Gluon and Ghost propagators in $SU(3)$ gluodynamics on large lattices

I.L. Bogolubsky (JINR, Dubna)
E.–M. Ilgenfritz, M. Müller-Preussker (Humboldt University, Berlin)
A. Sternbeck (CSSM, Univ. of Adelaide, Dubna)

Lomonosov Conference, Moscow, 2007, August 23-29

Introduction

- Nonperturbative studies of gluon and ghost propagators in the DSE and lattice approaches move forward in parallel, supporting each other.

- We are making next steps into the infrared region ($q \to 0$) with computation :

  i) of the gluon propagator:

$$D^{ab}_{\mu\nu} = \delta^{ab} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \frac{Z_{gl}(q^2)}{q^2}$$

  ii) and of the ghost propagator:

$$G^{ab} = \delta^{ab} \frac{Z_{gh}(q^2)}{q^2} \propto \sum_{x,y} \left\langle \mathrm{e}^{-ik\cdot(x-y)} [M^{-1}]^{ab}_{x,y} \right\rangle$$

  – defining running coupling constant and
  – needed for verification of the Gribov-Zwanziger confinement scenario;

- Infrared asymptotic behavior from solution of Dyson-Schwinger Eqs. [Alkofer et al. (1997), Fischer (2006)]

$$Z_{gh}(q^2) \propto (q^2)^{-\kappa} \quad \text{and} \quad Z_{gl}(q^2) \propto (q^2)^{2\kappa}$$

  with common exponent $\kappa \approx 0.595$.

Gauge fixing: standard approach

To fix the Landau gauge we apply to link variables $U_{x,\mu} \in SU(3)$ a gauge transformation $g(x)$ which has to maximize the functional

$$F_U[g] = \frac{1}{3N_{\text{Links}}} \sum_{x,\mu} \mathfrak{Re}\,\mathrm{Tr}\; {}^g U_{x,\mu}\,,$$

$\Rightarrow$ equivalent to $\partial_\mu A_\mu = 0\,,$
$\Rightarrow$ not unique: Gribov copies,
$\Rightarrow$ search for global maxima -
fundamental modular region (FMR)

Standard prescription:

i) $g(x)$ taken with periodic b.c.'s,

ii) maximize $F_U[g]$ with overrelaxation (OR) method,

Drawbacks of OR:

i) substantial slowing down of OR convergence with increasing lattice extension L

ii) its possibilities to find global maximum of $F_U[g]$ are strongly limited

Simulated annealing: the principle

(cf. poster of M. Müller-Preussker)

- Simulated annealing (SA) is a "stochastic optimization method" allowing quasi-equilibrium tunnelings through functional barriers, in the course of a "temperature" T decrease

- In principle - with infinitely slow cooling down - it allows to reach global extrema (contrary to OR, "tied" to the (initially chosen) local maximum)

- Control parameters at hand:

  i) N_{iter}, T_{max} and T_{min}

  ii) Schedule for temperature steps $T_i, \; i = 1, ..., N_{iter}$ can be optimized $(T_1 = T_{max}, \; T_{N_{iter}} = T_{min})$.

$\Rightarrow$ The larger N_{iter} the higher the local maxima. $N_{iter} \to \infty$: global maximum.

$\Rightarrow$ Schedule in practice: $T_{max} = 0.45, T_{min} = 0.01$, $N_{iter} = O(5 \cdot 10^3 - 15 \cdot 10^3)$ with tiny (larger) T-steps close to T_{max} (close to T_{min}).

Lattice Faddeev-Popov operator

Lattice Faddeev-Popov (F-P) operator can be written in terms of the (gauge-fixed) link variables $U_{x,\mu}$ as

$$M^{ab}_{xy} = \sum_{\mu} A^{ab}_{x,\mu}\,\delta_{x,y} - B^{ab}_{x,\mu}\,\delta_{x+\hat{\mu},y} - C^{ab}_{x,\mu}\,\delta_{x-\hat{\mu},y}$$

