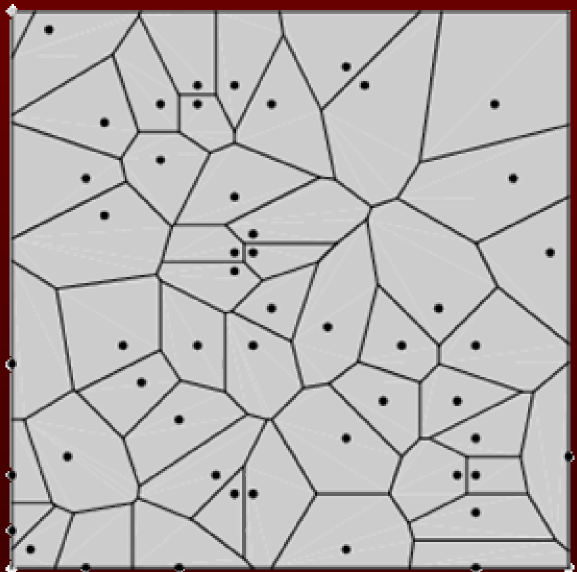
Effective Lagrangians
and
Field Theory on the lattice



Oleg V. Pavlovsky

(ITPM MSU, Moscow)

Lomonosov Conference, Moscow 2007

Method of the derivation of Chiral Effective Theories from Lattice QCD

- Why QCD on the lattice. Lattice QCD as regularization scheme.
- Analytical integration on the lattice – how it’s possible?
- First Step – Strong coupling regime
- Lattice and space-time symmetries: lattice artifacts
- Solution: Random Lattice!
- Restoration of space-time symmetries on the Random Lattice and zero-order chiral effective theory
- Derivation of Chiral Effective Theories from Lattice QCD – way is free!

Starting point – Wilson lattice action

$$Z = \int [DG][D\psi][D\overline{\psi}]e^{-S_G - S_\psi}$$

where

$$S_G = \beta \cdot \sum_P \left[1 - \frac{1}{N_c} \mathrm{Re}\left\{Tr\left(G_{x,\mu} G_{x-\mu,\nu} G^{+}_{x+\nu,\mu} G^{+}_{x,\nu}\right)\right\}\right]$$

$$S_\psi = \sum_{x,\mu} Tr\left(\bar{A}_{x,\mu} G_{x,\mu} + A_{x,\mu} G^{+}{}_{x,\mu}\right)$$

Analytical integration on the lattice? What happen with plaguet intergals?

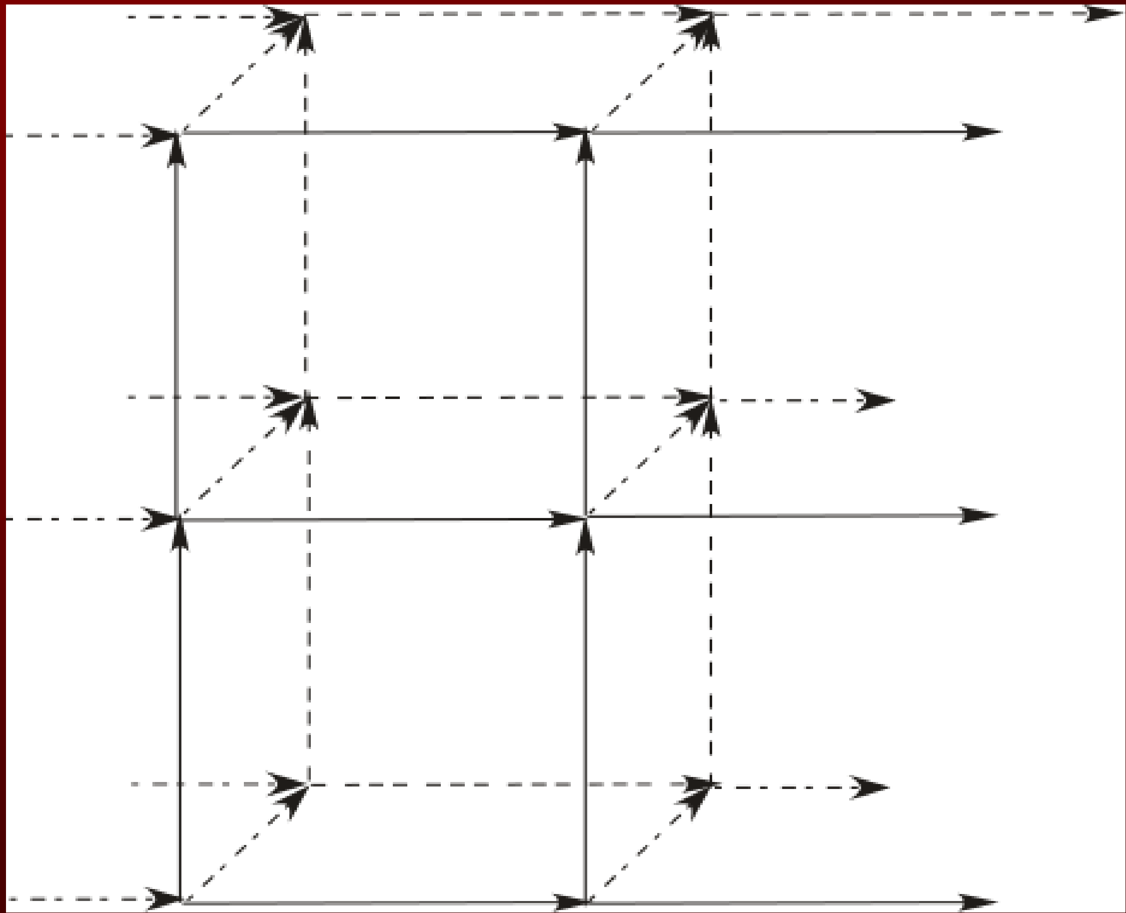
$$S_G = \beta \cdot \sum_P \left[1 - \frac{1}{N_c} \mathrm{Re}\left\{Tr\left(G_{x,\mu} G_{x-\mu,\nu} G^{+}_{x+\nu,\mu} G^{+}_{x,\nu}\right)\right\}\right]$$

