radial functions $R^{(1,2)}_{nlj}$ are well-known [Gordon; Darwin, 1928]. We don't demonstrate them for their complexity.

The degeneracy over l typical for this case is now removed, the splitting being $\lesssim 10^5$Hz (if $b_0 \lesssim 10^{-2}$eV). However, it increases linearly with j.

Approximations

The angular distribution of the dipole one-photon radiation for the transition $|i\rangle_{b_0} \to |f\rangle_{b_0}$, in the nonrelativistic and linear in b_0 approximation:

$$\frac{dw_{fi}}{d\Omega_{\boldsymbol{k}}} = \frac{k^3}{2\pi\hbar} \left| \boldsymbol{e}^{(\tau)*} \cdot \langle f|_0 \, \hat{\mathfrak{m}} \, |i\rangle_0 \right|^2; \qquad k = \frac{E - E'}{\hbar c},$$

$$\hat{\mathfrak{m}} = e\hat{\boldsymbol{r}} - \frac{ie}{2}(\boldsymbol{k} \cdot \hat{\boldsymbol{r}})\hat{\boldsymbol{r}} - \left[\frac{\boldsymbol{k}}{k} \times \hat{\boldsymbol{\mu}} \right],$$

$$\hat{\boldsymbol{\mu}} = \frac{e\hbar}{2m_e c}(\hat{\boldsymbol{l}} + \boldsymbol{\sigma}) + \hat{\boldsymbol{\mu}}_A, \quad \hat{\boldsymbol{\mu}}_A = \frac{eb_0}{m_e c^2}[\boldsymbol{\sigma}\hat{\boldsymbol{r}}].$$

$\boldsymbol{k}$ and $\boldsymbol{e}^{(\tau)}$ define the photon momentum and polarization (τ), $|i\rangle_0$ and $|f\rangle_0$ are the corresponding eigenstates in the absence of b_0; E, E' are their energies.

The Lorentz violation only modifies the electron magnetic moment operator $\hat{\boldsymbol{\mu}}$.

Transition: $\left|2p_{1/2},\, m_j = 1/2\right\rangle \to \left|1s_{1/2},\, m_j' = -1/2\right\rangle$

Distribution (summed over the photon polarizations):

$$\frac{dw}{d\Omega_{\boldsymbol{k}}} = \frac{256\alpha^3 R}{6561\pi}\left\{1 + \cos^2\theta - \frac{8b_0}{m_e c^2}\cos\theta\right\},$$

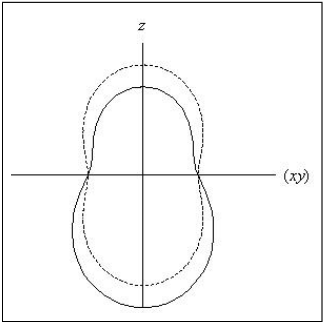
$R = \frac{\alpha^2 m_e c^2}{2\hbar^3}$ is the Rydberg constant, θ is the angle between $\boldsymbol{k}$ and the axis of the angular moment quantization (z).

- The asymmetry appears due to the parity-nonconserving radiation processes involving $\hat{\boldsymbol{\mu}}_A$ and the interference of the corresponding radiation with the electric dipole radiation.
- Within the linear order in b_0, the total radiation rates are unaffected.
- The factor of asymmetry is of the order $\frac{b_0}{m_e c^2} \lesssim 10^{-8}$
- For unpolarized atoms, a spherically-symmetric distribution is restored.

An example of a transition [2]

Transition: $\left|2p_{1/2},\, m_j = 1/2\right\rangle \to \left|1s_{1/2},\, m_j' = -1/2\right\rangle$

$$\frac{dw}{d\Omega_{\boldsymbol{k}}} = \frac{256\alpha^3 R}{6561\pi}\left\{1 + \cos^2\theta - \frac{8b_0}{m_e c^2}\cos\theta\right\},$$



To make the picture more vivid, we chose $b_0/m_e c^2 = 0.05$ $(b_0 > 0)$.
The distribution for $b_0 = 0$ is shown in dash.