with

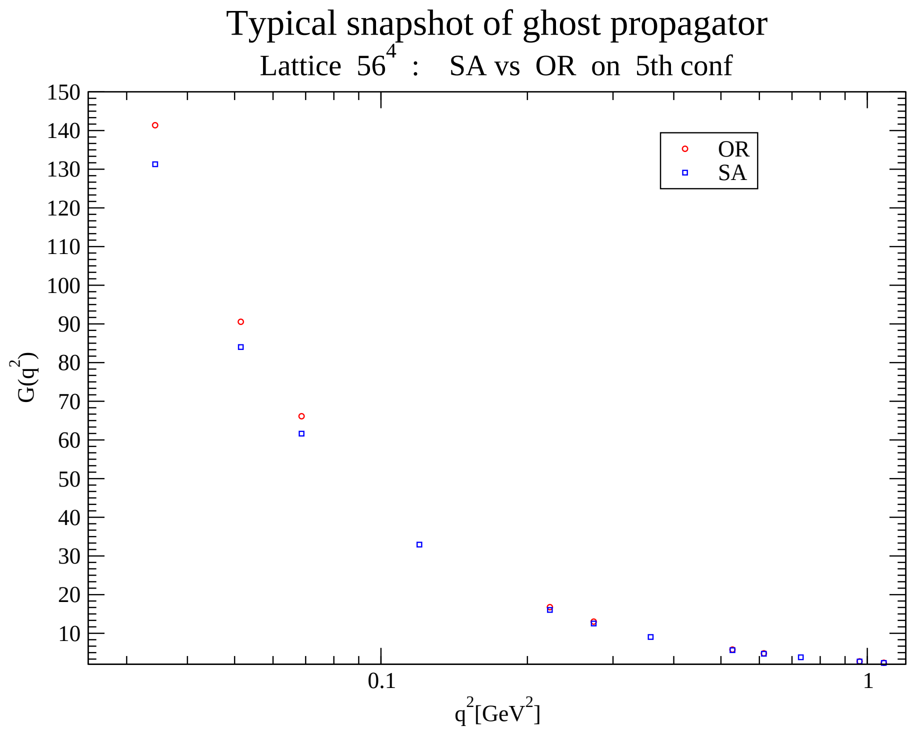
$$\begin{aligned} A^{ab}_{x,\mu} &= \quad \mathfrak{Re}\,\mathrm{Tr}\left[\{T^a, T^b\}(U_{x,\mu} + U_{x-\hat{\mu},\mu})\right], \\ B^{ab}_{x,\mu} &= 2\cdot\mathfrak{Re}\,\mathrm{Tr}\left[T^b T^a\, U_{x,\mu}\right], \\ C^{ab}_{x,\mu} &= 2\cdot\mathfrak{Re}\,\mathrm{Tr}\left[T^a T^b\, U_{x-\hat{\mu},\mu}\right] \end{aligned}$$

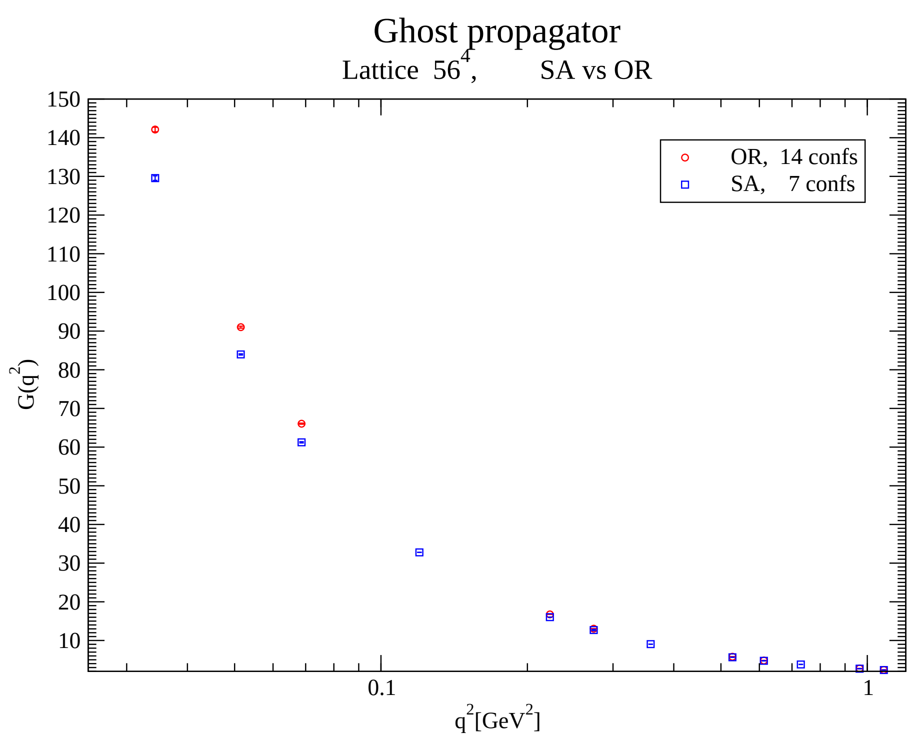
and T^a, $a = 1, \ldots, 8$ being the (hermitian) generators of the $\mathfrak{su}(3)$ Lie algebra satisfying $\mathrm{Tr}\,[T^a T^b] = \delta^{ab}/2$.

