13th Lomonosov conference
on Elementary Particles Physics
Moscow, 2007

K.V.Stepanyantz

Moscow State University
Department of Theoretical Physics

Application of higher covariant derivative regularization to calculations of quantum correction in $N=1$ supersymmetric theories

$N = 1$ supersymmetric theories

N=1 supersymmetric Yang-Mills theory with matter is described by the action

$$S = \frac{1}{2e^2} \mathrm{Re}\,\mathrm{tr} \int d^4x\, d^2\theta\, W_a C^{ab} W_b + \frac{1}{4} \int d^4x\, d^4\theta\, \Big(\phi^+ e^{2V} \phi + \\ + \tilde{\phi}^+ e^{-2V^t} \tilde{\phi}\Big) + \Big(\frac{1}{2} m \int d^4x\, d^2\theta\, \tilde{\phi}^t\, \phi + h.c.\Big),$$

where ϕ and $\tilde{\phi}$ are chiral scalar matter superfields, V is a real scalar gauge superfield, and the supersymmetric gauge field stress tensor is given by

$$W_a = \frac{1}{32} \bar{D}(1-\gamma_5) D \Big[e^{-2V}(1+\gamma_5) D_a e^{2V}\Big].$$

The action is invariant under the gauge transformations

$$e^{2V} \to e^{i\Lambda^+} e^{2V} e^{-i\Lambda}; \qquad \phi \to e^{i\Lambda}\phi; \qquad \tilde{\phi} \to e^{-i\Lambda^t}\tilde{\phi}.$$

We investigate quantum corrections to the two-point Green function of the gauge superfield and to some correlators of composite operators exactly to all orders of the perturbation theory.

Quantization

We use the background field method: $e^{2V} \rightarrow e^{2V'} \equiv e^{\mathbf{\Omega}^+} e^{2V} e^{\mathbf{\Omega}}$, where $\mathbf{\Omega}$ is a background field. Using the background gauge invariance it is possible to set $\mathbf{\Omega} = \mathbf{\Omega}^+ = \mathbf{V}$. Background covariant derivatives are given by

$$\boldsymbol{D} \equiv e^{-\mathbf{\Omega}^+} \frac{1}{2}(1 + \gamma_5) D e^{\mathbf{\Omega}^+}; \qquad \bar{\boldsymbol{D}} \equiv e^{\mathbf{\Omega}} \frac{1}{2}(1 - \gamma_5) D e^{-\mathbf{\Omega}};$$

$$\boldsymbol{D}_\mu \equiv -\frac{i}{4} (C\gamma^\mu)^{ab} \Big\{ \boldsymbol{D}_a, \bar{\boldsymbol{D}}_b \Big\}.$$

The gauge is fixed by adding the following term:

$$S_{gf} = -\frac{1}{32e^2} \operatorname{tr} \int d^4x \, d^4\theta \, \Big(V \boldsymbol{D}^2 \bar{\boldsymbol{D}}^2 V + V \bar{\boldsymbol{D}}^2 \boldsymbol{D}^2 V \Big).$$

The corresponding ghost Lagrangian is

$$S_c = i \operatorname{tr} \int d^4x \, d^4\theta \, \Big\{ (\bar{c} + \bar{c}^+) V \Big[(c + c^+) + \operatorname{cth} V (c - c^+) \Big] \Big\}.$$

Quantization

Also it is necessary to add the Nielsen-Kallosh ghosts

$$S_B = \frac{1}{4e^2}\mathrm{tr}\int d^4x\, d^4\theta\, B^+ e^{\mathbf{\Omega}^+} e^{\mathbf{\Omega}}\, B.$$

In order to calculate quantum corrections we also introduce additional sources

$$S_{\phi_0} = \frac{1}{4}\int d^4x\, d^4\theta \left(\phi_0^+ e^{2V}\phi + \tilde\phi_0^+ e^{-2V^t}\tilde\phi\right) + h.c.$$

where ϕ_0 and $\tilde\phi_0$ are not chiral superfields.

Differentiation with respect to additional sources allows calculating vacuum expectation value of some composite operators.

Higher derivative regularization

To regularize the theory we use the higher covariant derivative regularization.

A.A.Slavnov, Theor.Math.Phys. 23, (1975), 3; P.West, Nucl.Phys. B268, (1986), 113.

