Higgs decay to $\bar{b}b$: different approaches to resummation of QCD effects.

A.L.Kataev (INR, Moscow),V.T. Kim (PNPI, Gatchina) Main quantity under study $\Gamma(H^0 \to b\bar{b})=$ $=\Gamma_{H\bar{b}b}$ with the mass 115 GeV $\leq M_H \leq 2M_W$, calculated in the $\overline{MS}$ up to the α_s^4 corrections. This quantity is important, though at SMC and ATLAS at LHC H-decay to $\bar{b}b$ will have huge background, but may be seen clearly at TOTEM LHC detector (study of diffraction production). This decay mode is dominating in the sum of decay widths, and thus is dominating in the branching ratio of Higgs to $\gamma\gamma$. What is theoretical error of $\Gamma_{H\bar{b}b}$?

Cases to be considered in this talk

1. $\alpha_s(M_H)$, $\overline{MS}$-scheme, m_b-on shell **Kataev, Kim (94+new)**;
2. $\alpha_s(M_H)$ and $\overline{m}_b(M_H)$ $\overline{MS}$-scheme; calculated up to α_s^4-level; (α_s^4 massless term- **Baikov,Chetyrkin,Kuhn (06)**
3. invariant mass $\hat{m}_b$, the resummation of effects of analytic continuation within in β_0 approximation (β_0-first coefficient of the QCD β-function renormalon-inspired approximation) definition of special parameters in every order of PT (analog of **Shirkov, Solovtsev (96)** analatized perturbation theory) with fractional power of α_s, i.e. $\nu_0 = 2\gamma_0/\beta_0$, γ_0-first coefficient of anomalous dimension function- **Broadhurst, Kataev, Maxwell (01)**
4. invariant mass $\hat{m}_b$, the resummation of effects of analytic continuation within analatized perturbation theory with fractional power **Bakulev, Mikhailov, Stefanis (07)**

Some definitions in terms of $a_s = \alpha_s/\pi$

$$\Gamma_{Hb\bar{b}} = \Gamma_0^b \left(\beta^3 [1 + \delta_{NLO} a_s + \delta_{NNLO} a_s^2 + \delta_{N^3LO} a_s^3 + \delta_{N^4LO} a_s^4 \right) \quad (1)$$

$$\Gamma_0^b = \frac{3\sqrt{2}}{8\pi} G_F M_H m_b^2 \ ; \ \beta = \sqrt{1 - \frac{4m_b^2}{M_H^2}} \quad (2)$$

$$\delta_{NLO} = \frac{4}{3}\beta^2 A(\beta) + \frac{3 + 34\beta^2 - 13\beta^4}{16} \ln \frac{(1+\beta)}{(1-\beta)} + \beta \frac{3(-1+7\beta^2)}{8} \quad (3)$$

$$A(\beta) = (1+\beta^2)\left[4Li_2\left(\frac{1-\beta}{1+\beta}\right) + 2Li_2\left(-\frac{1-\beta}{1+\beta}\right) - 3\ln\frac{2}{1+\beta}\ln\frac{1+\beta}{1-\beta} \right. \quad (4)$$

$$\left. -2\ln\frac{1+\beta}{1-\beta}\ln\beta \right] - 3\beta\ln\frac{4}{1-\beta^2} - 4\beta\ln\beta \quad (5)$$

$Li_2(x) = \int_0^x (dt/t)\ln(1-t)$ only massive dependence δ_{NNLO}-term is known up to m_b^2/M_H^2

The relation of the m_b-pole case with $\overline{m}_b(M_H)$-case and $a_s(M_H)$ is

$$\begin{aligned}\Gamma_{Hb\bar{b}} &= \Gamma_0^{(b)}\frac{\overline{m}_b{}^2}{m_b^2}\Bigg[\Bigg(1+\Delta\Gamma_1 a_s+\Delta\Gamma_2 a_s^2+\Delta\Gamma_3 a_s^3+\Delta\Gamma_4 a_s^4\Bigg) \qquad (6)\\ &- 6\frac{\overline{m}_b^2}{M_H^2}[1+\Delta\Gamma_1^m a_s+\Delta\Gamma_2^m a_s^2]\Bigg]\end{aligned}$$

