CONSTRUCTION OF EXACT SOLUTIONS IN TWO-FIELDS MODELS

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based on
I.Ya. Aref'eva, A.S. Koshelev, S.V.
Phys. Rev. **D72** (2005) 064017; astro-ph/0507067
S.V., astro-ph/0612487

To specify different types of cosmic fluids one usually uses a phenomenological relation between the pressure density p and the energy density ϱ, corresponding to each component of fluid

$$p = w\varrho,$$

where w is the state parameter.

Experiments: $w_{DE} = -1 \pm 0.2.$

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$$w_{DE} = -1 - \frac{2}{3}\frac{\dot{H}}{H^2}. \tag{1}$$

H is the Hubble parameter.

- The first case, $w_{DE} > -1$, is achieved in quintessence models.
- The second case, $w_{DE} = -1$, is realized by means of the cosmological constant.
- The third case, $w_{DE} < -1$ can be realized due to a scalar field with a ghost (phantom) kinetic term.

Two-fields models with the crossing of the cosmological constant barrier include one phantom scalar field and one usual scalar field.

We consider the case $w_{DE} < -1$.

All natural energy conditions are violated and there are problems of instability at classical and quantum levels. A possible way to evade the instability problem for models with $w < -1$ is to yield a phantom model as an effective one, arising from a more fundamental theory.

A model with higher derivatives such as $\phi e^{-\Box}\phi$ i in the simplest approximation: $\phi e^{-\Box}\phi \simeq \phi^2 - \phi\Box\phi$ gives a kinetic term with a ghost sign.

Such a possibility does appear in the string field theory framework:

I.Ya. Aref'eva, astro-ph/0410443, 2004.

Nonlocal cosmological models:

I.Ya. Aref'eva, A.S. Koshelev, 2006; A.S. Koshelev, 2007;

I.Ya. Aref'eva, L.V. Joukovskaya, S.V., 2007; L.V. Joukovskaya, 2007

1 TWO-FIELDS MODEL

We consider a model of Einstein gravity interacting with a single phantom scalar field ϕ and one standard scalar field ξ in the spatially flat Friedmann Universe:

$$ds^2 = -\,dt^2 + a^2(t)\left(dx_1^2 + dx_2^2 + dx_3^2\right).$$

The equations of motion are as follows ($m_p^2 = constant$):

$$2\dot{H} = \frac{1}{m_p^2}\left(\dot{\phi}^2 - \dot{\xi}^2\right), \qquad (2)$$

$$3H^2 = \frac{1}{m_p^2}\left(-\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\xi}^2 + V\right), \qquad (3)$$

$$\ddot{\phi} + 3H\dot{\phi} = \frac{\partial V}{\partial \phi}, \qquad \ddot{\xi} + 3H\dot{\xi} = -\,\frac{\partial V}{\partial \xi}. \qquad (4)$$

2 THE SFT INSPIRED MODELS

ϕ is an open string tachyon.
ξ is the closed string tachion.

The string theory provides asymptotic conditions for solutions.

The phantom field $\phi(t)$ smoothly rolls from the unstable perturbative vacuum ($\phi = 0$) to a nonperturbative one, say, $\phi = 1$ and stops there.

The field $\xi(t)$ goes asymptotically to zero in the infinite future.

In other words we assume

$$\phi(0) = 0, \tag{5}$$

$$\phi(+\infty) = 1, \tag{6}$$

$$\xi(+\infty) = 0. \tag{7}$$

The form of the potential is assumed to be given from the string field theory within the level truncation scheme. An exact form of the open-closed tachyon interaction is not known and we consider the simplest polynomial interaction.

More exactly we impose the following restriction on $V(\phi,\xi)$:

- the potential is the sixth degree polynomial:

$$V(\phi,\xi)=\sum_{k=0}^{6}\sum_{j=0}^{6-k}c_{kj}\phi^k\xi^j, \tag{8}$$

- coefficient in front of 5-th and 6-th powers are of order $1/m_p^2$ and the limit $m_p^2\to\infty$ gives a nontrivial 4-th degree potential,
- the potential is even: $V(\phi,\xi)=V(-\phi,-\xi)$. It means that if $k+j$ is odd, then $c_{kj}=0$.