$$G_{x,\mu} = P \exp\left\{ig \int_{link} dx_\mu a_\mu\right\}$$

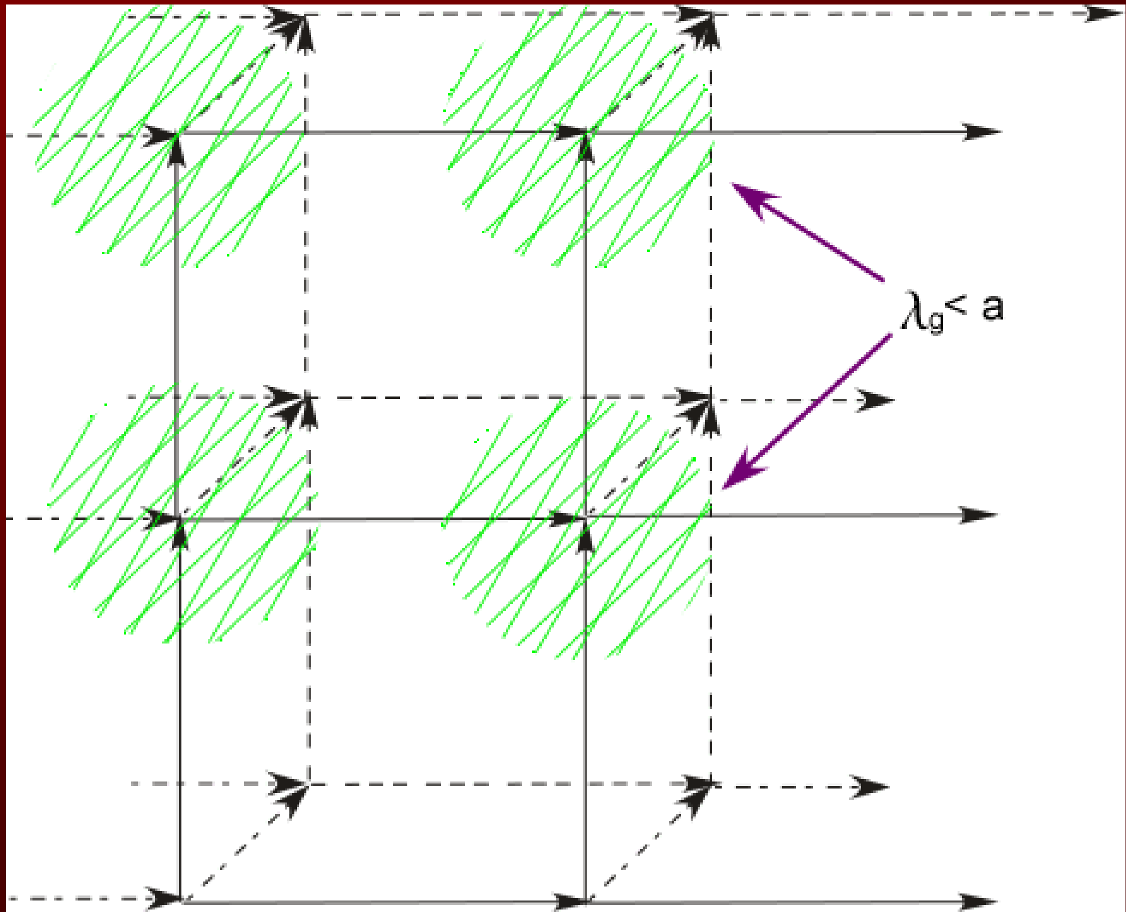
Integration on gluonic configuration is very complicated! And lattice-dependent :O(

Let us consider the perturbation theory by respect to small β **!**

Strong-coupling and gluon correlation length



Strong-coupling and gluon correlation length



Spacing a > 0.2 fm

Strong coupling regime = Link integrals domination

$$Z_0(\beta \to 0) = \int [DG][D\psi][D\overline{\psi}]e^{-S_\psi}$$

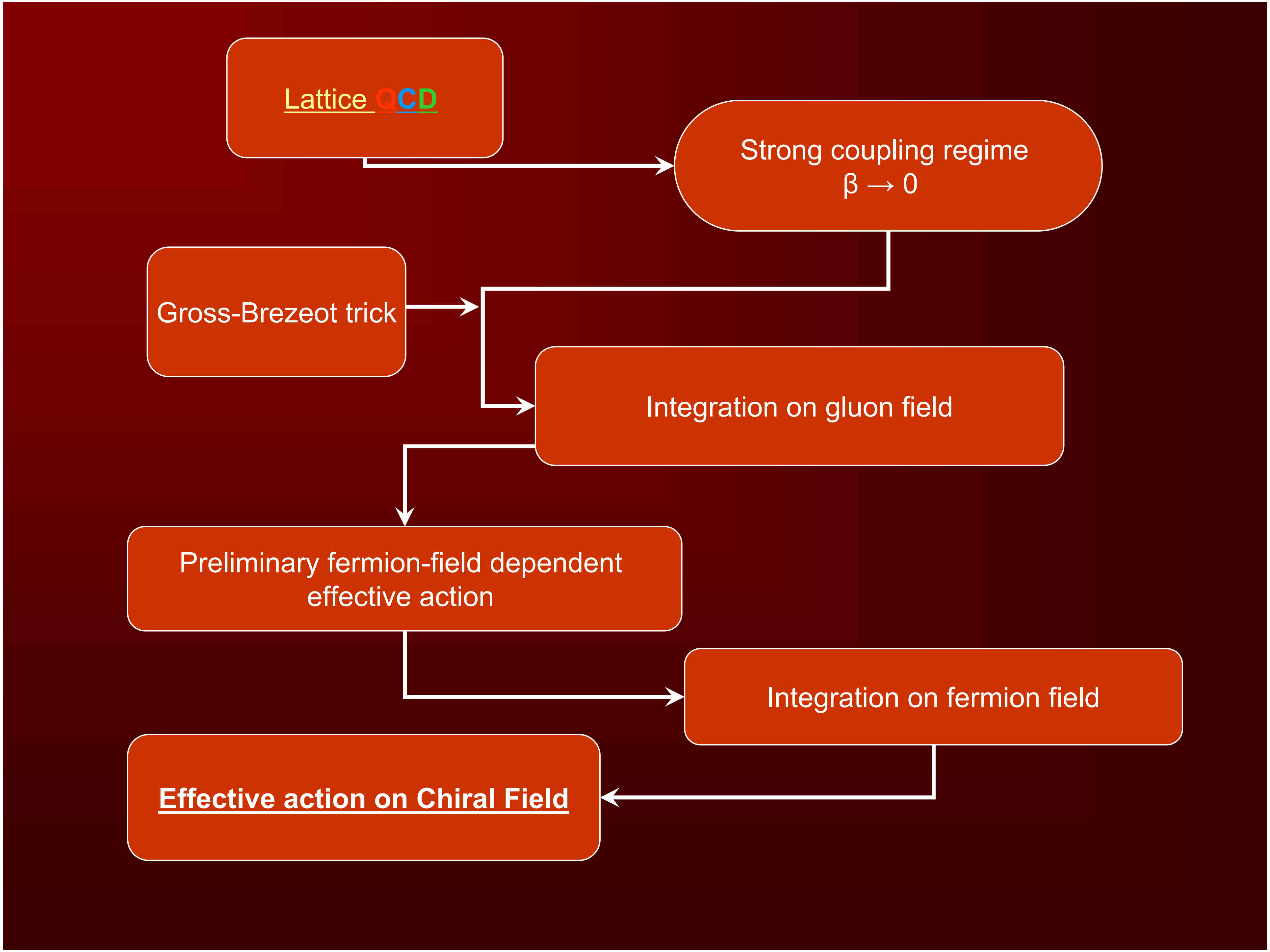
where

$$S_\psi = \sum_{x,\mu} Tr\left(\overline{A}_{x,\mu} G_{x,\mu} + A_{x,\mu} G^{+}{}_{x,\mu}\right)$$