Relations among different definitions of masses in QCD : running quark masses in different renormalization points are defined as

$$\overline{m}_b(\mu)=\overline{m}_b(\overline{m}_b)\exp\Bigg[-\int_{\alpha_{\mathrm{s}}(\overline{\mathrm{m}}_{\mathrm{b}})}^{\alpha_{\mathrm{s}}(\mu)}\frac{\gamma_{\mathrm{m}}(\mathrm{x})}{\beta(\mathrm{x})}\mathrm{dx}\Bigg] \quad .\text{where} \qquad (7)$$

the renormalization group functions are known up to 4-loop level

$$\mu^2\frac{da_s}{d\mu^2}=\beta(a)=-\beta_0 a_s^2-\beta_1 a_s^3-\beta_2 a_s^4-\beta_3 a_s^5+O(a_s^6) \qquad (8)$$

$$\frac{d\ln\overline{m}_b}{d\ln\mu^2}=\gamma_m(a_s)=-\gamma_0 a_s-\gamma_1 a_s^2-\gamma_2 a_s^3-\gamma_3 a_s^4+O(a_s^5) \qquad (9)$$

The relations are thus

$$\overline{m}_b(\mu) = \overline{m}_b(\overline{m}_b)\left(\frac{a_s(\mu)}{a_s(\overline{m}_b)}\right)^{\gamma_0/\beta_0} \frac{AD(a_s(\mu))}{AD(a_s(\overline{m}_b))} \tag{10}$$

$$AD(a_s) = \left[1+P_1 a_s+\left(P_1^2+P_2\right)\frac{a_s^2}{2}+\left(\frac{1}{2}P_1^3+\frac{3}{2}P_1 P_2+P_3\right)\frac{a_s^3}{3}\right] \tag{11}$$

$$\begin{aligned} P_1 &= -\frac{\beta_1\gamma_0}{\beta_0^2}+\frac{\gamma_1}{\beta_0} \quad , P_2 = \frac{\gamma_0}{\beta_0^2}\left(\frac{\beta_1^2}{\beta_0}-\beta_2\right)-\frac{\beta_1\gamma_1}{\beta_0^2}+\frac{\gamma_2}{\beta_0} \\ P_3 &= \left[\frac{\beta_1\beta_2}{\beta_0}-\frac{\beta_1}{\beta_0}\left(\frac{\beta_1^2}{\beta_0}-\beta_2\right)-\beta_3\right]\frac{\gamma_0}{\beta_0^2}+\frac{\gamma_1}{\beta_0^2}\left(\frac{\beta_1^2}{\beta_0}-\beta_2\right)-\frac{\beta_1\gamma_2}{\beta_0^2}+\frac{\gamma_3}{\beta_0} \end{aligned} \tag{12}$$

The **invariant** mass $\hat{m}_b$ as defined by **Becchi et al (81)** is simpler to translate to

$$\hat{m}_b = \overline{m}_b(\mu)\left[(a_s(\mu))^{\frac{\gamma_0}{\beta^0}} AD(a_s(\mu))\right]^{-1} \tag{13}$$

Let us define the coefficients in the basic formula in terms of $\overline{m}_b(M_H)$ and $a_s(M_H)$ for N_f=5.

$$\begin{aligned}
\Gamma_{Hb\bar{b}} &= \Gamma_0^{(b)} \frac{\overline{m}_b{}^2}{m_b^2} \Bigg[\Bigg(1 + \Delta\Gamma_1 a_s + \Delta\Gamma_2 a_s^2 + \Delta\Gamma_3 a_s^3 + \Delta\Gamma_4 a_s^4 \Bigg) \qquad (14) \\
&- \frac{6\overline{m}_b^2}{M_H^2} [1 + \Delta\Gamma_1^{(m)} a_s + \Delta\Gamma_2^{(m)} a_s^2] \Bigg] \\
\Delta\Gamma_1 &= \frac{17}{3} = 5.667 \quad , \Delta\Gamma_2 = d_2^E - \gamma_\mathbf{0}(\beta_\mathbf{0} + \mathbf{2}\gamma_\mathbf{0})\pi^\mathbf{2}/\mathbf{3} = \mathbf{29.147} \qquad (15) \\
\Delta\Gamma_3 &= d_3^E - \Big[d_1(\beta_\mathbf{0} + \gamma_\mathbf{0})(\beta_\mathbf{0} + \mathbf{2}\gamma_\mathbf{0}) + \beta_\mathbf{1}\gamma_\mathbf{0} + \mathbf{2}\gamma_\mathbf{1}(\beta_\mathbf{0} + \mathbf{2}\gamma_\mathbf{0}) \Big] \pi^\mathbf{2}/\mathbf{3} = \mathbf{41.178} \\
\Delta\Gamma_4 &= d_4^E - \Big[d_2(\beta_\mathbf{0} + \gamma_\mathbf{0})(\mathbf{3}\beta_\mathbf{0} + \mathbf{2}\gamma_\mathbf{0}) + \mathbf{d_1}\beta_\mathbf{1}(\mathbf{5}\beta_\mathbf{0} + \mathbf{6}\gamma_\mathbf{0})/\mathbf{2} + \mathbf{4d_1}\gamma_\mathbf{1}(\beta_\mathbf{0} + \gamma_\mathbf{0}) \\
&\quad + \beta_2\gamma_0 + 2\gamma_1(\beta_1 + \gamma_1) + \gamma_2(3\beta_0 + 4\gamma_0) \Big] \pi^2/3 \qquad (16) \\
&\quad + \gamma_\mathbf{0}(\beta_\mathbf{0} + \gamma_\mathbf{0})(\beta_\mathbf{0} + \mathbf{2}\gamma_\mathbf{0})(\mathbf{3}\beta_\mathbf{0} + \mathbf{2}\gamma_\mathbf{0})\pi^\mathbf{4}/\mathbf{30} = -\mathbf{825.7}
\end{aligned}$$