Ghost propagator $\propto M^{-1}$: inversion with conjugate gradient method and plane wave sources.

Gauge fixing: SA vs. OR

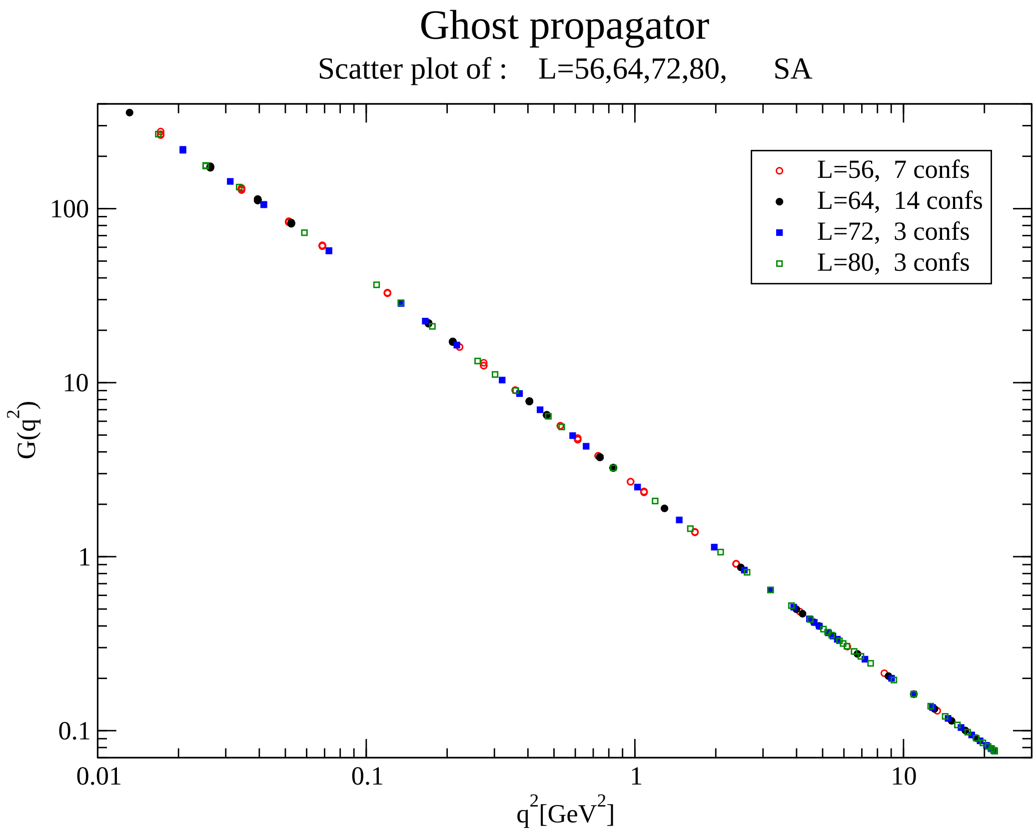
$SU(3)$ ghost propagator for $\beta = 5.70$.





$\Rightarrow$ Influence of Gribov copies small for $L \geq 56$.