$$A_{x,\mu} = \overline{\psi}_{x,\mu} P_\mu^{+} \psi_x$$

$$P_\mu^{\pm} = \frac{1}{2}\left(r \pm \gamma_\mu\right)$$

$$\overline{A}_{x,\mu} = \overline{\psi}_x P_\mu^{-} \psi_{x,\mu}$$



Lattice QCD
Strong coupling regime
β → 0
Gross-Brezeot trick
Integration on gluon field
Preliminary fermion-field dependent effective action
Integration on fermion field
Effective action on Chiral Field

Link intergal: integration on gluon field

$$Z_0\left(\beta=0\right)=\int[D\psi][D\overline{\psi}]\int[DG]\exp\left\{-\sum_{x,\mu}Tr\left(\overline{A}_{x,\mu}G_{x,\mu}+A_{x,\mu}G^{+}_{x,\mu}\right)\right\}$$

$$=\int[D\psi][D\overline{\psi}]\exp\{-W(\overline{A}A)\}$$

Gross-Brezeot trick

Phys.Lett. 97B(1980) 120

$$W(\lambda)=Tr\left[1-\sqrt{1-\lambda}\right]-Tr\left[\log\left(\left(1+\sqrt{1-\lambda}\right)/2\right)\right]$$

$$\lambda=-M(x)P_\mu^-M(x+\mu)P_\mu^+ \qquad M_{\alpha\beta}(x)=\frac{1}{N_c}\psi^a_\alpha\overline{\psi}^a_\beta$$

Integration on fermion field: color-chiral change of variables

$$Z_0 = \int [D\psi][D\overline{\psi}] \exp\left\{-\sum_{x,\mu} W(M)\right\} = \int [DM] \exp\left\{-\sum_{x,\mu} \tilde{W}(M)\right\}$$

Kawamoto-Smith method **Nucl.Phys. B192 (1981) 100**

<u>Representation of bosonic matrix *M* in terms of chiral fields</u>

$$M(x) = u_0 \exp\{i\gamma_5 \pi_i \tau_i / F_\pi\} = u_0 \left[U(x)\left(1+\gamma_5\right)/2 + U^+(x)\left(1-\gamma_5\right)/2\right]$$

$$U(x) = u_0 \exp\{i\pi_i \tau_i / F_\pi\}$$

Bosonic matrix M:

$M_{\alpha\beta}(x)$ is a metrix in the spin and flavor space representing an effective bosonic field

$$M_{\alpha\beta}(x) = M_0 \exp\{iS(x) + iP(x)\gamma_5 + iV_\mu(x)\gamma_\mu + iA_\mu(x)\gamma_\mu\gamma_5 + iT_{\mu\nu}(x)\sigma_{\mu\nu}\}$$

SU(2) Chiral scenario means the neglecting of contribution from Scalar, Vector, Axial-vector and Tensor mesons. Just only Pseudoscalers are taking into account!

$$M_{\alpha\beta}(x) = M_0 \exp\{iP(x)\gamma_5\}$$

Stationary point of action

Interaction on fermion degrees of freedom: stationary-point expansion

(S. Myint, C. Rebbi hep-lat/9401009, hep-lat/9401010)

$$S_{eff}(U) = -\sum_{k=1}^{\infty} \frac{W^{(k)}(\lambda_0)}{k!} \sum_{x,v} Tr\left[(\lambda_v(x) - \lambda_0)^k\right]$$

$$Tr\left[(\lambda_v(x)-\lambda_0)\right] = -2\lambda_0 Tr(\alpha)$$

$$\alpha = a^2 \nabla_v U \nabla_v U^+ + O(a^4)$$

$$Tr\left[(\lambda_v(x)-\lambda_0)^2\right] = 2\lambda^2{}_0 Tr(\alpha^2) - 4\lambda^2{}_0 Tr(\alpha)$$

$$U = \exp(i\vec{\phi}\vec{\tau} / f_\pi)$$

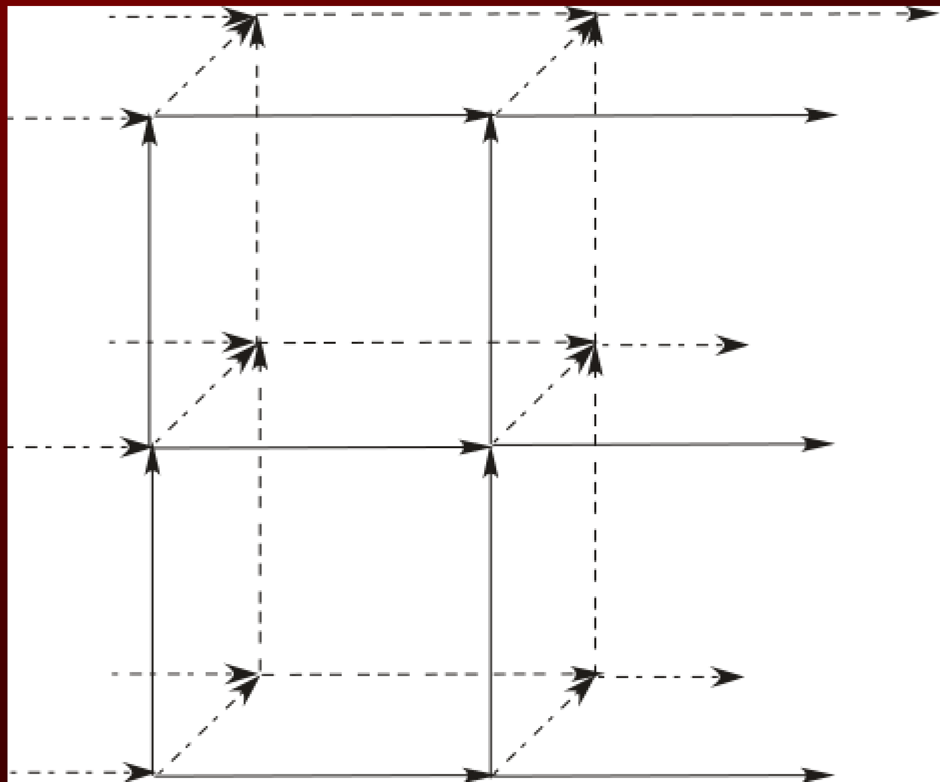
$$Tr\left[(\lambda_v(x)-\lambda_0)^3\right] = -2\lambda^3{}_0 Tr(\alpha^3) + 6\lambda^3{}_0 Tr(\alpha^2)$$

$$Tr\left[(\lambda_v(x)-\lambda_0)^4\right] = 2\lambda^4{}_0 Tr(\alpha^4) - 8\lambda^4{}_0 Tr(\alpha^3) + 4\lambda^3{}_0 Tr(\alpha^2)$$

$$Tr\left[(\lambda_v(x)-\lambda_0)^5\right] = -2\lambda^5{}_0 Tr(\alpha^5) + 10\lambda^5{}_0 Tr(\alpha^4) + \ldots\ldots\ldots\ldots$$

Integration on hyper-cubical lattice: O(4) symmetries violation!