It is also possible to get transformation from $m_b(M_H)$ to m_b -pole, known at the α_s^3 level : to translate first from $\overline{m}_b(M_H)$ to $\overline{m}_b(m_b)$ then use

$$\overline{m}_b^2(m_b) = m_b^2 \Bigg(1 - 2.67 a_s(m_b) - 18.57 a_s(m_b)^2 - 175.79 a_s^3(m_b) \Bigg)$$

So, one has **Approach N1**:
Truncated series for $\Gamma_{Hb\bar{b}}$, the dependence from M_H in $\overline{m}_b(M_H)$ and $\alpha_s(M_H)$, which we are expressing as

$$\alpha_s(\mu)_{NLO} = \frac{\pi}{\beta_0 \mathrm{Log}}\left[1 - \frac{\beta_1 \ln(\mathrm{Log})}{\beta_0^2 \mathrm{Log}^2}\right] \quad (17)$$

$$\alpha_s(\mu)_{NNLO} = \alpha_s(M_H)_{NLO} + \Delta\alpha_s(M_H)_{MNLO}$$

$$\alpha_s(\mu)_{N^3LO} = \alpha_s(M_H)_{NNLO} + \Delta\alpha_s(M_H)_{N^3LO} \quad (18)$$

$$\Delta\alpha_s(M_H)_{NNLO} = \frac{\pi}{\beta_0^5 \mathrm{Log}^3}\left(\beta_1^2 \ln^2(\mathrm{Log}) - \beta_1^2 \ln(\mathrm{Log}) + \beta_2\beta_0 - \beta_1^2\right)$$

$$\Delta\alpha_s(\mu)_{N^3LO} = \frac{\pi}{\beta_0^7 \mathrm{Log}^4}\left[\beta_1^3\left(- \ln^3(\mathrm{Log}) + \frac{5}{2}\ln^2(\mathrm{Log}) + 2\ln(\mathrm{Log}) - \frac{1}{2}\right)\right.$$
$$\left. -3\beta_0\beta_1\beta_2 \ln(\mathrm{Log}) + \beta_0^2\frac{\beta_3}{2}\right] \text{ where } \mathrm{Log} = \ln(\mu^2/\Lambda_{\overline{\mathrm{MS}}}^{(\mathrm{f}=5)\ 2})$$

and using $m_b = 4.7$ GeV and $\overline{m}_b(\overline{m}_b) = 4.34$ GeV [**Penin, Steinhauser (02)**] and NLO $\Lambda^{(5)}_{\overline{\rm MS}} = 253^{+163}_{-74}$MeV, NNLO $\Lambda^{(5)}_{\overline{\rm MS}} = 220^{+53}_{-31}$MeV, N^{3}LO $\Lambda^{(5)}_{\overline{\rm MS}} = 220^{+39}_{-23}$MeV which come from the analysis of CCFR data by **Kataev, Sidorov, Parente (01-03)**

Approach N2: Use trancated m_b parameterization

$$\begin{aligned} \Gamma_{Hb\bar{b}} &= \Gamma_0^{(b)}\Bigg[\Bigg(1 + \Delta\tilde{\Gamma}_1 a_s + \Delta\tilde{\Gamma}_2 a_s^2 + \Delta\tilde{\Gamma}_3 a_s^3 + \Bigg] \\ &- 6\frac{m_b^2}{M_H^2}[1 + \Delta\tilde{\Gamma}_1^{(m)} a_s + \Delta\tilde{\Gamma}_2^{(m)} a_s^2]\Bigg] \end{aligned} \tag{19}$$

$\Delta\Gamma_1 = \Bigg(3 - 2L\Bigg)$; $\Delta\Gamma_1^{(m)} = (\frac{4}{3} - 4L)$, where $L = ln(M_H^2/m_b^2)$.