Ghost propagator: SA results, $\beta = 5.70$



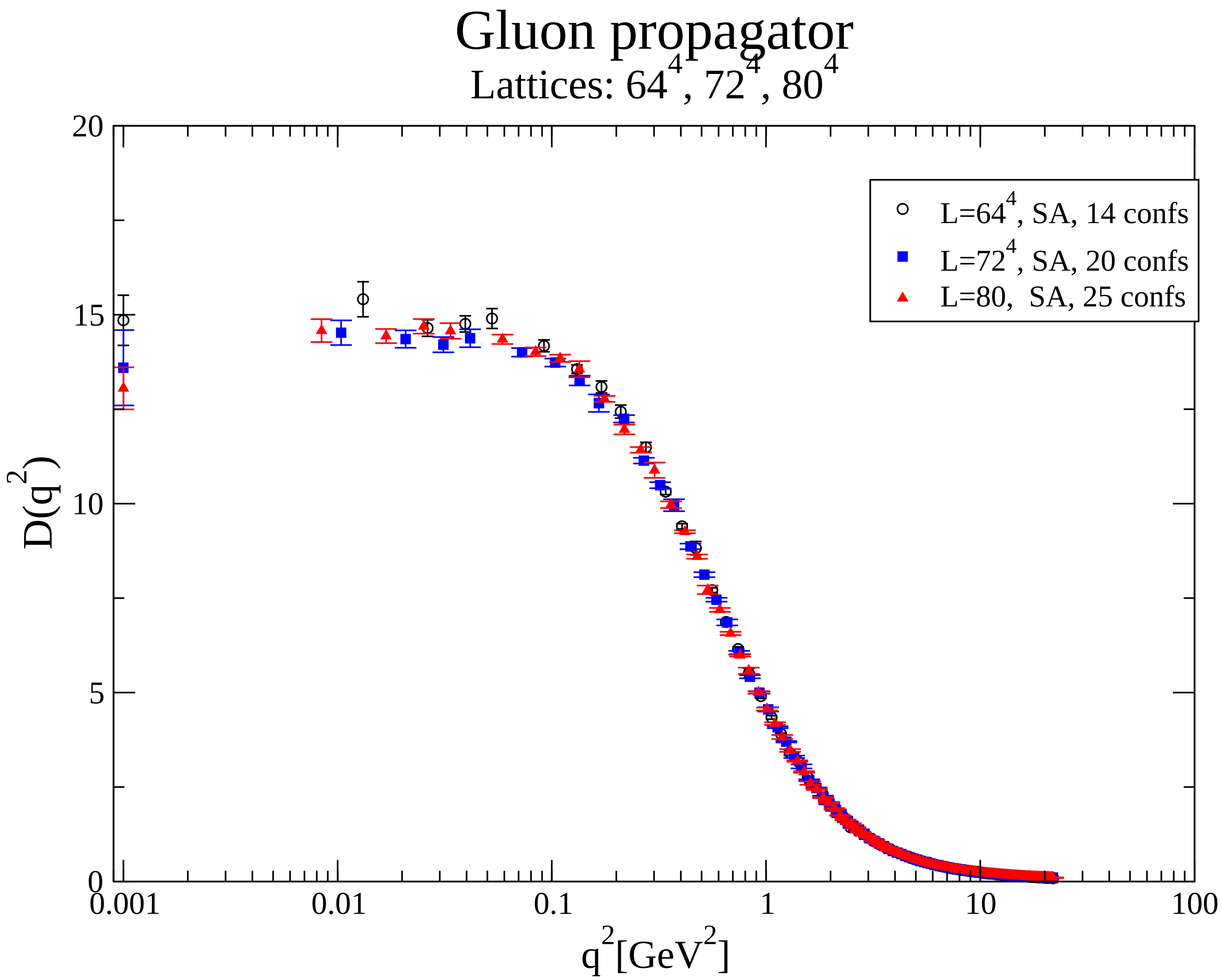
$\Rightarrow$ Weak estimator's dependence on MC configuration.

$\Rightarrow$ Our results for various L coincide within 1-2 percent, i.e. finite-size effects on ghost propagator are unexpectedly small, and not agreeing with finite-volume DSE results (Fischer et al (2007)).

$\Rightarrow$ Linear log-log-fit for infrared exponent: $\kappa = 0.06$. strongly differing from DSE result $(\kappa = 0.595)$.

$\Rightarrow$ Further penetration into IR region desirable: hopefully it will resolve the contradiction.