A short summary

We have studied the b_0-induced Lorentz violation both in its nonrelativistic and relativistic regimes. The eigenstate problem in a spherically symmetric potential was reduced to that without Lorentz violation.

The presence of b_0 causes the violation of atomic parity, which results in the additional energy splitting over l quantum number and in the existence of the b_0-induced anapole moment of the orbital.

The radiation of polarized atoms, in turn, demonstrates a specific asymmetry.

The Model

- Quantum treatment of electron motion within extended QED with b_μ
- $b^\mu = \{b_0, \mathbf{0}\}$ in the observer frame
- Classical constant homogeneous external magnetic field $\boldsymbol{H}$, $H \ll H_c = \frac{m_e^2}{e} \approx 4.41 \cdot 10^{13}$Gauss
- Anomalous magnetic moment (AMM) $\mu \approx \frac{\alpha}{2\pi}\frac{e}{2m_e}$ since $H \ll H_c$

$$\mathcal{L} = \bar{\psi}\left(i\gamma^\mu D_\mu - m_e + \frac{\mu}{2}\sigma^{\alpha\beta}F_{\alpha\beta} - \gamma_5\gamma^\mu b_\mu\right)\psi,$$

$$\gamma_5 = -i\gamma^0\gamma^1\gamma^2\gamma^3,\ \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu,\gamma^\nu],\ D_\mu = \partial_\mu - ieA_\mu,\ e > 0\ (q_e = -e).$$

Eigenstate problem

Let $\boldsymbol{H} = H\boldsymbol{e}_z$, $H = const > 0$, $A^\mu(x) = \{0, \frac{1}{2}[\boldsymbol{H}\boldsymbol{r}]\}$ so that $A_z = 0$.

For an eigenstate $|\psi\rangle$, the wavefunction $\psi(x) = e^{-iEt}\Psi(\boldsymbol{r})$, and we come to the modified Dirac eigenstate problem:

$$\hat{H}_{\mathrm{D}}\Psi(\boldsymbol{r}) = E\Psi(\boldsymbol{r}),$$

where

$$\hat{H}_{\mathrm{D}} = \boldsymbol{\alpha}\hat{\boldsymbol{P}} + \gamma^0 m_e + \mu H \gamma^0 \Sigma_3 - b_0\gamma_5.$$

$[\hat{H}_{\mathrm{D}}, \hat{p}_z] = 0$, so let us resort to a subspace with definite p_z where

$$\Psi(\boldsymbol{r}) = \frac{e^{ip_z z}}{\sqrt{2\pi}}\phi(\rho, \varphi),$$

$\boldsymbol{r} \leftrightarrow \{\rho, \varphi, z\}$ are the cylindrical coordinates.

Transformation to effective quantities

Lorentz-violating γ_5-term in the Hamiltonian can be effectively removed with the help of a unitary transformation:

$$\phi = e^{-\frac{\vartheta}{2}\gamma^3}\tilde{\phi}, \qquad \hat{\tilde{H}}_{\mathrm{D}}\tilde{\phi} = E\tilde{\phi}; \qquad \vartheta = \arctan\frac{b_0}{\mu H},$$

$$\hat{\tilde{H}}_{\mathrm{D}} = \boldsymbol{\alpha}\hat{\tilde{\boldsymbol{P}}} + \gamma^0\tilde{m}_e + \tilde{\mu}H\gamma^0\Sigma_3, \qquad \hat{\tilde{\boldsymbol{P}}} = \{\hat{P}_1, \hat{P}_2, \tilde{p}_z\}.$$