$\Delta\Gamma_2 = \Bigg(-4.52 - 18.138L + 0.084L^2\Bigg)$; $\Delta\Gamma_2^{(m)}$-included and

$\Delta\Gamma_3 = \Bigg(-316.906 - 133.421L - 1.153L^2 + 0.05L^3\Bigg)$ only now.

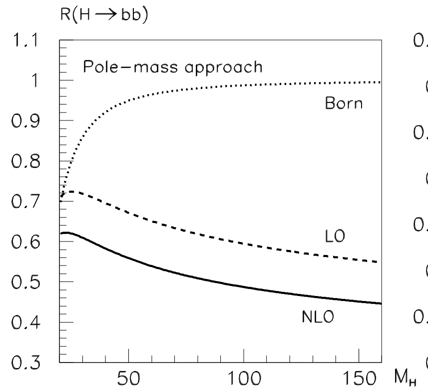


Fig.1

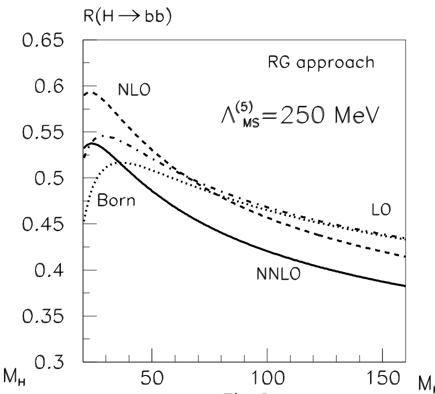


Fig.2

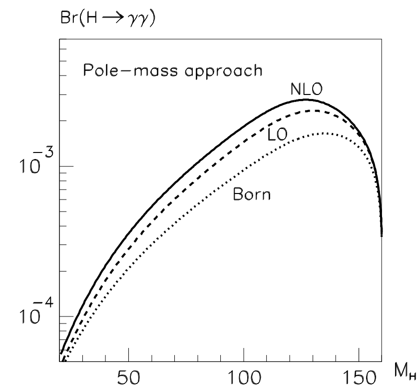


Fig.3

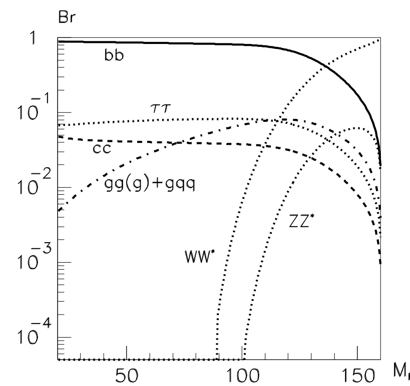
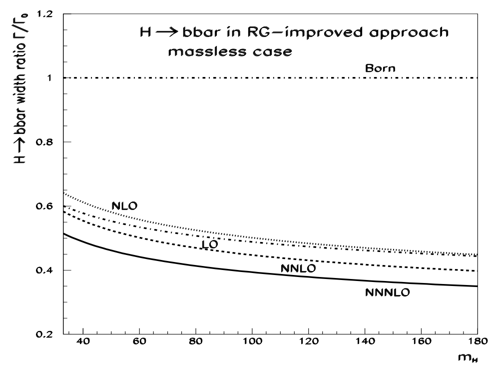
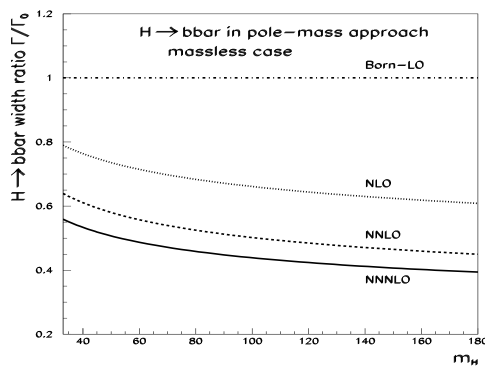


