New theoretical Results in Synchrotron Radiation

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We recall that the SR is created by charged particles, which are moving with velocities v along circles of radius R in an uniform magnetic field H,

$$R\frac{\beta E}{eH} = \frac{m_0c^2}{eH}\sqrt{\gamma^2 - 1}\,,\ \beta = \frac{v}{c},\ \ \gamma(1-\beta^2)^{-1/2} = \frac{E}{m_0c^2} > 1\,. \tag{1}$$

Here E is the particle energy, e is the charge, and m_0 the rest mass. The radiation frequencies $\omega_\nu = \nu\omega_0$, $\nu = 1, 2, ...$, are multiples of the synchrotron frequency $\omega_0 = ceH/\,E$. The spectral SR-intensity (SR-intensity for a fixed radiation frequency) has maximum for harmonics with $\nu \sim \gamma^3$. Two limiting cases, the non-relativistic $(\beta \ll 1,\ E \simeq m_0c^2)$ and the relativistic limits $(\beta \sim 1,\ E \gg m_0c^2)$, are of particular interest. In the non-relativistic case, only the first harmonic $\omega_1 = \omega_0$ is effectively emitted. The SR-intensity has a maximum in the direction of the magnetic field. In the relativistic case, the integral SR-intensity (spectral SR-intensity summed over the spectrum) is concentrated in the orbital plane within a small interval $\Delta\theta \sim 1/\,\gamma \ll 1$ of the angle θ that we have chosen to measure from the direction of the magnetic guide field which is normal to the orbital plane. Thus, as the electron energy increases, the integral SR-intensity tends to be concentrated in the orbit plane. Any polarization component of the integral SR-intensity has the same behavior. These results were first derived in the framework of classical theory.

In the SR theory one introduces polarization components W_i , $i = 0, \pm 1, 2, 3$ of the integral SR-intensity. Here $W_{\pm 1}$ are the integral SR-intensities of the right $(+1)$ and the left (-1) circular polarization components respectively, whereas W_2 and W_3 are the so called "σ" and "π" linear polarization components. The total integral SR-intensity W_0 is defined as $W_0 = W_1 + W_{-1} = W_2 + W_3$. In the framework of the classical theory of SR one can find:

$$W_i = V_0 \Phi_i(\beta), \quad \Phi_i(\beta) = \int_0^\pi F_i(\beta, \theta) \sin\theta d\theta, \;\; F_i(\beta, \theta) = \sum_{\nu=1}^{\infty} f_i(\nu, \beta; \theta) \,,$$

$$f_0(\nu, \beta; \theta) = f_{-1}(\nu, \beta; \theta) + f_1(\nu, \beta; \theta) = f_2(\nu, \beta; \theta) + f_3(\nu, \beta; \theta), \qquad (2)$$

$$V_0 = \frac{ce^2\beta^4}{R^2} = \frac{e^4 H^2 \beta^2 (1-\beta^2)}{m_0^2 c^3}.$$

Here θ is the angle between the z-axis and the radiation direction. The particle orbit is placed in the plane $z = 0$, which corresponds to $\theta = \pi/2$. In some works a different set of angles is used for the SR description. In particular, z-axis is selected to coincide with the direction of the instant particle velocity.

The sum over ν is just the sum over the spectrum, such that the expressions inside the sum represent spectral distributions. The functions $f_i(\nu, \beta; \theta)$ have the form:

$$f_{\mp 1}(\nu, \beta; \theta) = \frac{\nu^2}{2} \left[J_\nu'(x) \mp \frac{\cos\theta}{\beta \sin\theta} J_\nu(x) \right]^2, \quad x = \nu\beta\sin\theta,$$
$$f_2(\nu, \beta; \theta) = \nu^2 J_\nu'^{\,2}(x), \quad f_3(\nu, \beta; \theta) = \frac{\nu^2 \cos^2\theta}{\beta^2 \sin^2\theta} J_\nu^2(x). \tag{3}$$

Here $J_\nu(x)$ are Bessel functions of integer indices. The following simple properties hold true:

$$f_k(\nu, \beta; \theta) = f_k(\nu, \beta; \pi - \theta), \quad k = 0, 2, 3;$$
$$f_{-1}(\nu, \beta; \theta) = f_1(\nu, \beta; \pi - \theta)\,. \tag{4}$$

Thus, it is enough to study the functions $f_k(\nu, \beta; \theta), k = 0, 2, 3$, at the interval $0 \leq \theta \leq \pi/2$ only, and between the functions $f_{\pm 1}$ it is enough to study f_1 only.