Lattice basis = orthogonal vectors $\nu = i, j, k, t$



Integration on hyper-cubical lattice: O(4) symmetries violation!

Lattice basis = orthogonal vectors $\nu = i, j, k, t$

$$U = \exp(i\vec{\phi}\vec{\tau} / f_\pi)$$

1) $\nabla_\nu U \nabla_\nu U^+ = \nabla_i U \nabla_i U^+$

$$Tr\left[(\lambda_\nu(x) - \lambda_0)\right] = -2\lambda_0 Tr(\alpha) \longrightarrow \sim Tr(\partial_i U \partial_i U^+)$$

$$Tr\left[(\lambda_\nu(x) - \lambda_0)^2\right] = 2\lambda^2{}_0 Tr(\alpha^2) - 4\lambda^2{}_0 Tr(\alpha) \longrightarrow \cancel{\sim Tr(\partial_i U \partial_i U^+ \partial_i U \partial_i U^+)}$$

$$Tr\left[(\lambda_\nu(x) - \lambda_0)^3\right] = -2\lambda^3{}_0 Tr(\alpha^3) + 6\lambda^3{}_0 Tr(\alpha^2)$$

$$Tr\left[(\lambda_\nu(x) - \lambda_0)^4\right] = 2\lambda^4{}_0 Tr(\alpha^4) - 8\lambda^4{}_0 Tr(\alpha^3) + 4\lambda^3{}_0 Tr(\alpha^2)$$

$$Tr\left[(\lambda_\nu(x) - \lambda_0)^5\right] = -2\lambda^5{}_0 Tr(\alpha^5) + 10\lambda^5{}_0 Tr(\alpha^4) + \ldots\ldots\ldots\ldots$$

We have a problem with reproducing of space-time symmetry!

Cause: Violation of space-time symmetry to discrete group

Sequence: Lattice Artifacts!

Possible solutions: Try to find more symmetrical lattice!

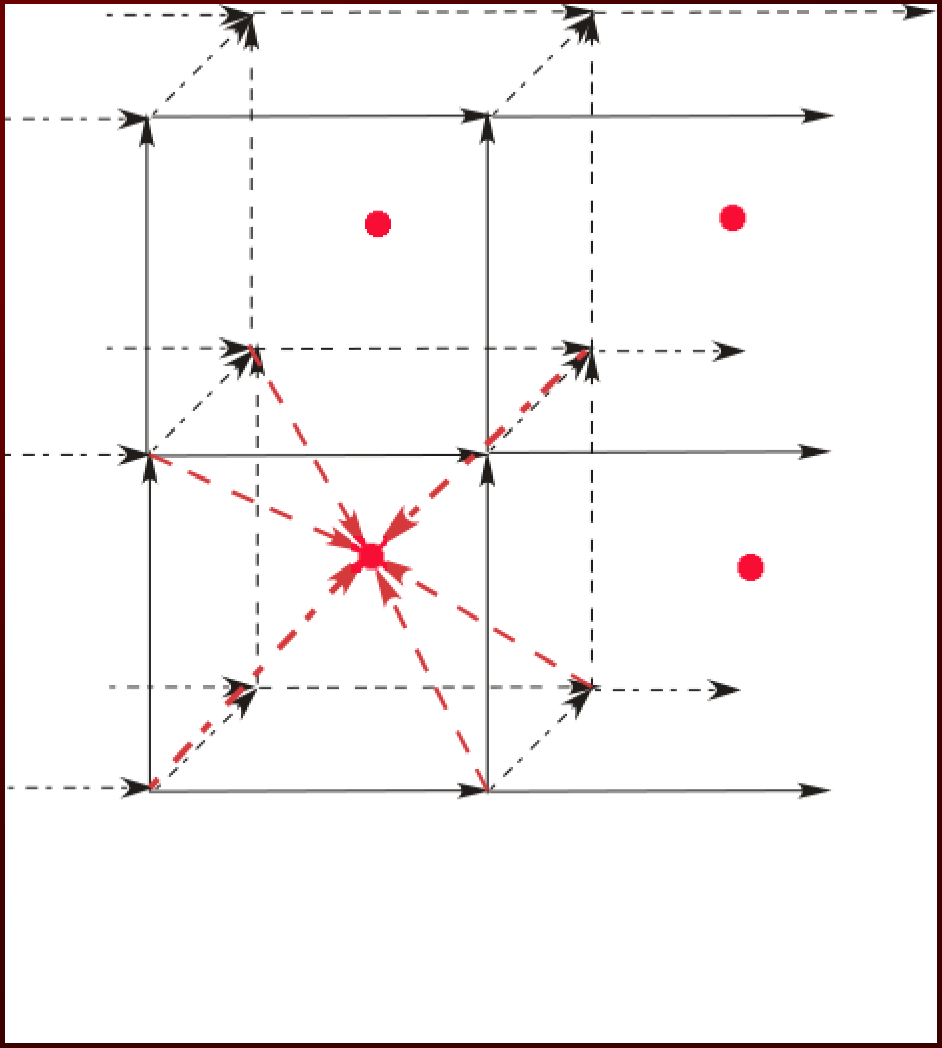
More symmetrical lattice in 4-dim:

Body Centered Hyper Cubical Lattice

Integration on hyper-cubical lattice with center element as step on the way to general solution

Basis vectors of the hyper-cubical lattice with center element

$$v_{ij}^{\pm} = \left(e_i \pm e_j\right) / \sqrt{2}$$



Integration on hyper-cubical lattice with center element as step on the way to general solution

Basis vectors of the hyper-cubical lattice with center element

$$v_{ij}^{\pm} = \left(e_i \pm e_j\right)/\sqrt{2}$$

$$\nabla_v U \nabla_v U^+ = \nabla_i U \nabla_i U^+ \qquad Tr\left[(\lambda_v(x)-\lambda_0)\right] \longrightarrow \sim Tr(\partial_i U \partial_i U^+)$$

$$Tr\left[(\lambda_v(x)-\lambda_0)^2\right]$$

$$\downarrow$$

$$\sim Tr(\partial_i U \partial_i U^+ \partial_j U \partial_j U^+ + \partial_i U \partial_j U^+ \partial_i U \partial_j U^+ + \partial_i U \partial_j U^+ \partial_j U \partial_i U^+)$$

$$Tr\left[(\lambda_v(x)-\lambda_0)^3\right] = -2\lambda^3{}_0 Tr(\alpha^3) + 6\lambda^3{}_0 Tr(\alpha^2) \longrightarrow$$

Non- invariance again!!