This fact, however, does not indicate that the Lorentz violation is nonphysical. Instead, effective quantities arise in the effective Hamiltonian $\hat{\tilde{H}}_{\mathrm{D}}$:

effective AMM $\tilde{\mu}$, such that $\tilde{\mu}H = \sqrt{(\mu H)^2 + b_0^2}$,

effective mass and z-momentum $\begin{pmatrix}\tilde{m}_e \\ \tilde{p}_z\end{pmatrix} = \begin{pmatrix}\cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta\end{pmatrix}\begin{pmatrix}m_e \\ p_z\end{pmatrix}.$

Except for the change $(m_e, p_z, \mu) \to (\tilde{m}_e, \tilde{p}_z, \tilde{\mu})$, the eigenstates $\tilde{\phi}$ are the same as those found in [Ternov, Bagrov, Zhukovsky, 1966].

Energy Spectrum and Integrals of Motion

Full set of quantum numbers and the corresponding integrals of motion (in the initial representation):

quantum number	int. of motion	eigenvalue
$\epsilon = \pm 1$	$\hat{H}_D$	$E = \epsilon\sqrt{(\Pi + \tilde{\mu}H)^2 + \tilde{p}_z^2}$
$\zeta = \pm 1,$ $n = 0, 1, 2, \dots$	$\hat{\Pi} = \hat{\Pi}_\perp \cos\vartheta + \hat{\Pi}_\parallel \sin\vartheta$	$\Pi = \zeta\sqrt{\tilde{m}_e^2 + 2eHn}$
$p_z \in \mathbb{R}$	$\hat{p}_z = -i\frac{\partial}{\partial z}$	p_z
$s = 0, 1, 2, \dots$	$\hat{J}_z = -i\frac{\partial}{\partial\varphi} + \frac{\Sigma_3}{2}$	$n - s - 1/2$

Here, $\hat{\Pi}$ is the operator of electron polarization properties, which contains a transversal and a longitudinal parts:

$$\hat{\Pi}_\perp = m_e\Sigma_3 + i\gamma^0\gamma^5[\boldsymbol{\Sigma} \times \hat{\boldsymbol{P}}]_z, \qquad \hat{\Pi}_\parallel = \boldsymbol{\Sigma}\hat{\boldsymbol{P}}.$$

When $b_0 \neq 0$, the transversal polarization is no more conserved and the electron eigenstates possess a 'mixed' (partially longitudinal) polarization.

Eigenfunctions [1]

After the inverse unitary transformation is performed, the solutions for Ψ have the form:

$$\Psi(\rho,\varphi,z) = \frac{e^{ip_z z}}{\sqrt{2\pi}} \frac{e^{i(n-s-1/2)\varphi}}{\sqrt{2\pi}} \sqrt{eH} \begin{pmatrix} c_1 \, e^{-i\varphi/2} \, I_{n-1,s}(\chi) \\ ic_2 \, e^{i\varphi/2} \, I_{n,s}(\chi) \\ c_3 \, e^{-i\varphi/2} \, I_{n-1,s}(\chi) \\ ic_4 \, e^{i\varphi/2} \, I_{n,s}(\chi) \end{pmatrix}, \quad \chi \equiv \frac{eH}{2}\rho^2,$$

Laguerre functions : $I_{n,s}(\chi) = \sqrt{\frac{s!}{n!}}\, e^{-\chi/2}\chi^{(n-s)/2} L_s^{n-s}(\chi),$

Laguerre polynomials : $L_s^l(\chi) = \frac{1}{s!} e^{\chi}\chi^{-l} \frac{d^s}{d\chi^s}\left(e^{-\chi}\chi^{s+l}\right).$

Eigenfunctions [2]