Exact analytic expressions for the functions $F_k(\beta,\theta), k=0,2,3$ have the following form:

$$F_2(\beta,\theta)=\frac{7-3\varepsilon}{16\varepsilon^{5/2}},\quad \varepsilon=1-\beta^2\sin^2\theta,\quad \gamma^{-2}\le\varepsilon<1\,,$$
$$F_3(\beta,\theta)=\frac{(\gamma^2\varepsilon-1)(5-\varepsilon)}{16(\gamma^2-1)\varepsilon^{7/2}},$$
$$F_0(\beta,\theta)=\frac{(3-4\gamma^2)\varepsilon^2+6(2\gamma^2-1)\varepsilon-5}{16(\gamma^2-1)\varepsilon^{7/2}}\,. \tag{5}$$

Expressions for the functions $F_{\pm1}$ can be found in the form:

$$F_{\pm1}(\beta,\theta)=\frac{1}{2}F_0(\beta,\theta)\pm\Psi(\beta\sin\theta)\cos\theta\,,$$
$$\Psi(x)=\frac{1}{2x}\frac{d}{dx}\sum_{\nu=1}^{\infty}\nu J_\nu^2(\nu x)\,. \tag{6}$$

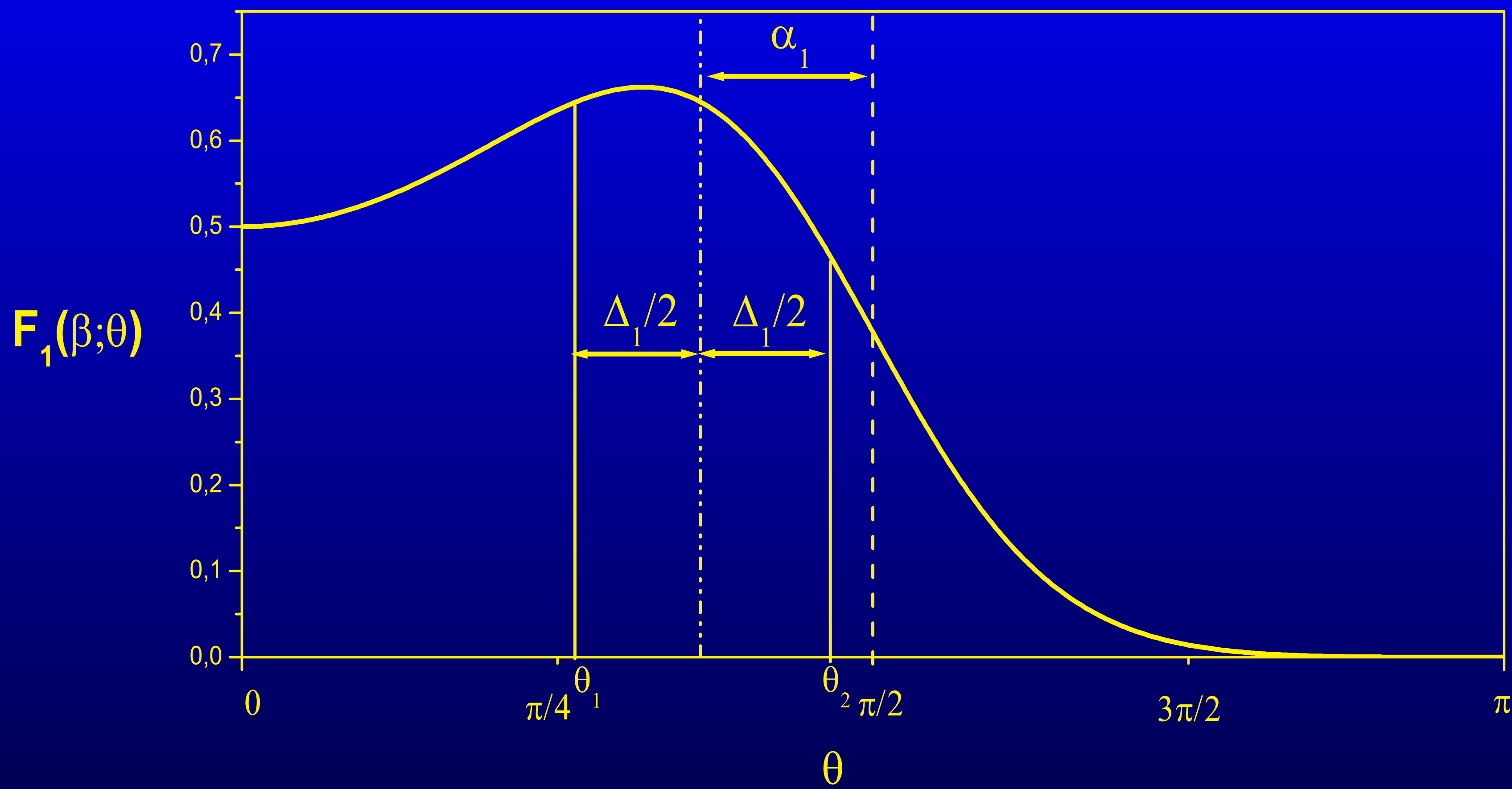


Figure 1: To definition of effective angle and deviation angle for the function $F_1(\beta;\theta)$