Unfortunately, Body Centered Hyper Cubical Lattice is not what we seek…

We need More symmetrical Lattice!

Unfortunately, there are no more symmetrical Lattice in 4-dim than BCHC!

No more REGULAR Lattice…

OK, no more regular lattice!

Let us to consider Random Lattice!

Random Lattice: historical remark

- Georgy Voronoi

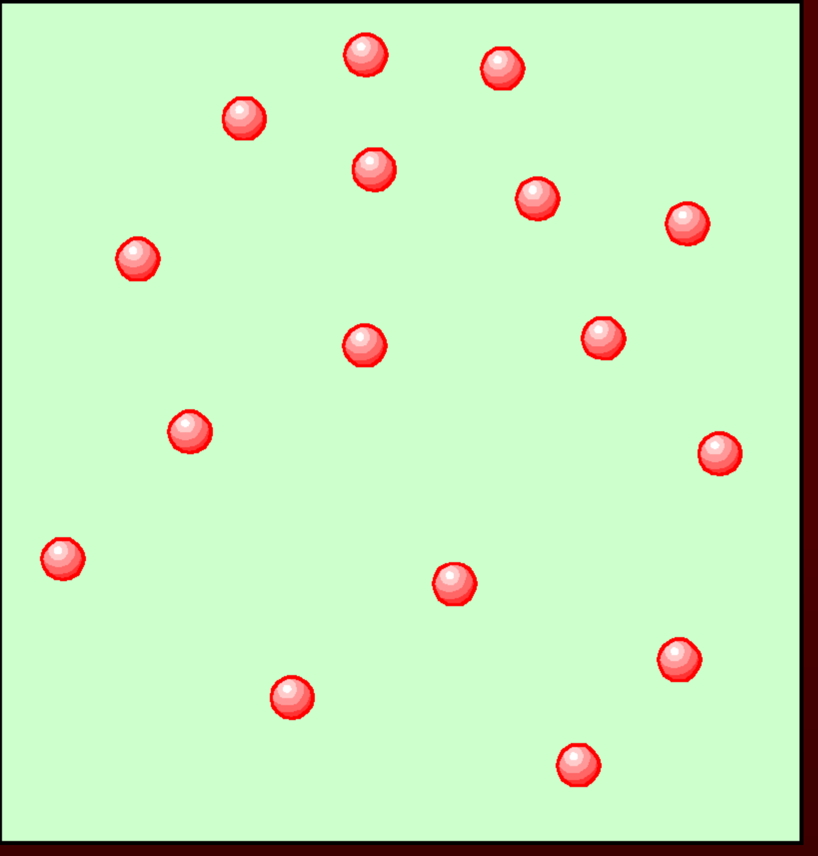
1868-1909 г

- Boris Delaunay

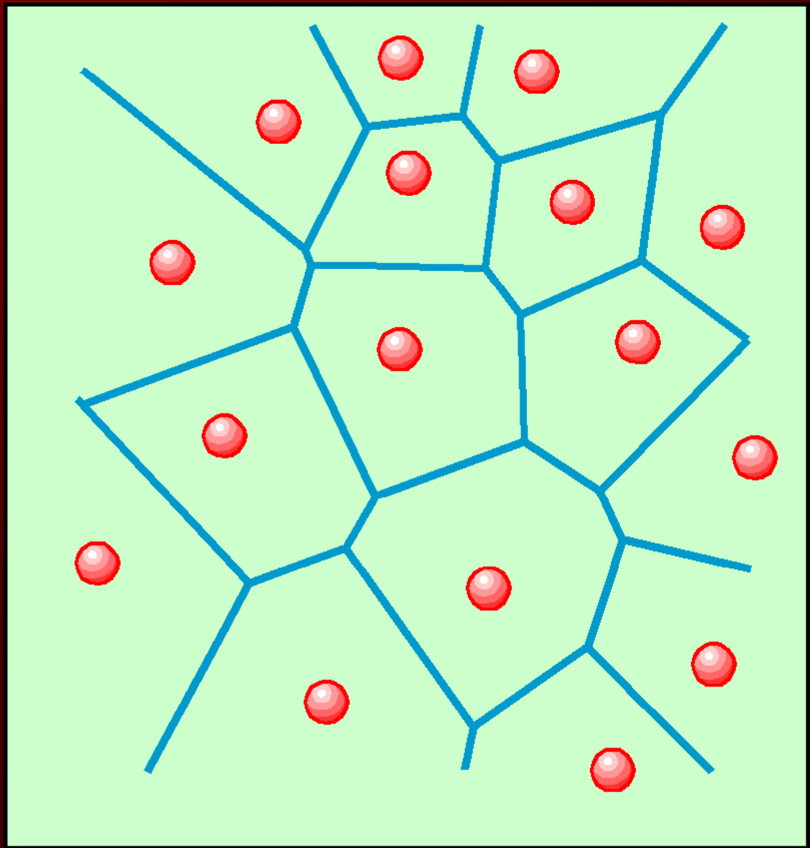
1890-1980 г

Random Lattice: step 1

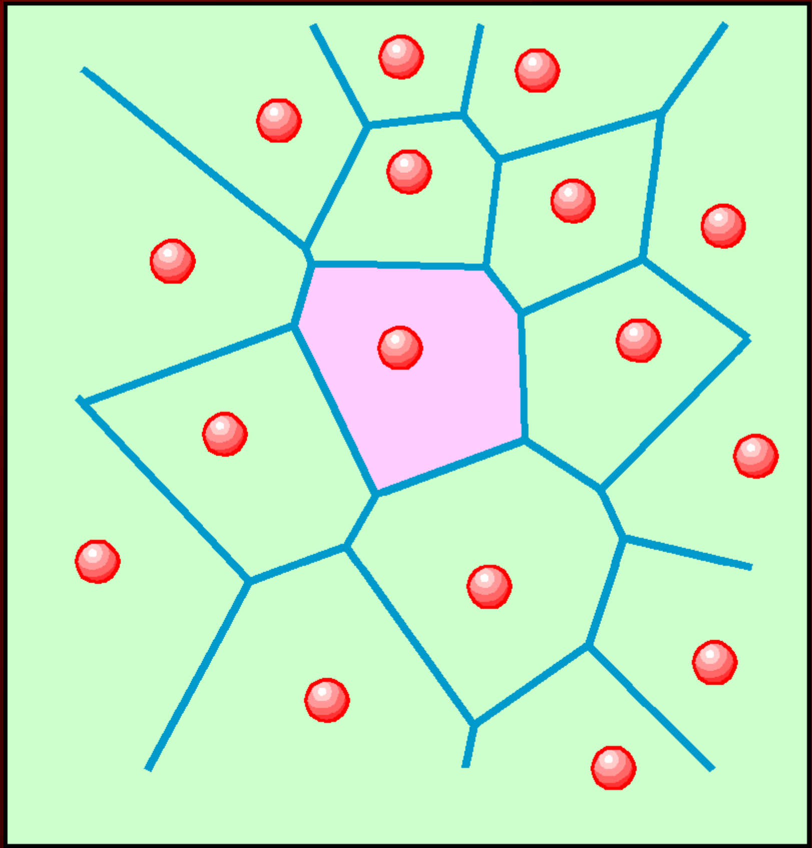
Let us consider Random distribution of centers