Fig.4

The results for $R_{H\bar{b}b} = \Gamma_{H\bar{b}b}/\Gamma_0^{(b)}$ $(\Gamma_0^{(b)} = \frac{3\sqrt{2}}{8\pi} G_F M_H m_b^2)$ at α_s^2-level. Approach 2 is compared with Aproach 1. At the next page the comparison is made at the α_s^3 level in the massless case





1) Results for "running approach" are rather stable, effects of coefficient functions are not very large. 2) In the on-shell scheme large logs are important and are decreasing the results, making them comparable with the "running case", In this case the corrections to RG function and coefficient functions are seen more clearly, in particular in the approaches with resummations of the π^2 terms (**Krasnikov, Pivovarov (82), Radyushkin (82), Shirkov (00)**)

Consider the resummation of π^2-terms in $\Gamma_{Hb\bar{b}}$ in the case of running or invariant masses. In these cases the RG-evolution is starting from $\alpha_s(s)^{2\frac{\gamma_0}{\beta_0}}$. The summation of the π^2 leading terms with fractional power were used by **Gorishny, Kataev, Larin (84)**. It was considered more carefully in **BKM (01)** and in more detail by **BMS (07)**. Using notations of this paper let us define

$$\tilde{R}_S(M_H) = \frac{8\pi}{\sqrt{2}G_F M_H}\Gamma(H \to b\bar{b}) \tag{20}$$

In the $\overline{MS}$-scheme

$$\tilde{R}_S(M_H) = 3\overline{m}_b^2(M_H)\left[1 + \sum_{i=1}^{4} \Delta\Gamma_i a_s(M_H)^i\right] \tag{21}$$

The BKM expression with 1-loop coupling constant is

$$
\begin{aligned}
\widetilde{R}_{\mathrm{S}}^{\mathrm{BKM}} &= 3\,\hat{m}_b^2\,(a_s)^{\nu_0}\left[A_0^{\mathrm{BKM}}(a_s)+\sum_{n\geq 1} d_n\,A_n^{\mathrm{BKM}}(a_s)\right],\\
A_n^{\mathrm{BKM}}(a_s) &= \frac{4}{\beta_0\,\pi\,\delta_n}\left[1+\left(\frac{\beta_0\,\pi\,a_s}{4}\right)^2\right]^{-\delta_n/2}(a_s)^{n-1}\sin\left(\delta_n\arctan\left(\frac{b_0\,\pi\,a_s}{4}\right)\right),\\
\delta_n &= n+\nu_0-1\,,\quad \nu_O=2(\gamma_0/\beta_0).
\end{aligned}
$$

In the Fractional Analytic Perturbation Theory BMS obtained

$$
\widetilde{R}_{\mathrm{S}}^{(l)\mathrm{BMS}} = 3\,\hat{m}_{(l)}^2\left[\mathfrak{a}_{\nu_0}^{(l)}+\sum_{n\geq 1}^{l} d_n\,\mathfrak{a}_{n+\nu_0}^{(l)}+\sum_{m\geq 1}^{l+4}\Delta_m^{(l)}\,\mathfrak{a}_{m+\nu_0}^{(l)}\right]
$$

The terms $\mathfrak{a}_{n+\nu_0}^{(l)}$ are summing β_0, γ_0 terms $(1\leq l\leq 4)$ and proportional to them π^2, higher orders in γ_i and β_i are accumulated in the coefficients $\Delta_m^{(l)}\ \mathfrak{a}_{n+\nu_0}^{(l)}=(a_s)^{\frac{2\gamma_0}{\beta_0}}A_n(a_s)$, the letter are rather closed to $A_n^{\mathrm{BKM}}(a_s)$. Next figure is from BMS paper.

