Triangle anomaly and radiatively-induced Lorentz and CPT violation in electrodynamics

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Main points

- New regularization based on the Mellin representation of Feynman amplitudes
- Connection between diagrams of electrodynamics with vector and axial vertexes and scalar theory ϕ^3
- Application to extended electrodynamics with Lorentz and CPT violation

Feynman rules for the triangle diagram

$$T_{\kappa\lambda\mu}(p_1, p_2) = S_{\kappa\lambda\mu}(p_1, p_2) + S_{\lambda\kappa\mu}(p_2, p_1)$$

$$S_{\kappa\lambda\mu}(p_1, p_2) =$$
$$-i\int \frac{d^4k}{(2\pi)^4}\, tr\left(\gamma_\kappa \frac{i(\not{k} - \not{p}_1 + m)}{(k-p_1)^2 - m^2}\gamma_\mu\gamma_5 \frac{i(\not{k} + \not{p}_2 + m)}{(k+p_2)^2 - m^2}\gamma_\lambda \frac{i(\not{k} + m)}{k^2 - m^2}\right)$$

Exponential representation

$$S_{\kappa\lambda\mu}(p_1, p_2) =$$

$$\frac{1}{4\pi^2 i}\int\limits_0^{+\infty} d\alpha_1 d\alpha_2 d\alpha_3 \,\frac{e^{i\frac{D}{C}}}{C^2}\left\{m^2\varepsilon_{\lambda\kappa\alpha\mu}\Big[(1-A_1)p_1^\alpha + (A_2-1)p_2^\alpha\Big] + \right.$$

$$\frac{i}{C}\,\varepsilon_{\lambda\kappa\alpha\mu}\left[(3A_1-1)p_1^\alpha + (1-3A_2)p_2^\alpha\right] -$$

$$\Big[\varepsilon_{\lambda\kappa\alpha\mu}\left((A_1-1)p_1^\alpha - A_2\,p_2^\alpha\right)\left(A_1^2 p_1^2 - A_1(1-2A_2)(p_1 p_2) - (1-A_2)\,A_2\,p_2^2\right) +$$

$$p_1^\alpha p_2^\beta\big(-\varepsilon_{\mu\alpha\lambda\beta}A_1((A_1-1)p_{1\kappa} - A_2 p_{2\kappa}) - \varepsilon_{\kappa\alpha\lambda\mu}A_1((A_1-1)p_{1\mu} - A_2 p_{2\mu}) +$$

$$\left.\varepsilon_{\kappa\alpha\mu\beta}A_2(2A_1 p_{1\lambda} - (1-2A_2)p_{2\lambda}) + \varepsilon_{\kappa\alpha\mu\beta}(A_1-1)(A_1 p_{1\lambda} - A_2 p_{2\lambda})\big)\Big]\right\}$$

Parameters of the exponential representation

$$C = \alpha_1 + \alpha_2 + \alpha_3,\ A_1 = \frac{\alpha_1}{C},\ A_2 = \frac{\alpha_2}{C},$$

$$D = \alpha_1(\alpha_2 + \alpha_3)p_1^2 + \alpha_2(\alpha_1 + \alpha_3)p_2^2 + 2\alpha_1\alpha_2(p_1 p_2) - m^2 C^2.$$

Mellin transformation

$$\nu = \frac{m'^2}{m_0^2} \quad z = \frac{\delta m^2}{m_0^2} \quad \nu + z = \frac{m^2}{m_0^2}$$

$$S_{\kappa\lambda\mu}(s) \; = \; \int_0^{+\infty} dz\, z^{s-1} S_{\kappa\lambda\mu}(z) =$$

$$= \frac{1}{4\pi^2 i} \int_0^{+\infty} d\alpha_1 d\alpha_2 d\alpha_3 \, \frac{exp\left\{\, i\frac{D}{C}\,\right\}}{C^2} \times$$

$$\times \left\{ \Gamma(s)(iC)^{-s} \mathcal{A}_{\kappa\lambda\mu} \; + \; \Gamma(s+1)(iC)^{-s-1} \mathcal{B}_{\kappa\lambda\mu} \right\}$$

Regularization

- Scale transformation of parameters of the exponential representation

$$\alpha_i \; \rightarrow \; \rho\alpha_i,$$

$$\int_0^{+\infty} d\alpha_1 d\alpha_2 d\alpha_3 \; \rightarrow \; \int_0^{+\infty} d\rho \int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \, \rho^2 \delta(1-\alpha_1-\alpha_2-\alpha_3)$$

- Integration over ρ
- Inverse Mellin transformation

$$S_{\kappa\lambda\mu}(z)_{reg} \; = \; \frac{1}{2\pi i} \int_{s_0-i\infty}^{s_0+i\infty} ds \, z^{-s} S_{\kappa\lambda\mu}(s)$$

Resulting expression for the triangle anomaly

$$
S_{\kappa\lambda\mu}(p_1, p_2) \;=\; \pi^2 \int\limits_0^1 dxdy\, \theta(1-x-y) \times
$$

$$
\left\{ \left\{ (p_1 - p_2)^\alpha \left[-1 + \frac{z+\nu}{-D+z+\nu} \right] - (p_1+p_2)^\alpha \, \frac{x^2 p_1^2 - y^2 p_2^2}{-D+z+\nu} \right\} \varepsilon_{\mu\kappa\lambda\alpha} + \right.
$$

$$
\left. \frac{p_{1\alpha} p_{2\beta}}{-D+z+\nu} \left\{ 2y\left(x p_{1\lambda} + (1-y) p_{2\lambda} \right) \varepsilon_{\mu\kappa\alpha\beta} - 2x \left(y p_{2\kappa} + (1-x) p_{1\kappa} \right) \varepsilon_{\mu\lambda\alpha\beta} \right\} \right.
$$

Regularization based on the Mellin representation

- Simple computational structure useful for analytical computer calculation
- Anomalies don't cause any additional complications
- We like it

Connection between spinor and scalar diagrams

$$
\begin{aligned}
S_{\kappa\lambda\mu}(p_1, p_2) \; = & \\
& \pi^2 \left\{ \left[\varepsilon_{\mu\kappa\lambda\alpha}\, p_2^\alpha p_1^\beta \frac{\partial}{\partial p_1^\beta} - \varepsilon_{\mu\lambda\alpha\beta}\, p_2^\alpha p_1^\beta \frac{\partial}{\partial p_1^\kappa} \right] - \right. \\
& \left. - \left[\varepsilon_{\mu\kappa\lambda\alpha}\, p_1^\alpha p_2^\beta \frac{\partial}{\partial p_2^\beta} - \varepsilon_{\mu\kappa\alpha\beta}\, p_2^\alpha p_1^\beta \frac{\partial}{\partial p_1^\lambda} \right] \right\} \times \\
\times \int_0^1 dxdy\, \theta(1-x-y) \ln \left\{ m^2 - p_1^2 x(1-x) - p_2^2 y(1-y) - 2xy(p_1 p_2) \right\} &
\end{aligned}
$$

Fermionic lagrangian of extended electrodynamics with an axial vector term

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - e\not{A} - m - \not{b}\gamma_5)\psi$$

Summary

We used a new regularization based on the Mellin transformation, revealed the connection between diagrams of renormalizable spinor theory and superrenormalizable scalar theory and applied this construction to the extended electrodynamics with Lorenz and CPT violation. Although we presented these results for the triangle anomaly diagram, this approach is much more general.