Questions:

1. Can we construct model with an exact solution, which satisfies the asymptotic and boundary conditions?

2. Is this solution unique?

3. Is $w_{DE} > -1$ or $w_{DE} < -1$ at late time?

3 POTENTIAL AND SUPERPOTENTIAL

To find the potential $V(\phi, \xi)$ we assume that $H(t)$ is a function (superpotential) of $\phi(t)$ and $\xi(t)$: $H(t) = W(\phi(t), \xi(t))$. This allows us to rewrite eq. (2) as

$$\frac{\partial W}{\partial \phi}\dot{\phi} + \frac{\partial W}{\partial \xi}\dot{\xi} = \frac{1}{2m_p^2}\left(\dot{\phi}^2 - \dot{\xi}^2\right).$$

Equations (2)–(4) are solved provided the relations

$$\frac{\partial W}{\partial \phi} = \frac{1}{2m_p^2}\,\dot{\phi}, \qquad \frac{\partial W}{\partial \xi} = -\frac{1}{2m_p^2}\,\dot{\xi}, \tag{9}$$

$$V = 3m_p^2 W^2 + 2m_p^4\left(\left(\frac{\partial W}{\partial \phi}\right)^2 - \left(\frac{\partial W}{\partial \xi}\right)^2\right) \tag{10}$$

are satisfied (O. DeWolfe, D.Z. Freedman, S.S. Gubser, A. Karch, Phys. Rev. **D62** (2000) 046008, hep-th/9909134).

4 MODEL WITH EXACT SOLUTIONS

Let us construct a potential, which corresponds to fields

$$\phi(t) = \tanh(t), \qquad \text{and} \qquad \xi(t) = \frac{\sqrt{2(1+b)}}{\cosh(t)} \equiv \frac{B}{\cosh(t)}. \quad (11)$$

The functions $\phi(t)$ and $\xi(t)$ are solutions of the system

$$\begin{cases} \dot{\phi} = b\left(\phi^2 - 1\right) + \frac{1}{2}\xi^2, \\ \dot{\xi} = -\phi\xi. \end{cases} \quad (12)$$

The corresponding superpotential and potential are given by

$$H(t) = W(\phi, \xi) = -\frac{\phi}{6m_p^2}\left(b\left(3 - \phi^2\right) - \frac{3}{2}\xi^2\right), \quad (13)$$

$$V = \frac{1}{2}\left(b\left(\phi^2 - 1\right) + \frac{\xi^2}{2}\right)^2 - \frac{\phi^2\xi^2}{2} + \frac{\phi^2\left(b\left(3 - \phi^2\right) - \frac{3\xi^2}{2}\right)^2}{12m_p^2}.$$

Note that we have a freedom to choose the potential does not changing solutions. The same functions $\phi(t)$**,** $\xi(t)$ **(and consequently the Hubble parameter** $H(t)$**) can correspond to different potentials** $V(\phi, \xi)$.

1. System (12) is not unique: the functions $\phi(t)$ and $\xi(t)$ are solutions of the following differential equations:

$$\dot{\phi} = 1 - \phi^2, \qquad \dot{\xi} = \xi\sqrt{1 - \frac{\xi^2}{B^2}}. \tag{14}$$

2. The solution is not violated if we add to the potential V a function δV, which is such that δV, $\partial(\delta V)/\partial\phi$ and $\partial(\delta V)/\partial\xi$ are zero on the solution:

$$\delta V = A(\xi, \phi)\left[\phi^2 + \frac{1}{B^2}\xi^2 - 1\right]^2. \tag{15}$$

5 NEW SOLUTIONS

System

$$\begin{cases} \dot{\phi} = b\left(\phi^2 - 1\right) + \dfrac{1}{2}\xi^2, \\ \dot{\xi} = -\phi\xi, \end{cases} \tag{16}$$

has not only solutions (11).

The general solution is defined in quadratures.

In the case $b = -1/2$ we can write it in the explicit form:

$$\begin{aligned} \phi(t) &= \frac{\left((C^2+4)e^{t-t_0} - e^{-(t-t_0)}\right)}{(C^2+4)e^{t-t_0} + 2C + e^{-(t-t_0)}}, \\ \xi(t) &= \frac{4}{(C^2+4)e^{t-t_0} + 2C + e^{-(t-t_0)}}, \end{aligned} \tag{17}$$

where C and t_0 are arbitrary constant.

For all values of C and t:

$$\phi(\pm\infty) = \pm 1, \qquad \xi(\pm\infty) = 0. \tag{18}$$

If

$$t_0 = \frac{1}{2}\ln(C^2 + 4), \tag{19}$$

then

$$\phi(0) = 0. \tag{20}$$

So, we have constructed **the SFT inspired model with two-parameter set of exact solutions, which satisfy asymptotic and boundary conditions**.