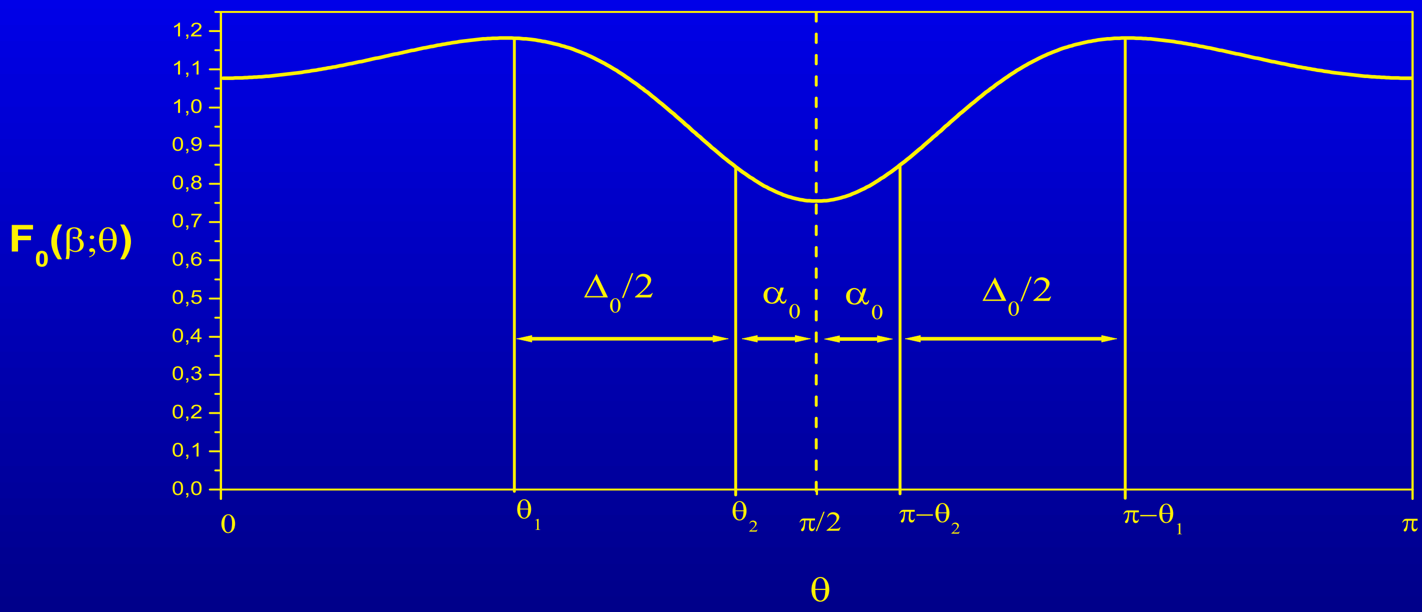


Figure 2: To definition of effective angle and deviation angle for the function $F_0(\beta;\theta)$

$$\int_{\theta_1^{(k)}}^{\theta_2^{(k)}} F_k(\beta;\theta)\,\sin\theta\,d\theta = \frac{1}{2}\int_0^{\pi/2} F_k(\beta;\theta)\,\sin\theta\,d\theta.$$

$$F_k(\beta;\theta_1^{(k)})\,\sin\theta_1^{(k)} = F_k(\beta;\theta_2^{(k)})\,\sin\theta_2^{(k)}. \quad (7)$$

$$0 \le \theta_1^{(k)} < \theta_2^{(k)} \le \pi/2;\;\; \Delta_k/2 = \theta_2^{(k)} - \theta_1^{(k)}, \alpha_k = \pi/2 - \theta_2^{(k)}.k = 0, 2, 3;$$