Higher derivative regularization

We add to the action the term

$$S_\Lambda = \frac{1}{2e^2}\,\mathrm{tr}\,\mathrm{Re}\int d^4x\, d^4\theta\, V\,\frac{(\boldsymbol{D}_\mu^2)^{n+1}}{\Lambda^{2n}} V.$$

Then divergences remain only in the one-loop approximation. In order to regularize them, it is necessary to introduce Pauli-Villars determinants into the generating functional

$$Z[J,\boldsymbol{\Omega}] = \int D\mu \prod_i \Big(\det PV(V,\mathbf{V},M_i)\Big)^{c_i} \times$$
$$\times \exp\Big\{iS + iS_\Lambda + iS_{gf} + iS_B + iS_{gh} + \mathrm{Sources}\Big\},$$

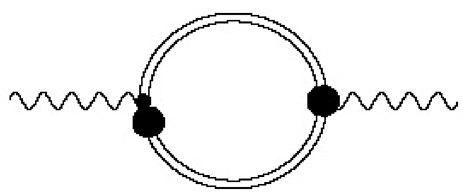
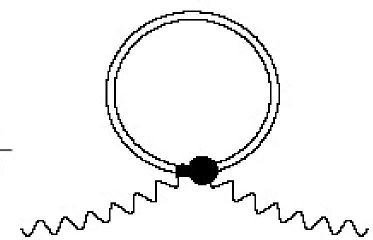
where the coefficients satisfy the conditions $\sum\limits_i c_i = 1; \qquad \sum\limits_i c_i M_i^2 = 0.$

The regularization breaks the BRST-invariance. Therefore, it is necessary to use a special subtraction scheme, which restores the Slavnov-Taylor identities.

A.A.Slavnov, Phys.Lett. B518, (2001), 195; Theor.Math.Phys. 130, (2002), 1;
A.A.Slavnov, K.Stepanyantz, Theor.Math.Phys. 135, (2003), 673; 139, (2004), 599.

Calculating matter contribution by Schwinger-Dyson equations and Slavnov-Taylor identities

Schwinger-Dyson equation for the two-point Green function of the gauge field can be graphically presented as

$$\frac{\delta^2\Gamma}{\delta \mathbf{V}_x \delta \mathbf{V}_y} =$$  $+$  $+$ gauge contribution

Here (in the massless case for simplicity) the vertexes are given by

$$\frac{\delta^2\Gamma}{\delta\phi^+_{0x}\delta\phi_y} = -\frac{1}{8}G(\partial^2)\bar{D}^2_x\delta^8_{xy}; \qquad \frac{\delta^3\Gamma}{\delta\mathbf{V}^a_x\delta\phi^+_z\delta\phi_w}; \qquad \frac{\delta^3\Gamma}{\delta\mathbf{V}^a_x\delta\phi^+_{0z}\delta\phi_w},$$

and the propagator is

$$\frac{\delta^2\Gamma}{\delta\phi^+_x\delta\phi_y} = -\frac{D^2_x\bar{D}^2_x}{4\partial^2 G(\partial^2)}\delta^8_{xy}.$$

Calculating matter contribution by Schwinger-Dyson equations and Slavnov-Taylor identities

Expressions for vertexes can be found by solving Slavnov-Taylor identities:

$$\frac{\delta^3\Gamma}{\delta \mathbf{V}_y^a \delta\phi_{0z}^+ \delta\phi_x}\Bigg|_{p=0} = eT^a\Big[-2F\partial^2\Pi_{1/2y}\Big(\bar{D}_y^2\delta_{xy}^8\delta_{yz}^8\Big) + \frac{1}{8} f D^b C_{bc}\bar{D}_y^2$$

$$\times\Big(\bar{D}_y^2\delta_{xy}^8 D_y^c\delta_{yz}^8\Big) + \frac{i}{16}\partial_x^\mu G'\bar{D}\gamma^\mu\gamma_5 D_y\Big(\bar{D}_y^2\delta_{xy}^8\delta_{yz}^8\Big) - \frac{1}{4}G\bar{D}_y^2\delta_{xy}^8\delta_{yz}^8\Big];$$

$$\frac{\delta^3\Gamma}{\delta \mathbf{V}_y^a \delta\phi_z^+ \delta\phi_x}\Bigg|_{p=0} = eT^a\Big[F\partial^2\Pi_{1/2y}\Big(\bar{D}_y^2\delta_{xy}^8 D_y^2\delta_{yz}^8\Big) -$$

$$-\frac{i}{32}\partial_x^\mu G'\bar{D}\gamma^\mu\gamma_5 D_y\Big(\bar{D}_y^2\delta_{xy}^8 D_y^2\delta_{yz}^8\Big) + \frac{1}{8}G\bar{D}_y^2\delta_{xy}^8 D_y^2\delta_{yz}^8\Big].$$

where all functions depend on ∂_x^2.