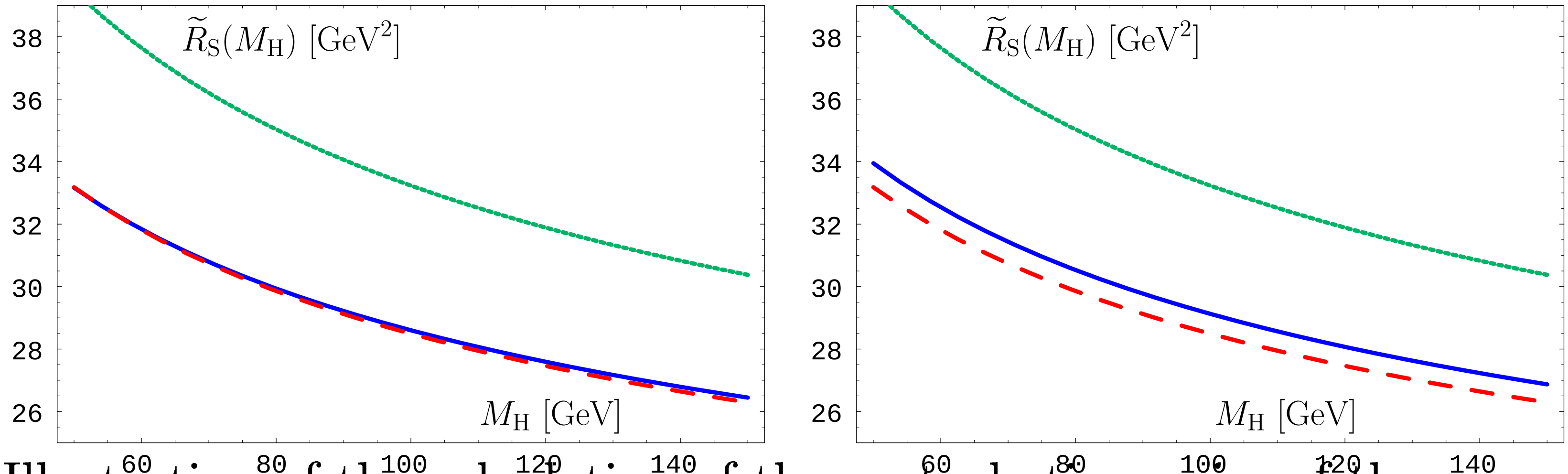


Illustration of the calculation of the perturbative series of the quantity $\widetilde{R}_{\rm S}(M_{\rm H}^2)$ in different approaches within the $\overline{MS}$ scheme: Standard perturbative QCD at the loop level $l = 4$ (dashed red line), BKM estimates, by taking into account the $O((a_s)^{\nu_0}\, A_4(a_s))$-terms, —(dotted green line), and finally MFAPT from for $N_f = 5$ (solid blue line), displayed for $l = 2$ (left panel) and $l = 3$ (right panel). These figures are from BMS (07) paper

In spite of different definitions of the mass parameters, the results in presented plots for

$$R_{Hb\bar{b}} = \frac{\widetilde{R}_{\rm S}(M_{\rm H})}{3m_b^2} \tag{22}$$

are in agreement with the results, given in the BMS (07) paper. Thus, calculated from BMS (07) results interval for $R_{Hb\bar{b}} \approx 0.48 - 0.42$ at $M_H = 120$ GeV should be compared with $NNLO$ $R_{Hb\bar{b}} = 0.42$ in case of on-shell parameterization, and $R_{Hb\bar{b}} = 0.4$ in case of slightly different parameterization of the QCD effects in the $\overline{MS}$-scheme.

Estimates of theoretical uncertainties of $\Gamma_{H\bar{b}b}$

The proposal: to estimate theoretical uncertainties by analyzing the difference of different theoretical approximations for $\Gamma_{H\bar{b}b}$.

In fact at M_H=120 GeV, and considering last two plots we get $\Delta\widetilde{R}_S(M_H) \sim 3$ GeV^2. On another hand we have:

$$\Gamma_{H\bar{b}b} = \frac{\sqrt{2}G_F}{8\pi} M_H \widetilde{R}_{\mathrm{S}}(M_H) \tag{23}$$

For $M_H = 120\ GeV$ and $G_F = 1.166 \times 10^{-5}\ GeV^{-2}$ we get $\Delta\Gamma_{H\bar{b}b} \approx 30 \times 10^{-5}\ GeV$ $\Gamma_{H\bar{b}b} \approx 236 \times 10^{-5}$ GeV Our proposed value of theoretical uncertainty $\Gamma_{H\bar{b}b}$ is $\pm 15 \times 10^{-5}\ GeV$.

Conclusions

- The results of different analysis of the effects of $O(\alpha_s^3)$ agree.
- The results of calculations of α_s^4 were rather useful and important. They clearly demonstrate that asymptotic of PT theory series are different in Eucledian and Minkowskian regions.
- The estimate of theoretical presision of $\Gamma_{H\bar{b}b}$ is proposed. It is possible to check its possible stability to higher order-effects α_s^4-effects.