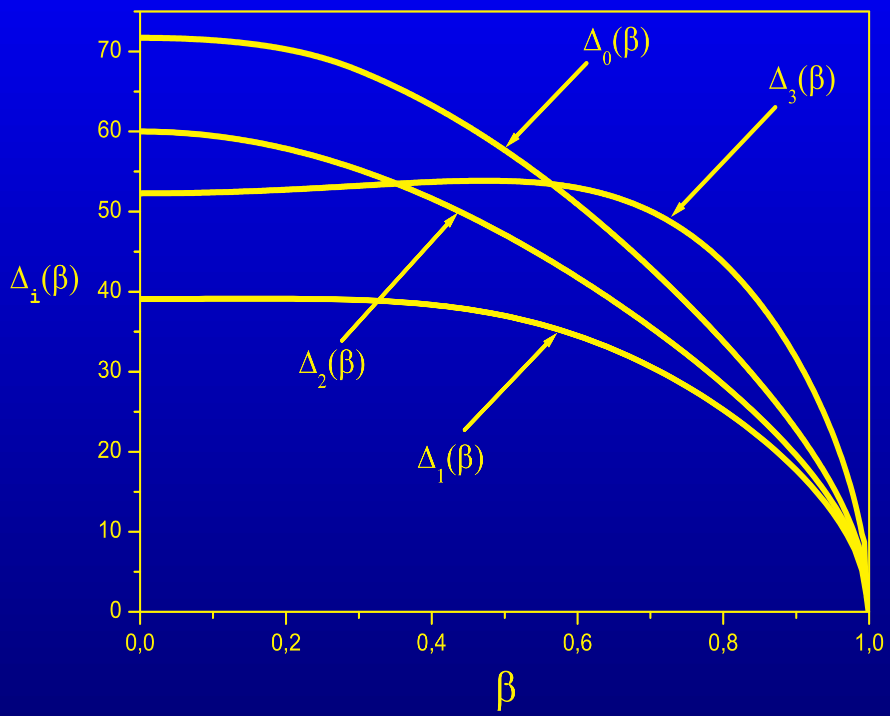


Figure 3: The effective angle dependence of β and polarization component.

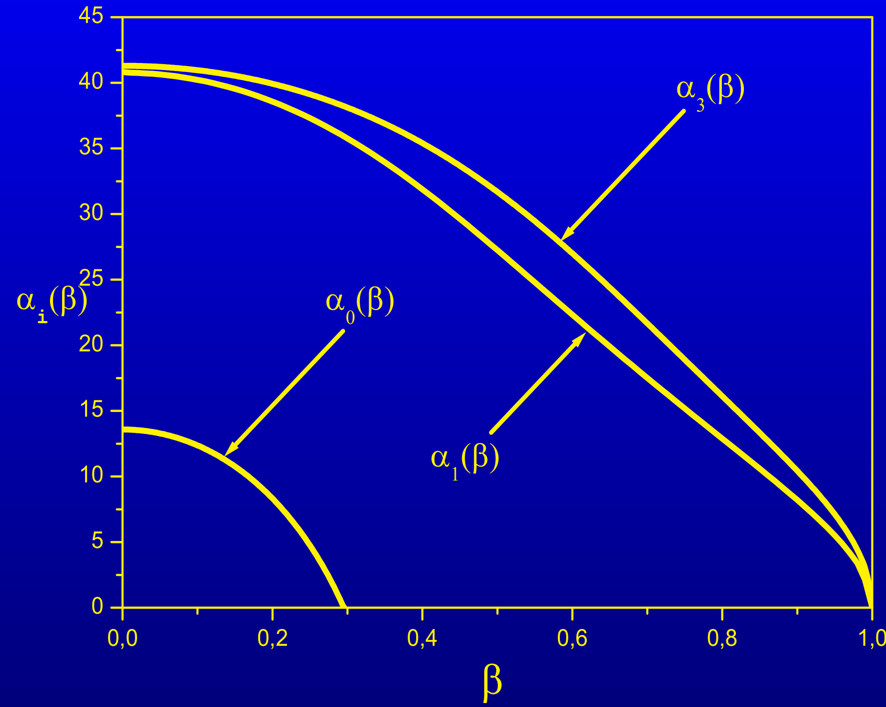


Figure 4: The deviation angle dependence of β and polarization component.