Both vertexes are defined by the same diagrams, but in the first case one of the external lines is not chiral.

Exact beta-function and new identity

Substituting the solution of Slavnov-Taylor identities to the Schwinger-Dyson equations (in the massless case) we find

$$\Gamma = -\frac{1}{8\pi}\mathrm{tr}\int d^4\theta\,\frac{d^4p}{(2\pi)^4}\mathbf{V}(-p)\,\partial^2\Pi_{1/2}\mathbf{V}(p)\,d^{-1}(\alpha,\mu/p),$$

Gell-Mann-Low function is then given by

$$\beta\Big(d(\alpha,\mu/p)\Big) = \frac{\partial}{\partial\ln p}d(\alpha,\mu/p).$$

We obtained that

$$\frac{d}{d\ln\Lambda}d^{-1}\Big|_{p=0} = -8\pi C(R)\frac{d}{d\ln\Lambda}\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2}\left(\frac{d}{dq^2}\ln(q^2G^2) - \frac{16f}{G}\right)$$

$-(PV) +$ gauge contribution.

(We did not calculate contribution of the gauge field and did not write contribution of Pauli-Villars fields).

Exact beta-function and new identity

Then the Gell-Mann-Low function differs from NSVZ beta-function

$$\beta(\alpha) = -\frac{\alpha^2\Big(3C_2 - 2C(R)(1-\gamma(\alpha))\Big)}{2\pi(1-C_2\alpha/2\pi)}$$

in the substitution

$$\gamma(\alpha) \to \gamma(\alpha) + \lim_{p\to 0}\frac{16f(p^2)}{p^2 G(p^2)}.$$

HOWEVER, explicit calculations with the higher derivative regularization in the three- and four-loop approximations

A.Soloshenko, K.Stepanyantz, Theor.Math.Phys. 140, (2004), 1264;
A.Pimenov, K.Stepanyantz, Theor.Math.Phys. 147, (2006), 687.

show that all in integrals, defining the two-point Green function of the gauge field in the limit $p\to 0$, has integrands which are total derivatives.

Exact beta-function and new identity

This leads to the following identity (massless case for simplicity):

$$\frac{d}{d\ln\Lambda}\int\frac{d^4q}{(2\pi)^4}\,\frac{f}{q^2G}=0.$$

This identity is also valid in non-Abelian theory

K.Stepanyantz, Theor.Math.Phys. 150, (2007), 377.

It is nontrivial only in the three-loops.

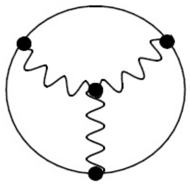
In the massive case this identity is much more complicated, but can be written in the simple functional form:

$$\frac{d}{d\ln\Lambda}\int d^8x\,d^8y\,\mathbf{V}_y D^a\mathbf{V}_x\frac{D_{a,z}\bar{D}_z^2}{\partial^2}\frac{\delta^3\Gamma}{\delta j_z^+\delta\mathbf{V}_y\delta\phi_{0,x}^+}\Bigg|_{x=z,p=0}=0,$$

where the sources are assumed to be expressed in terms of fields.

Three-loop verification of new identity in non-Abelian theory

In the non-Abelian case new identity and factorization of the integrands to total derivatives can be verified only for special groups of diagrams

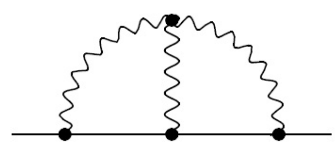


A.Pimenov, K.Stepanyantz, in preparation.

because a way of proving in the Abelian case was sketched in

K.Stepanyantz, Theor.Math.Phys. 146, (2006), 321.

The corresponding function f is defined by diagrams of the type

non-chiral ϕ_0 $\longrightarrow$  $\longleftarrow$ chiral ϕ

It is also necessary to attach a line of the background gauge field to the line of the matter superfield by all possible ways.