State-dependent spin coefficients $\{c_\mathfrak{a}\}$ for the normalized eigenfunctions:

$$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} = \frac{1}{2\sqrt{2}} \begin{pmatrix} A(P\alpha + \epsilon\zeta Q\beta) \\ -\zeta B(P\alpha - \epsilon\zeta Q\beta) \\ A(P\beta - \epsilon\zeta Q\alpha) \\ \zeta B(P\beta + \epsilon\zeta Q\alpha) \end{pmatrix},$$

$$A = \sqrt{1 + \frac{\tilde{m}_e}{\Pi}}, \quad B = \sqrt{1 - \frac{\tilde{m}_e}{\Pi}}, \quad P = \sqrt{1 + \frac{\tilde{p}_z}{E}}, \quad Q = \sqrt{1 - \frac{\tilde{p}_z}{E}},$$

$$\alpha = \cos\frac{\vartheta}{2} - \sin\frac{\vartheta}{2}, \quad \beta = \cos\frac{\vartheta}{2} + \sin\frac{\vartheta}{2}.$$

General theory

Total radiation power in the leading order in e^2 for the spontaneous transition from an eigenstate $|i\rangle$ to $|f\rangle$ with energies E and E', respectively:

$$W = \frac{e^2}{2\pi} \sum_{\tau=\sigma,\pi} \int d^3k \; \delta(E' - E - k) \left| \boldsymbol{e}^{(\tau)^*} \cdot \langle f | \boldsymbol{\alpha} e^{-i\boldsymbol{kr}} | i \rangle \right|^2 ,$$

$\mathbf{k}$ is the photon wave vector,
$\boldsymbol{e}^{(\tau)}$ is the vector describing the photon polarization (τ).

The ultimate goal is to obtain the spectral-angular radiation distribution (per unit length along z) summarized over all final states $|f\rangle$ with energies $E' < E$ (i.e. over all allowed transitions from the fixed initial state).

Used approximations

- ultrarelativistic electron: $m_e/E \equiv \lambda \ll 1$
- 'weak' magnetic field: $H \ll H_c$ (taken together, these two assumptions imply that $n \gg 1$, that corresponds to a quasi-classical electron motion)
- small electron AMM: $\tilde{\mu}H \ll E$; we assume $\tilde{\mu}H/E \approx 0$ (this is quite natural since in a typical laboratory $E \sim 1$ GeV, $H \sim 10^4$ Gauss, and $\tilde{\mu}H/E \ll m_e/E, \vartheta$ provided that $b \gg 10^{-20}$ eV)
- however, no assumption of smallness of ϑ

Calculation of the radiation distribution [1]

In the chosen zero-order approximation in $\tilde{\mu}H$, the presence of b_0 and μ affects only the spin coefficients of the eigenfunctions, i.e. the electron polarization:

$$\hat{H}_{\mathrm{D}} \to \boldsymbol{\alpha}\hat{\boldsymbol{P}} + m_e\gamma^0, \quad E \to \sqrt{m_e^2 + 2eHn + p_z^2}.$$

In this case, a conventional quasi-classical theory of synchrotron radiation can be applied (within the assumption that $p_z = 0$). However, initial and final spin polarization states should be treated as mixed ('longitudinal-transversal'), since ϑ is finite.

Calculation of the radiation distribution [2]

Total radiation power in a spherical coordinate system with the z-axis oriented along $\boldsymbol{H}$:

$$W = W_{cl} \int dy \, \sin\theta \, d\theta \, \frac{27}{64\pi^2} \, \frac{y^2}{\lambda^5 (1+\xi y)^4} \, \Phi, \quad W_{cl} = \frac{8}{27} (em\xi)^2,$$

where y is a dimensionless variable defining the radiation frequency, ξ is a parameter characterizing the role of quantum effects (in the quasi-classical theory of synchrotron radiation):

$$\frac{k}{E} = \frac{\xi y}{1+\xi y}, \quad 0 < y < +\infty; \quad \xi = \frac{3}{2} \frac{H}{H_c} \frac{1}{\lambda}.$$

The signature of the Lorentz violation is contained in Φ, which depends on the spin coefficients.