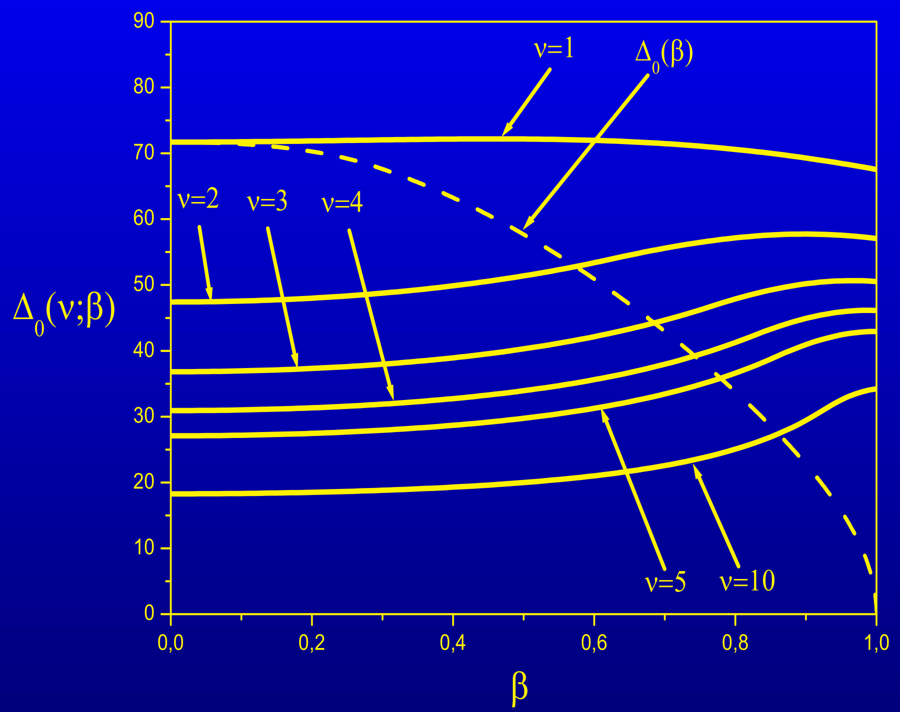


Figure 5: The dependence of effective angle $\Delta_0(\nu, \beta)$ for different harmonics ν.

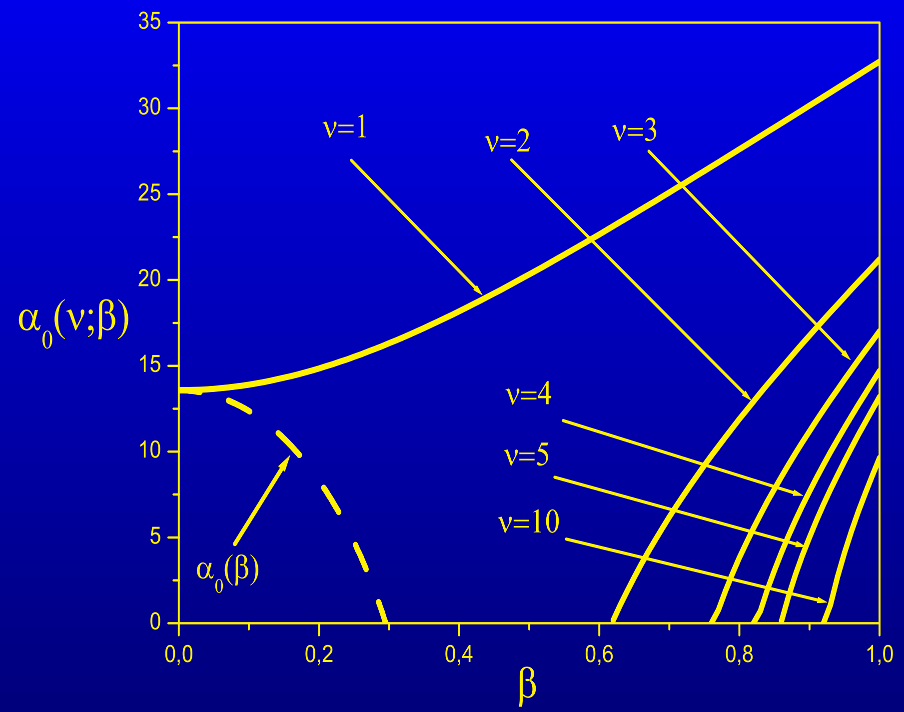


Figure 6: The dependence of deviation angle $\alpha_0(\nu, \beta)$ for different harmonics ν.