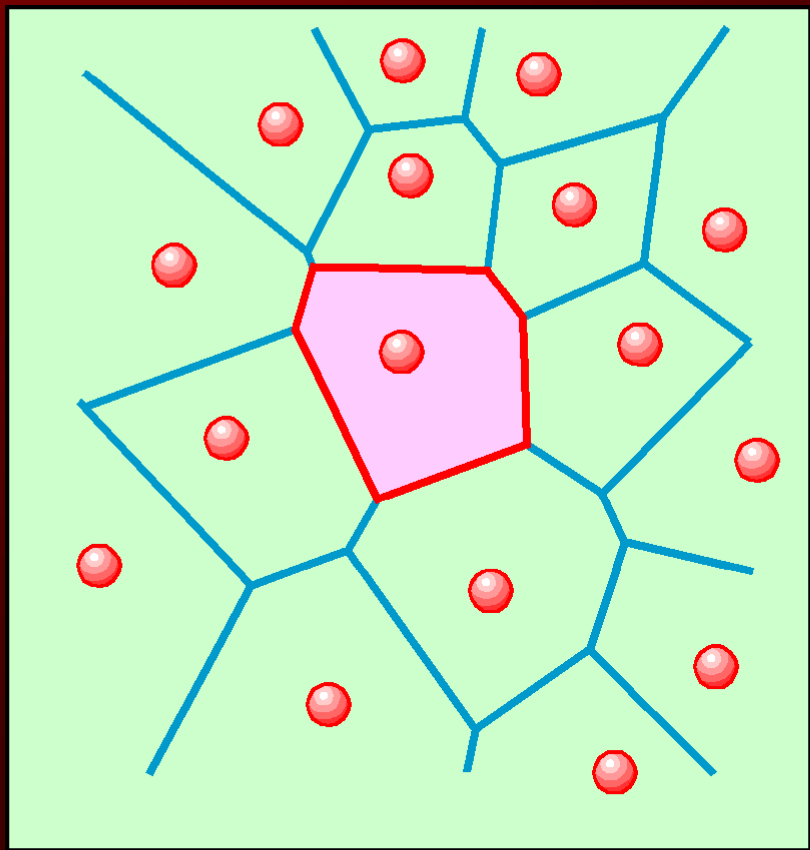
Random Lattice: step 2



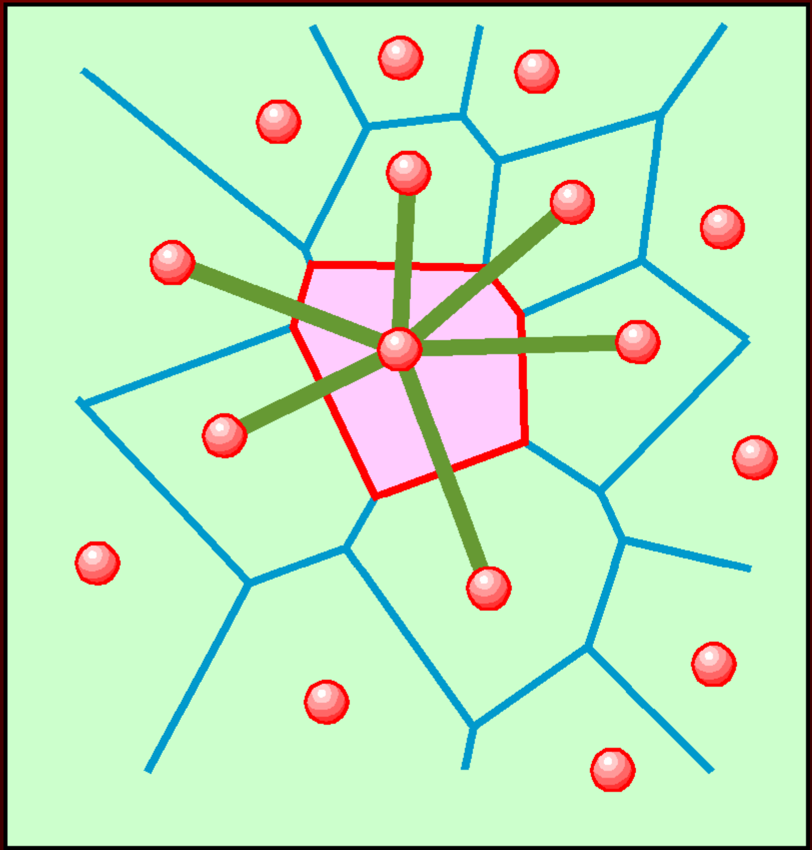
Random Lattice: step 3



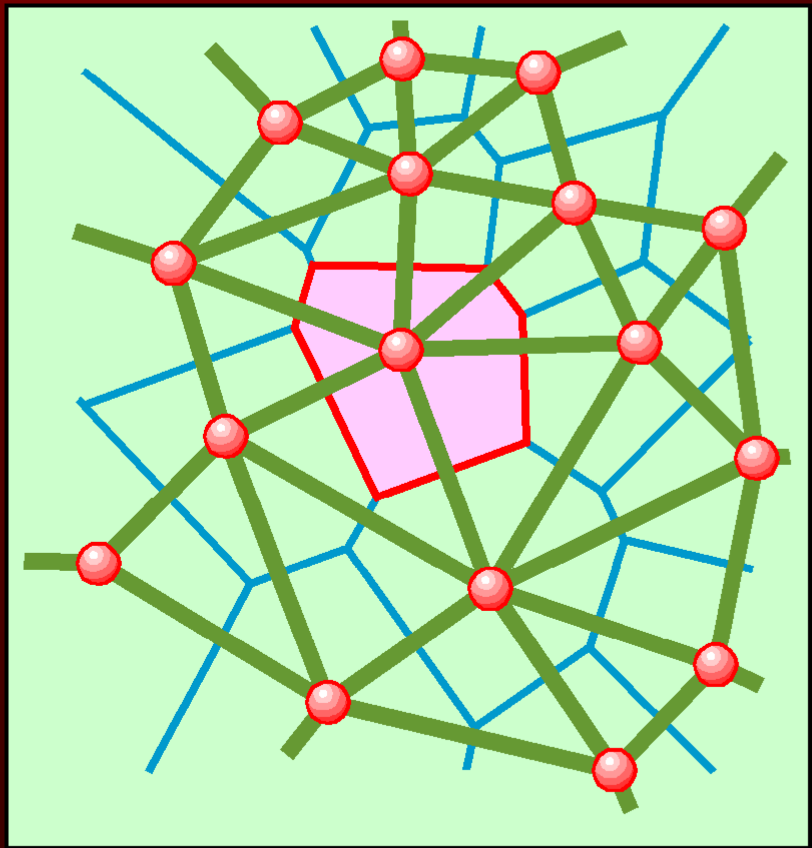
Random Lattice: step 4



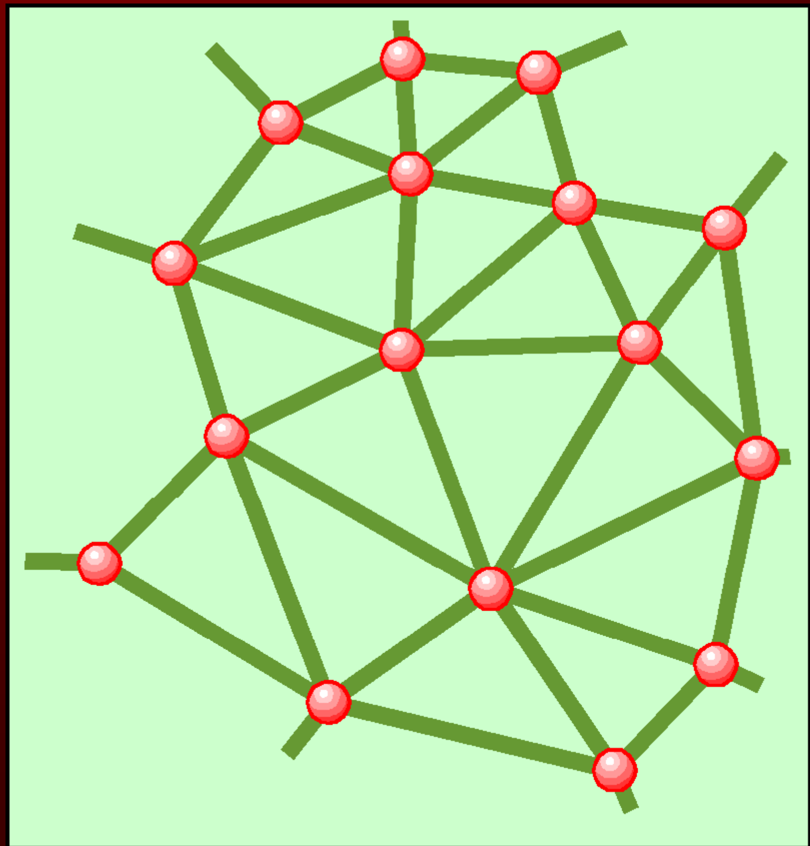
Random Lattice: step 5