Explicit form of spectral-angular distribution [1]

Asymptotic expressions for Φ, for the σ- and π-components of linear polarization of the radiation without a spin-flip ($\zeta' = \zeta$):

$$\Phi_\sigma^+ = \hat{\lambda}^2 \Big((2+\xi y)\hat{\lambda}\, K_{2/3}(z) - \zeta(\xi y)(\lambda \cos\vartheta - \cos\theta \sin\vartheta) K_{1/3}(z) \Big)^2,$$

$$\Phi_\pi^+ = \hat{\lambda}^2 \Big((2+\xi y)\cos\theta\, K_{1/3}(z) + \zeta(\xi y)\sin\vartheta\, \hat{\lambda}\, K_{2/3}(z) \Big)^2,$$

$$z = \frac{y}{2}\left(\hat{\lambda}/\lambda\right)^3, \quad \hat{\lambda}^2 = \cos^2\theta + \lambda^2 \sin^2\theta,$$

$K_\nu(z)$ are the Macdonald cylindrical functions.

Limits: $\vartheta = 0,\ \frac{\pi}{2}$ (correspond to $\Pi = \Pi_\perp,\ \Pi_\parallel$).

Explicit form of spectral-angular distribution [2]

Asymptotic expressions for Φ, for the σ- and π-components of linear polarization of the radiation with a spin-flip ($\zeta' = -\zeta$):

$$\Phi_\sigma^- = \hat{\lambda}^2 \Big((\xi y)(\cos\theta \cos\vartheta + \lambda \sin\vartheta) K_{1/3}(z) \Big)^2,$$

$$\Phi_\pi^- = \hat{\lambda}^2 \Big((\xi y) \big(\cos\vartheta \, \hat{\lambda} \, K_{2/3}(z) + \zeta \, \lambda \, K_{1/3}(z) \big) \Big)^2,$$

$$z = \frac{y}{2} \left(\hat{\lambda}/\lambda \right)^3, \quad \hat{\lambda}^2 = \cos^2\theta + \lambda^2 \sin^2\theta,$$

$K_\nu(z)$ are the Macdonald cylindrical functions.

Limits: $\vartheta = 0, \frac{\pi}{2}$ (correspond to $\Pi = \Pi_\perp, \Pi_\|$).

Explicit form of spectral-angular distribution [3]

Dominating effect: asymmetry of synchrotron radiation relative to the electron orbit plane. This asymmetry is absent when $\Pi = \Pi_\perp$ and appears due a longitudinal admixture to the electron polarization. In other words, it exists due to a non-conservation of the conventional integral of motion $\Pi_\perp$ and its modification stemming from the violation of Lorentz invariance:

$$\Pi_\perp \to \Pi_\perp \cos\vartheta + \Pi_{||} \sin\vartheta,$$

The asymmetry also maintains for unpolarized electrons.

Explicit form of spectral-angular distribution [4]

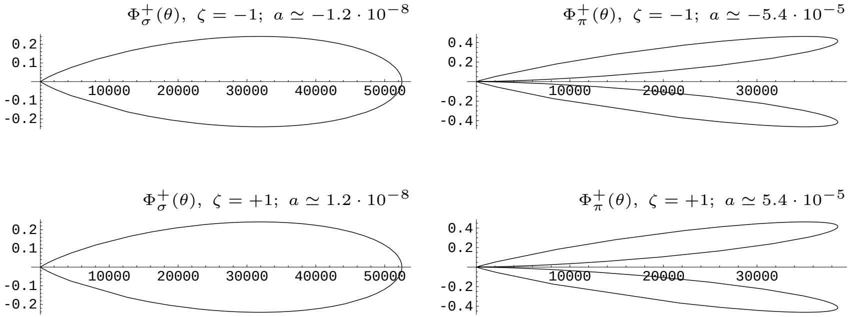


Рис.: Normalized angular distribution $\Phi^{+}_{i}(\theta)$ for $k = 1$ MeV, $\zeta = \pm 1$ in the case $H = 10^4$ Gauss, $E = 1$GeV, $b_0 \sim 10^{-9}$ eV, $\vartheta = 10^{-3}$.