Gluon propagator: SA results, $\beta = 5.70$



$\Rightarrow$ Flattening is clearly seen.

$\Rightarrow$ Convergence to $L = \infty$ seems very slow.

$\Rightarrow$ Gribov copy effect small
(here $\mathbf{Z}(3)$-flip sectors are still neglected !).

Conclusions

- MC configurations used for computation of propagators should be well thermalized

- Gribov effects are small but still visible for ghost propagator at L=56, becoming negligible for larger lattices,in accordance with Zwanziger's prediction.

- Finite-size effects are mild for ghost propagator but distortion of gluon propagator for small momenta is not excluded, though simulations on $L = 72$ and $L = 80$ do no resolve the problem.

- Estimator of ghost propagator is weakly dependent $(\leq 1 - 2\%)$ on choice of MC configuration, and statistical errors are pretty small.

- Message from ghost: we haven't reach yet the "deep" infrared region with awaited $k \approx 0.5$, hopefully are near the entrance to it.

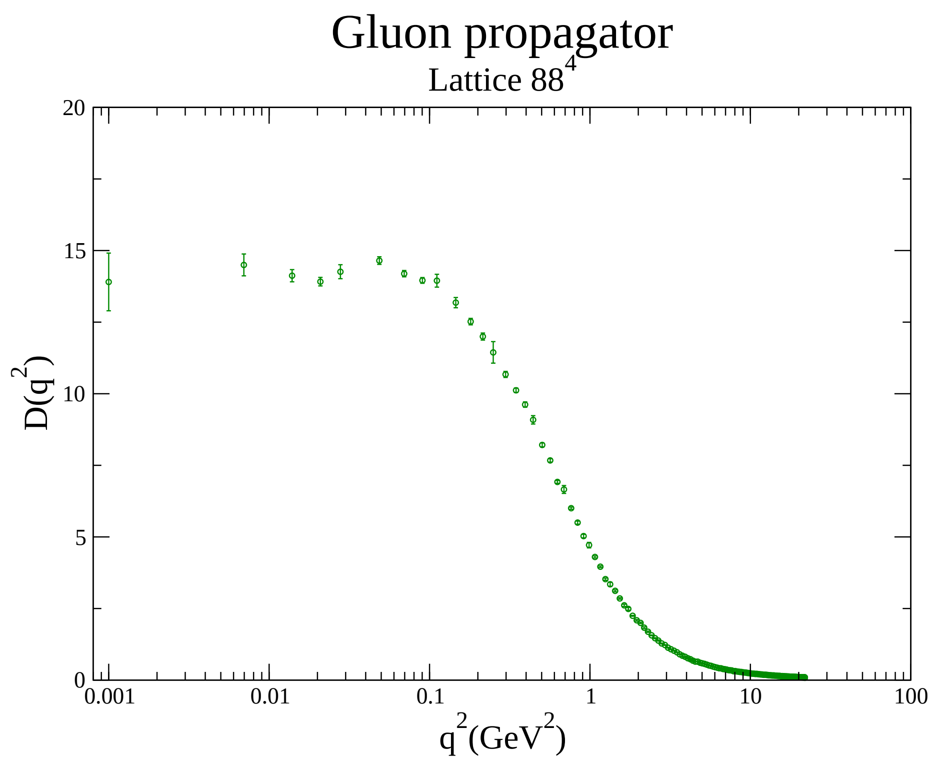
- We see flattenning of the gluon propagator, but haven't found numerical evidence of its maximum yet.