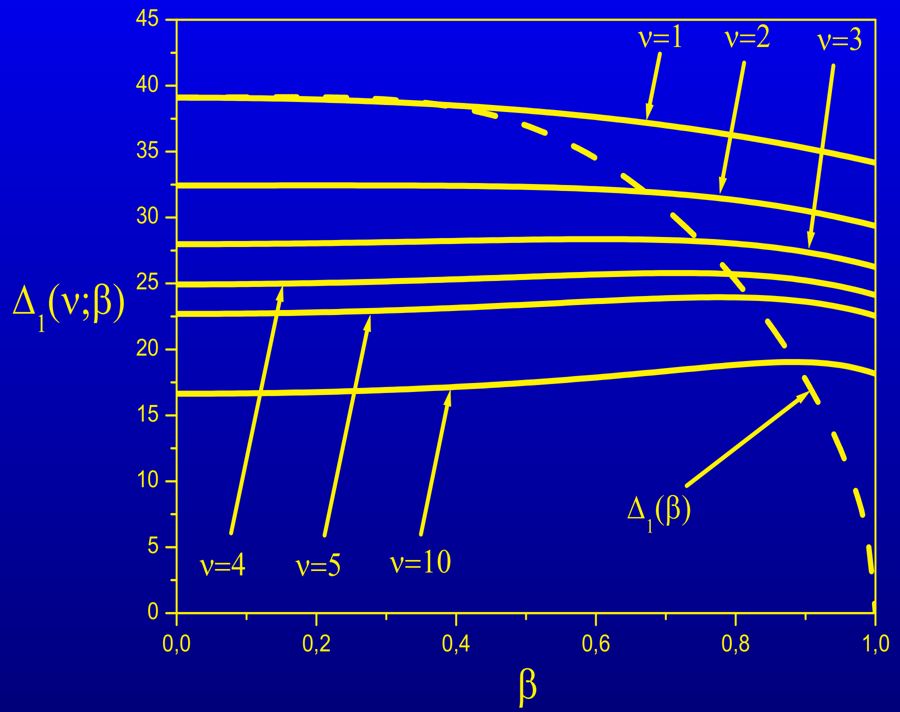


Figure 7: The dependence of effective angle $\Delta_1(\nu, \beta)$ for different harmonics ν.

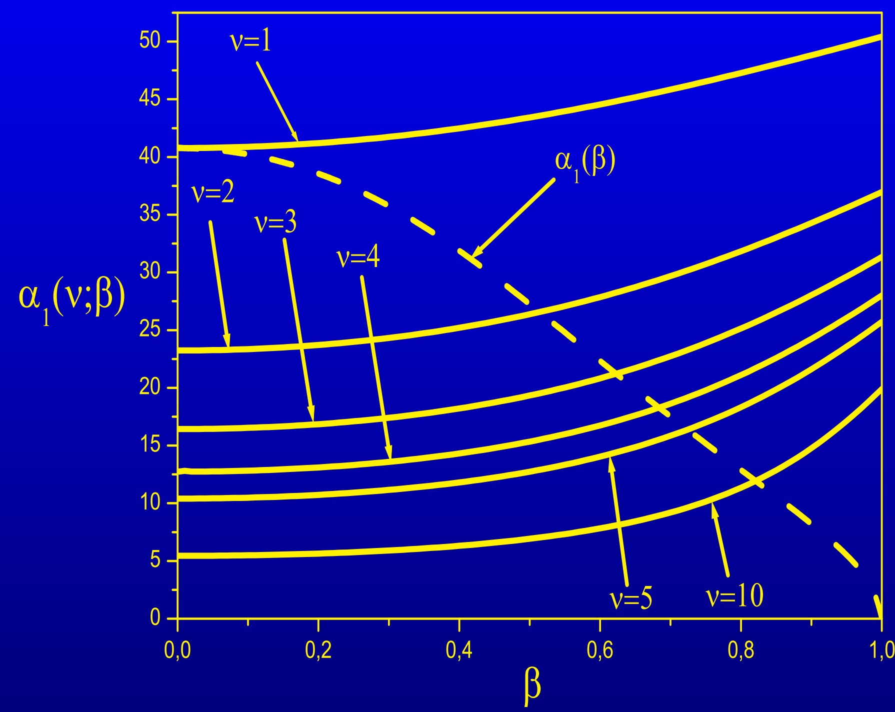
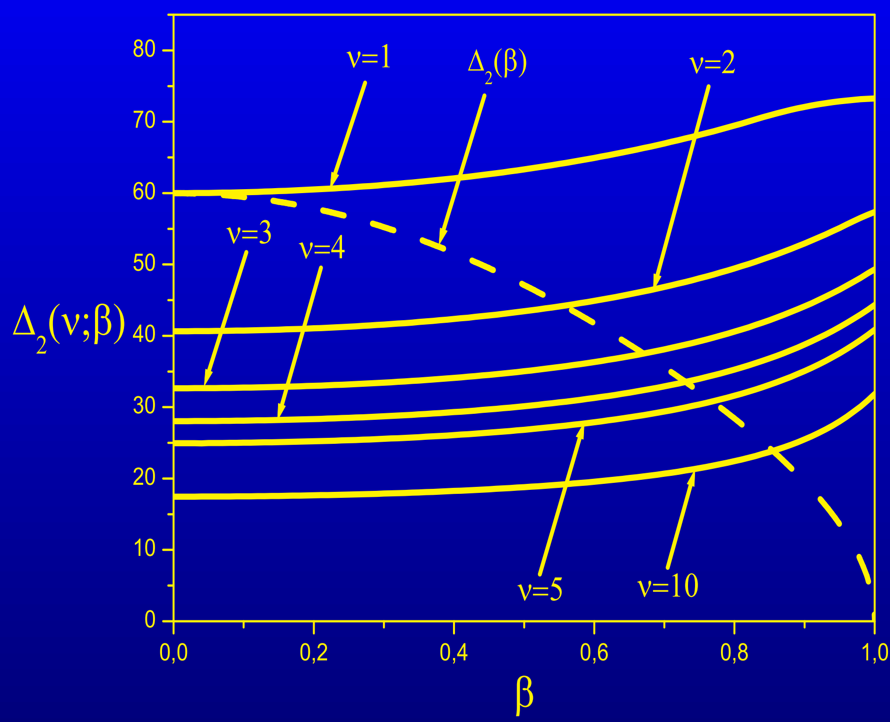