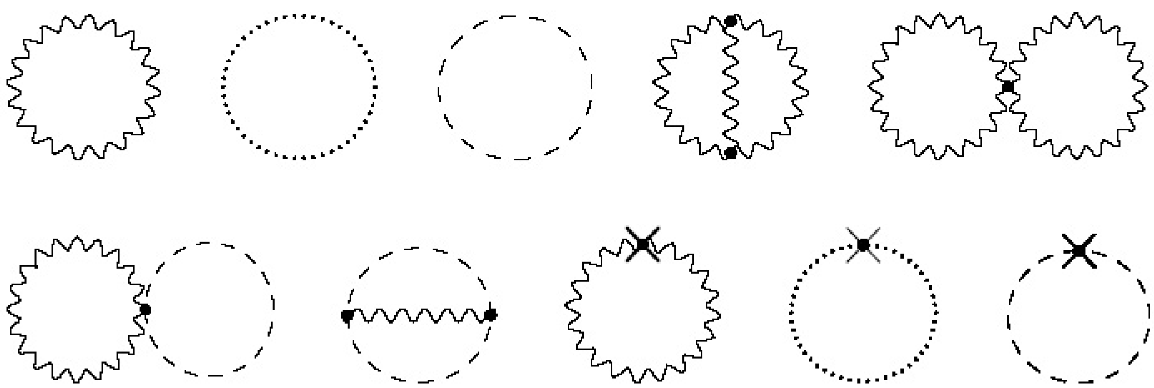
The result is again an integral of a total derivative! Therefore, the new identity is also valid in this case.

Structure of quantum correction in $N = 1$ supersymmetric Yang-Mills theory

Are integrands, defining the two-loop function of N=1 SYM, also reduced to total derivatives with the higher covariant derivative regularization?

A.Pimenov, K.Stepanyantz, hep-th/0707.4006.

Two-loop Gell-Mann-Low function of N=1 SYM (without matter) is defined by the diagrams



Usual diagrams are obtained by adding two external lines of the background field by all possible ways.

Structure of quantum correction in $N=1$ supersymmetric Yang-Mills theory

In the limit $p\to 0$ two-loop contribution is $d^{-1}(\alpha,\Lambda/p) = d_2 \ln\dfrac{\Lambda}{p} + \mbox{const}$, where

$$d_2 = 48\pi^2\alpha_0\,\frac{d}{d\ln\Lambda}\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2}\,\frac{d}{dk^2}\int\frac{d^4q}{(2\pi)^4}\Big(q^2(1+q^{2n}/\Lambda^{2n})\Big)^{-1}$$
$$\times\Big((q+k)^2(1+(q+k)^{2n}/\Lambda^{2n})\Big)^{-1}\Big[2(n+1)\Big(1+k^{2n}/\Lambda^{2n}\Big)^{-1} -$$
$$-2n\Big(1+k^{2n}/\Lambda^{2n}\Big)^{-2}\Big].$$

The integrand is again a total derivative in the four dimensional spherical coordinates! Really,

$$\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2}\,\frac{d}{dk^2}f(k^2) = \frac{1}{16\pi^2}\Big(f(k^2=\infty) - f(k^2=0)\Big).$$

Structure of quantum correction in $N = 1$ supersymmetric Yang-Mills theory

The corresponding two-loop Gell-Mann-Low function coincides with the well known expression

$$\beta(\alpha) = -\frac{3C_2\alpha^2}{2\pi} - \frac{3\alpha^3 C_2^2}{(2\pi)^2} + O(\alpha^4).$$

Therefore, factorization of integrands to total derivatives seems to be a general feature of all supersymmetric theories, although the reason is so far unclear.

Also it is interesting to note that with the higher derivative regularization there are divergences only in the one-loop approximation.

(This is similar to M.Shifman, A.Vainstein, Nucl.Phys. B277, (1986), 456.)

Conclusion and open questions

- ✓ With the higher derivtive regularizations integrals, defining the two-point Green function of the gauge field in the limit $p \longrightarrow 0$, can be easily taken, because the integrands are total derivatives. It is a general feature of $N=1$ supersymmetric theories. Why it is so?

- ✓ There is a new identity in both Abelian and non-Abelian $N=1$ supersymmetric theories the matter superfields, for example,

$$\frac{d}{d\ln\Lambda}\int\frac{d^4q}{(2\pi)^4}\,\frac{f}{q^2G}=0.$$

  It is not a consequence of the supersymmetric or gauge Slavnov-Taylor identities. The corresponding terms in the effective action are invariant under rescaling. What symmetry leads to this identity?

- ✓ New identity is nontrivial starting from the three-loop approximation.