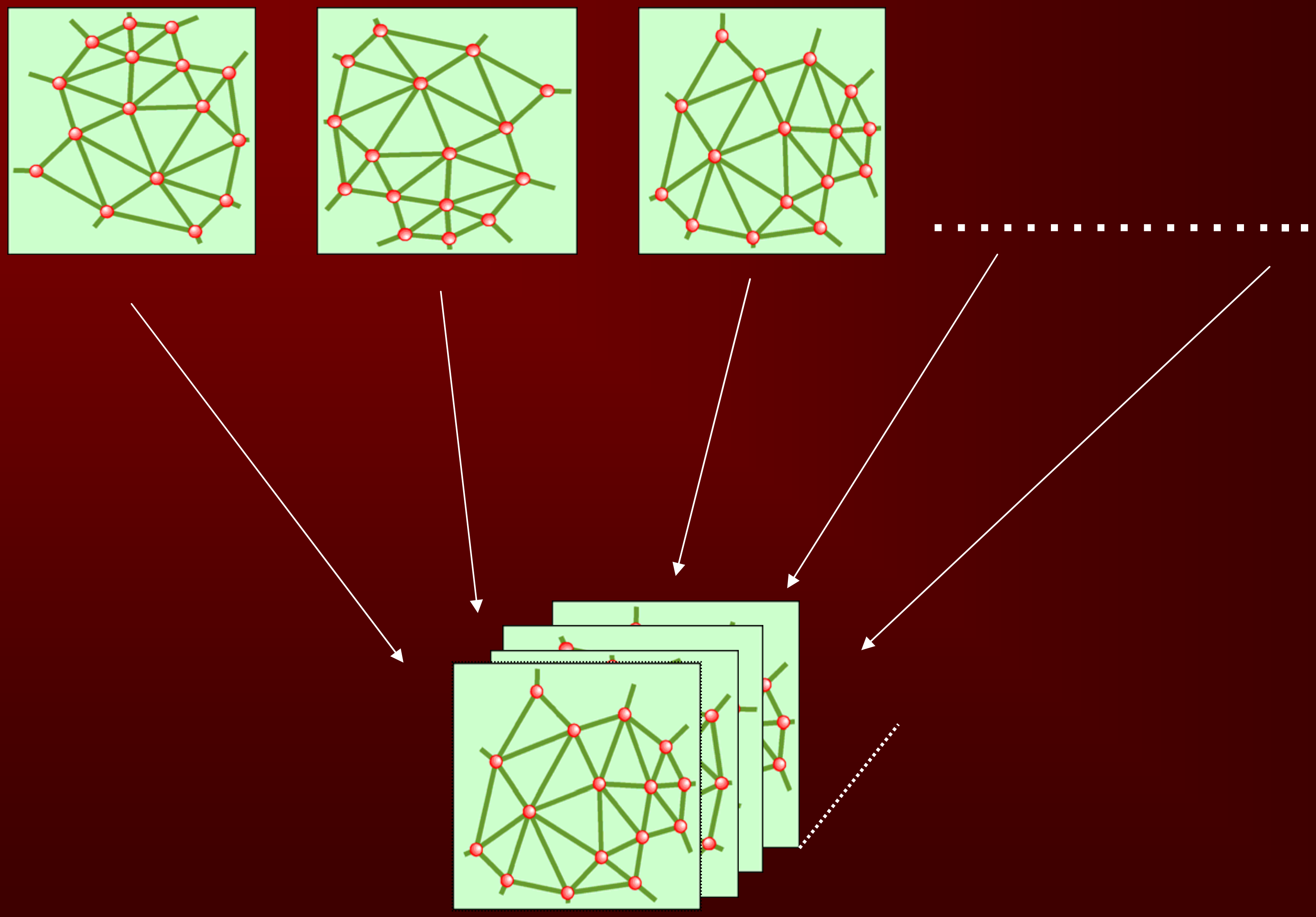
Random Lattice: step 6



Random Lattice: step 7



Random Lattice averaging = restoration of O(D=4) invariance



CFL formalism for random lattice

(Christ, Friedberd, Lee Nucl.Phys. B202 (1982) 89; B210 (1982) 310; 337)