Figure 8: The dependence of deviation angle $\alpha_1(\nu, \beta)$ for different harmonics ν.



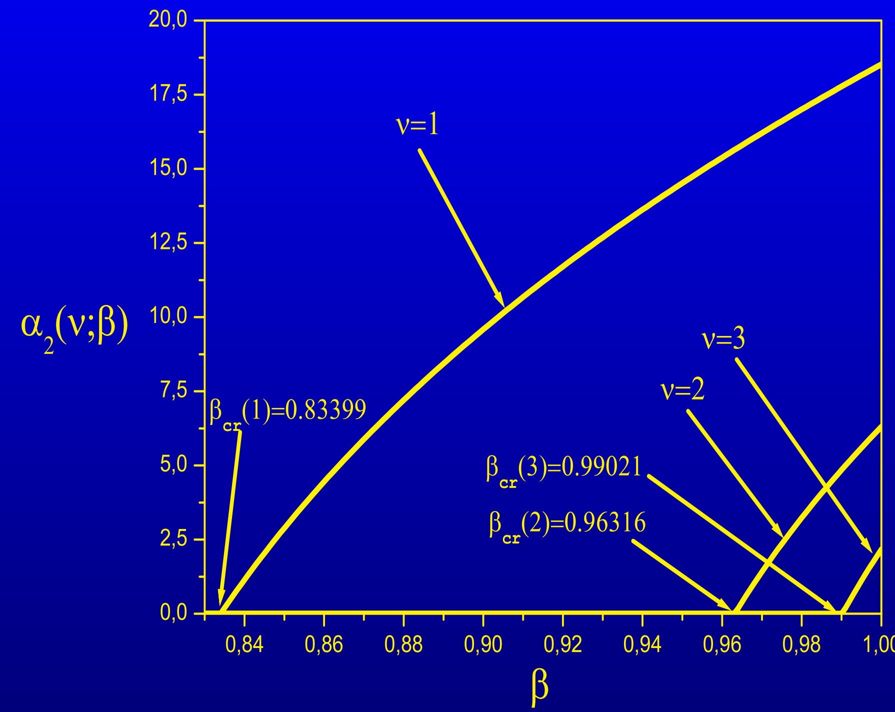
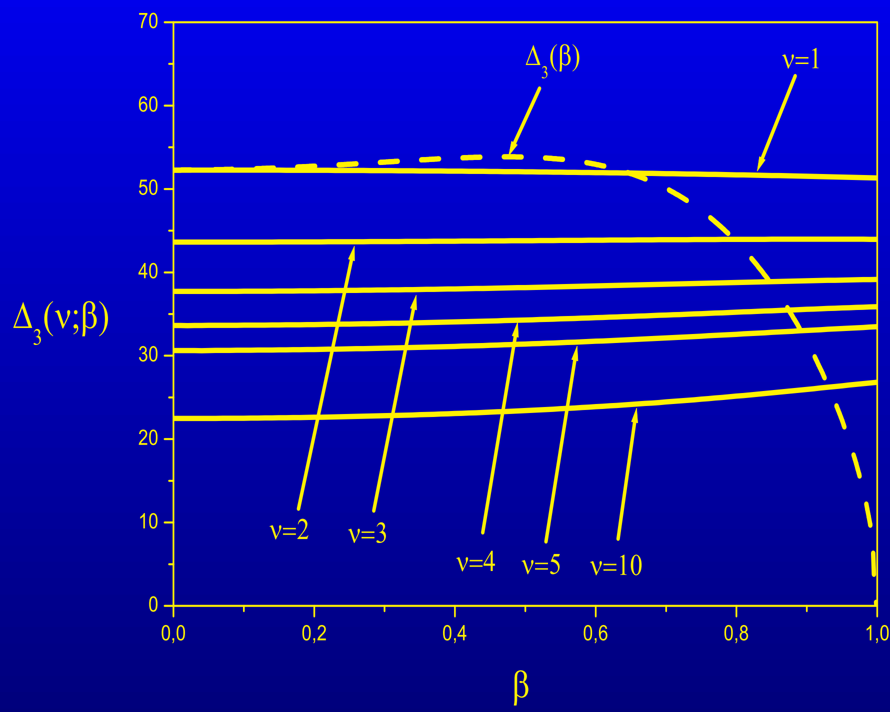