In the case $C = 0$ we obtain two one-parameter sets of solutions

$$\phi_0(t) = \tanh(t - t_C), \tag{21}$$

$$\xi_0(t) = \pm\frac{1}{\cosh(t - t_C)}. \tag{22}$$

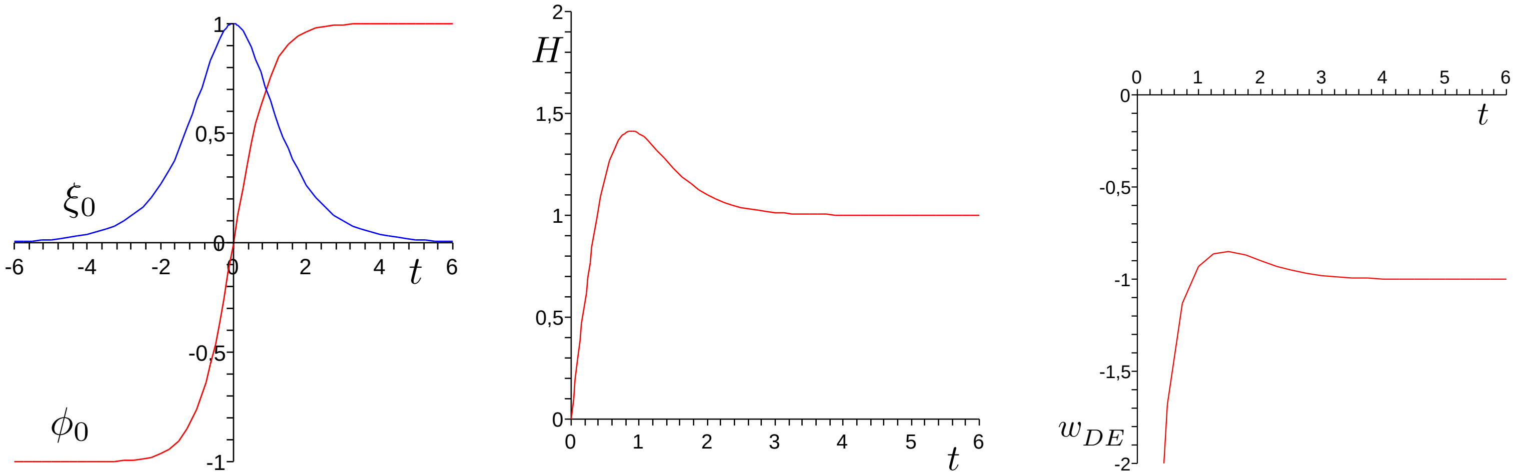

Figure 1: ϕ and ξ (left), H (center) and w_{DE} (right) at $C = 0$ and $m_p^2 = 1/6$.

If $C > 0$ then the straightforward calculations give $\dot{H}(t) = 0$ at 4 points. Two of these points are not real numbers. Other two points are real if and only if $C < 2$.

$H(t)$ is a monotonic function at $C \geqslant 2$ and has a maximum and a minimum at $0 < C < 2$.