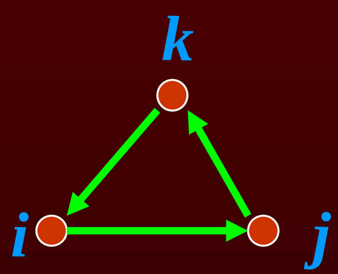
1. Any element or object on Random lattice correspond to same element or object of Dual Random lattice
2. Any element or object on Random lattice has statistical weighs

Center i → D-dim value of dual Vonoi cell

Link i

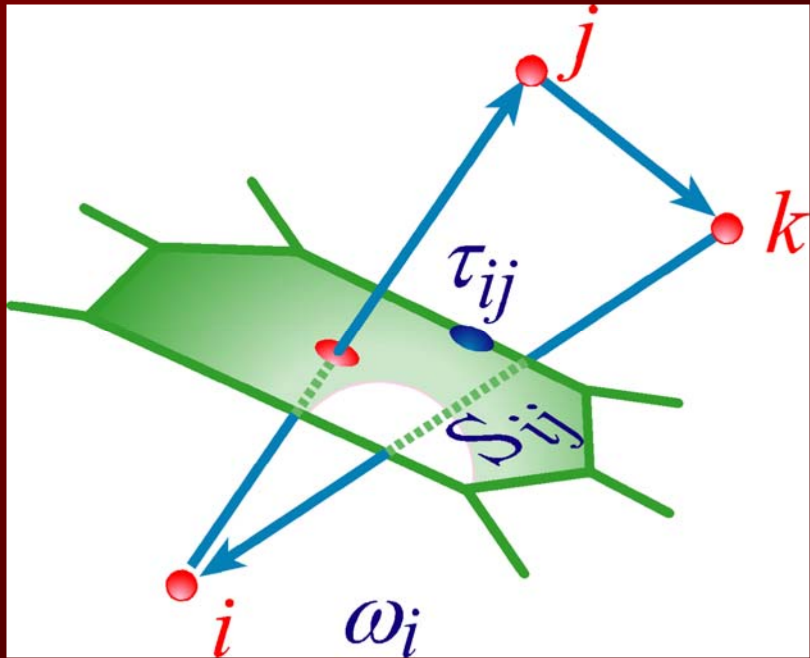
→ D-1 – dim value of the plane of the Voronoi cell

Plaquette



→ D-2 - dim value of correspondent dual object

Fermion on Random Lattice



Lattice action for fermion on the lattice

$$S_{\psi} = \frac{1}{2}\sum_{x,\mu} Tr\left(\bar{\psi}\underbrace{\gamma^{\mu} l_{ij}^{\mu}}_{\nu\gamma_{\nu}}\lambda_{ij}\psi\right) \qquad \lambda_{ij} = S_{ij}/l_{ij}$$

Restoration of O(4) symmetry on Random Lattice

$\boxed{\mathrm{T}}$ (CFL)

$$L^{\mu\nu} = \left(\sum_{ij} l^{\mu}_{ij} l^{\nu}_{ij} \lambda^{2}_{ij} \right)_{[\Omega]} = \delta^{\mu\nu}$$

$$L^{\mu\nu\rho\sigma} = c_2 \left(\delta^{\mu\nu}\delta^{\rho\sigma} + \delta^{\mu\rho}\delta^{\nu\sigma} + \delta^{\mu\sigma}\delta^{\rho\nu} \right)$$

$$L^{i_1 i_2 \cdots i_{2N}} = c_N \left(\sum_{\text{permutation}} \prod \delta^{i_l i_k} \cdots \delta \right)$$

$$c_N = \frac{1}{2^N (N+1)!}$$

Restoration of O(4) symmetry on Random Lattice

$$\left(\sum_{\nu} a_{\nu} b_{\nu} \right)_{[\Omega]} = a_i b_i$$

$$\left(\sum_{\nu} a_{\nu} b_{\nu} c_{\nu} d_{\nu} \right)_{[\Omega]} = \frac{1}{6}\left(a_i b_i c_j d_j + a_i b_j c_i d_j + a_i b_j c_j d_i \right)$$

Effective theory for chiral field in strong coupling regime

Random lattices ensemble averaging => restoration of space symmetry on zero step of perturbation!!!

$$L = \frac{F_\pi^2}{2} Tr(\partial_i U \partial_i U^+) +$$

$$+\tilde{C}_4 Tr(\partial_i U \partial_i U^+ \partial_j U \partial_j U^+ + \partial_i U \partial_j U^+ \partial_i U \partial_j U^+ + \partial_i U \partial_j U^+ \partial_J U \partial_i U^+) + \tilde{C}_6 \frac{1}{F_\pi^2} L_6 + \cdots$$

$$p < F_\pi \sim 100 MeV$$

Shock-wave solutions of Chiral Born-Infeld Theory

$$L_{ChBI} = -f_\pi{}^2\beta^2 Tr\left(1-\sqrt{1-\left(L_\mu L^\mu/2\beta^2\right)}\right)$$

$f_\pi \to 0$

$\beta \to 0$

$$L_H = 1-\sqrt{1-(\varphi_t{}^2-\varphi_{\vec{x}}{}^2)}$$

$$L_W = -\frac{f_\pi{}^2}{4} Tr\left(L_\mu L^\mu\right)$$

$$L_\mu = U^+\partial_\mu U$$

$$U = \exp(i\vec{\phi}\vec{\tau}/f_\pi)$$

Conclusions:

- The method of derivation of chiral effective theories from Lattice QCD was considered.
- Basis of the method – conception of the random lattice ensemble averaging
- Zero-order perturbation theory in strong coupling regime was considered. Restoration of O(4) space symmetry was studied.
- This method could be used for any order of perturbation, for any plaquet cotributions.