Figure 9: The dependence of effective angle $\Delta_2(\nu, \beta)$ for different harmonics ν.

Figure 10: The dependence of deviation angle $\alpha_2(\nu, \beta)$ for different harmonics ν.



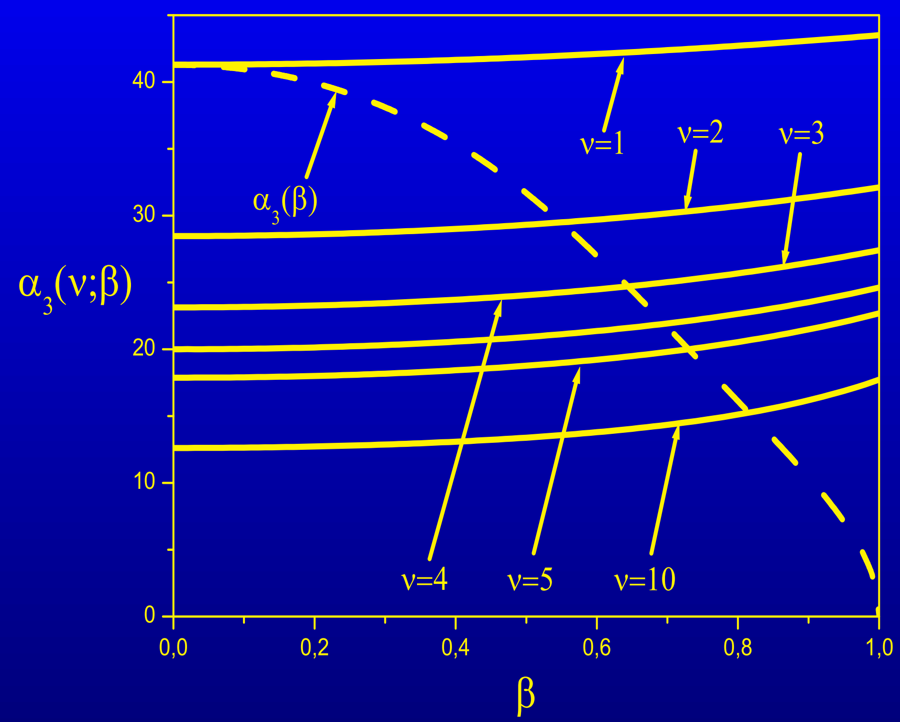


Figure 11: The dependence of effective angle $\Delta_3(\nu, \beta)$ for different harmonics ν.

Figure 12: The dependence of deviation angle $\alpha_3(\nu, \beta)$ for different harmonics ν.

The phenomena of angular deconcentration of single harmonic radiation is established in terms of classical theory.
It is known that the phenomena of the angular concentration of total (summed up spectrum) SR consists in the following fact: when the particles energy E increases all the radiation concentrates in the orbits plane area and the effective angle $\Delta\theta$, which contains main output of SR, decreases down to zero as $\Delta\theta \sim 1/\gamma$. One considered that the angular distribution of separate spectral components of SR acts the same way in quantitative sense.
However, the attempt to show that the angular distribution of separate spectral harmonic treats itself completely different was definitely successful.
When γ increases the maximums of the angular distribution move away from orbits plane while the effective angle decreases and moves, at the high value of γ- characteristic, to its finite limit $\Delta\theta(\nu)$, which depends on the number of harmonic ν.
This is, in fact, the deconcentration phenomena of SR. It seems, that the concentration and the deconcentration of SR contradict each other. However, they are perfectly agreed. Indeed, the high value of γ-characteristic corresponds with the fact, that the maximum in SR spectrum accounts for the harmonic $\nu_{max} \sim \gamma^3$. But as for high harmonic numbers, the approximation gives $\Delta\theta(\nu) \sim \nu^{-1/3}$. Therefore, $\Delta\theta(\nu_{max}) \sim 1/\gamma$, and that is an expectant result.

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