Factor of asymmetry: $a = \frac{w_{up} - w_{down}}{w_{up} + w_{down}}$, where $w_{up} = \int_0^{\frac{\pi}{2}} \sin\theta \, d\theta \, \Phi$, $w_{down} = \int_{\frac{\pi}{2}}^{\pi} \sin\theta \, d\theta \, \Phi$.

Explicit form of spectral-angular distribution [5]

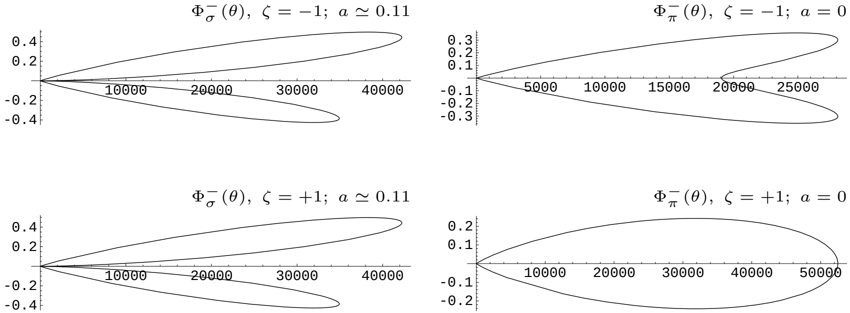


Рис.: Normalized angular distribution $\Phi_i^-(\theta)$ for $k = 1$ MeV, $\zeta = \pm 1$ in the case $H = 10^4$ Gauss, $E = 1$GeV, $b_0 \sim 10^{-9}$ eV, $\vartheta = 10^{-3}$.

Factor of asymmetry: $a = \frac{w_{up} - w_{down}}{w_{up} + w_{down}}$, where $w_{up} = \int_0^{\frac{\pi}{2}} \sin\theta \, d\theta \, \Phi$, $w_{down} = \int_{\frac{\pi}{2}}^{\pi} \sin\theta \, d\theta \, \Phi$.

Obtained constraints on b_0

- Experimental evidence confirms the 'transversality' of electron states, therefore we can conclude that $\vartheta \ll 1$. Taken in the laboratory conditions ($E \sim 1$GeV, $H \sim 10^4$ Gauss), this gives:

$$|b_0| \ll \mu H \sim 10^{-6}\,\text{eV}$$

- If *reliable* data would be obtained for the observation of the radiation of the electron anomalous magnetic moment, demonstrating no signature of $\vartheta \neq 0$, that would imply $\vartheta \lesssim \tilde{\mu} H/E$, and thus,

$$|b_0| \lesssim 10^{-20}\,\text{eV}.$$

A short summary

By solving the eigenstate problem, we have found that the nonperturbative interaction between the electron AMM and the Lorentz-violating condensate b_0 can affect both the spectrum and the polarization properties of the electron, the latter acquiring a longitudinal contribution.

This effect, causes, in turn, a specific asymmetry of the synchrotron radiation of an ultrarelativistic electron. For a polarized electron, the asymmetry becomes observable even for minuscule values of b_0.

Using the predicted radiation distribution, we have obtained the new stringent constraints on b_0.

The investigation of the two systems discussed above showed that the Lorentz-violating interaction with b_0 expresses itself in:

- the modified electron spectrum and integrals of motion (parity or polarization),
- the nonpertubative interaction with its AMM,
- the asymmetry of its radiation, especially for polarized particles,
- the contribution to the anapole moment of the electron orbital.

The results obtained seem promising in suggesting new experiments, and even now gave us new stringent constraints on b_0.

References:

[1] *O.G.Kharlanov and V.Ch.Zhukovsky*, `arXiv:0705.3306 (hep-th)`

[2] *I.E.Frolov and V.Ch.Zhukovsky*, J.Phys.A **40**, 10625-10640 (2007), `arXiv:0705.0882 (hep-th)`

Thanks for your attention!