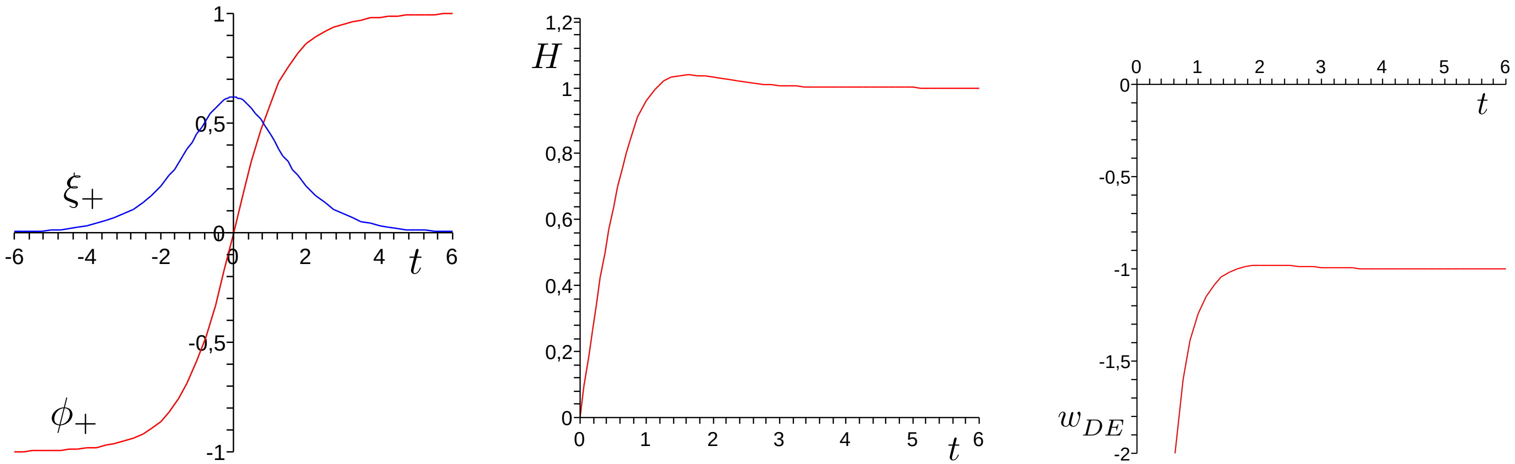

Figure 2: ϕ and ξ (left), H (center) and w_{DE} (right) at $C = 1$ and $m_p^2 = 1/6$.

To consider the case $C < 0$.

The corresponding $\dot{H}$ is equal to zero at 4 points: two points are always real numbers, other two points are real at $C < -2$.

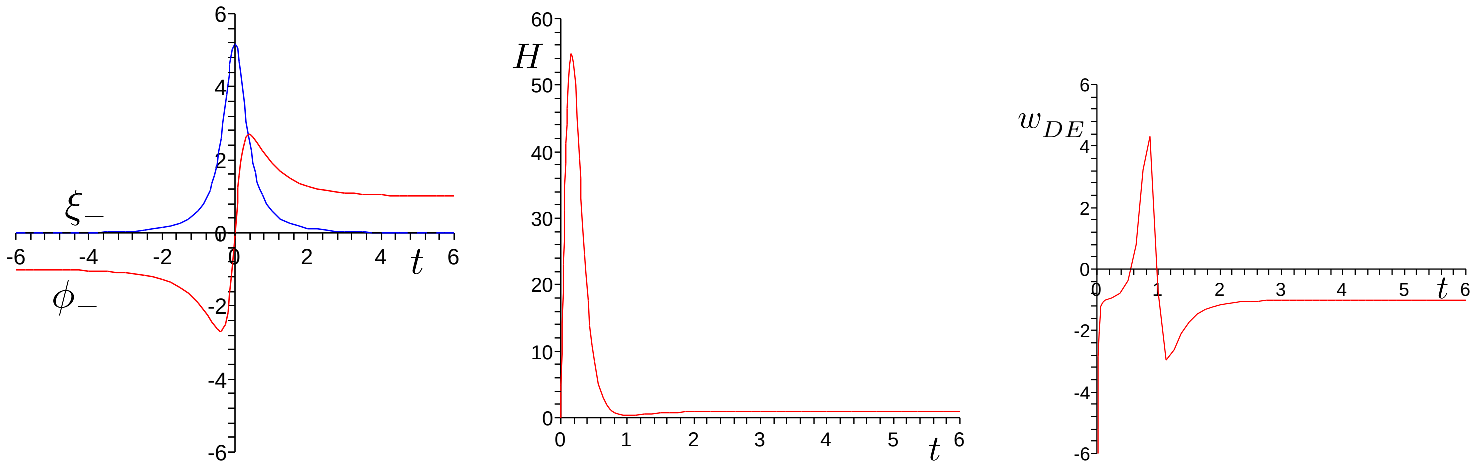

Figure 3: ϕ and ξ (left), H (center) and w_{DE} (right) at $C = -5$ and $m_p^2 = 1/6$.

So, we have obtained the model with the polynomial potential:

$$V = \frac{1}{8}\Big(1 - \phi^2 + \xi^2\Big)^2 - \frac{1}{2}\phi^2\xi^2 + \frac{\phi^2}{36m_p^2}\Big(\Big(3 - \phi^2\Big) + 3\xi^2\Big)^2, \quad (23)$$

which has a two-parameter set of exact solutions.

This set can separated into two subsets, one of which corresponds to the quintessence large time behaviour, another one corresponds to the phantom large time behaviour.

The obtained solutions have one and the same asymptotic conditions.

So, we can conclude that both quintessence and phantom large time behaviors are possible to obtain from the SFT inspired effective model with the polynomial potential.

6 CONCLUSIONS

We consider the model with a phantom scalar field + an ordinary scalar field and obtain:

- $H(t)$ **is not a monotonic function. The state parameter** w_{DE} **crosses the barrier** $w_{DE} = -1$
- **We have a freedom to choose the potential for the given solutions.**
- **Using the superpotential method we generalize a one-parameter solutions to two-parameter solutions.**
- **Both quintessence and phantom late time behaviors are possible to obtain from the SFT inspired effective model with one and the same potential.**