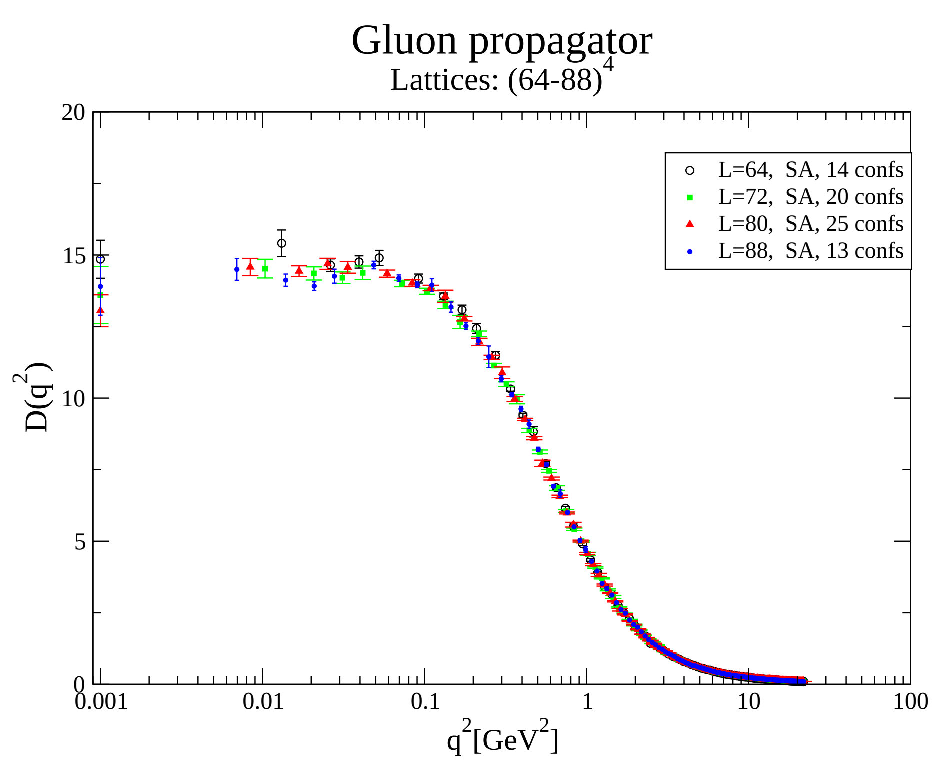
References

1. V.N. Gribov, NPB 139 (1978) 1; D. Zwanziger, NPB 412 (1994) 657.
2. T. Kugo, I. Ojima, Prog. Theor. Phys. Suppl. 66 (1979) 1.
3. D. Zwanziger, PRD 69 (2004) 016002.
4. L. von Smekal, R.Alkofer, A.Hauk, PRL 79 (1997) 3591.
5. L. von Smekal, A.Hauck, R.Alkofer, Ann. Phys. 267 (1998) 1.
6. A. Sternbeck, E.-M. Ilgenfritz, M. Müller-Preussker, A. Schiller, PRD 72 (2005) 014507.
7. A. Sternbeck, Ph.D.Thesis (2006), hep-lat/0609016, and references therein.
8. A. Sternbeck, E.-M. Ilgenfritz, M. Müller-Preussker, A. Schiller, I.L. Bogolubsky, PoS LAT2006 (2006) 076.
9. E.-M.Ilgenfritz, M. Müller-Preussker, A. Sternbeck, A. Schiller, I.L. Bogolubsky, Brazilian Journal of Physics 37, N1B (2007) 193.
10. I.L. Bogolubsky, V.G. Bornyakov, G. Burgio, E.-M. Ilgenfritz, M. Müller-Preussker, V.K. Mitrjushkin (2007), arXiv:0707.3611 [hep-lat].
11. F.D.R. Bonnet, P.O.Bowman, D.B.Leinweber, A.G.Williams, PRD 62 (2000) 051501.
12. F.D.R. Bonnet, P.O.Bowman, D.B.Leinweber, A.G.Williams, J.M.Zanotti, PRD 64 (2001) 034501.
13. S. Furui, H. Nakajima, PRD 69 (2004) 074505.
14. P.Boucaud et al. (2005), hep-ph/0507104.
15. A. Maas, A. Cucchieri, T. Mendes, Braz. J. Phys. 37N1B (2007) 219.
16. C.S. Fischer, A. Maas, J.M. Pawlowski, L. von Smekal (2007), hep-ph/0701050.

Post LATTICE-2007 results

$SU(3)$ Gluon propagator: $\beta = 5.70$.





$\Rightarrow$ Maximum of Gluon propagator observed !

Perspectives of Propagators' Simulation

- Supercomputers of next generations
- Simulation on asymmetric lattices, $N_s^3 \times N_t$
- Nonstandard new approaches are of interest
- New approaches to numerical inversion of Faddeev-Popov lattice operator are highly desirable!