A New Generalization of Dirac's Equation for Nucleons

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Generalization of Dirac's equation

$$[\Gamma^n (i\hbar c \partial_n - \beta e_p A_n) - \alpha (mc^2 + \Phi)]\Psi = 0.$$

Ψ is a 12-component wave function,

$\Gamma^n = \gamma^n_{ij} \cdot 1,\ \gamma^n_{ij}$ are elements of Dirac matrices,

1 is the unit 3×3 matrix,

e_p is proton's charge, m is the mass of a nucleon,

A_n, Φ are electromagnetic and nuclear potentials,

α, β are matrices of the quark structure of nucleons.

Main requirements

- 1) Wave functions of a free nucleon should satisfy the Klein-Gordon equation
- 2) Charge conservation equations for nucleons and quarks should follow from the proposed equation.
- 3) The matrices α and β should reflect the quark structure of nucleons.

Main properties

Properties of matrices α, β :

$$\alpha^2 = 1, \ \alpha\beta = \beta\alpha, \ \alpha^+ = \alpha, \ \beta^+ = \beta.$$

Conservation equations:

$$\partial_n \left(\overline{\Psi} \Gamma^n q \Psi \right) = 0, \ \overline{\Psi} = \Psi^+ \Gamma^0,$$

$$q\alpha = \alpha q, \ q\beta = \beta q, \ q^+ = q.$$

Matrices of quark structure

Quark matrices α, β for nucleons:

$$\alpha_p = \begin{pmatrix} \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ -\frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \end{pmatrix}, \qquad \beta_p = \alpha_p,$$

$$\alpha_n = \alpha_p, \qquad \beta_n = \begin{pmatrix} -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \end{pmatrix}.$$

Quark currents

$J^n = e_p \overline{\Psi} \Gamma^n \beta \Psi$ are nucleon's current densities.

$$\partial_n J^n = 0, \quad J^n = \sum_{l=1}^{3} \theta_l J_l^{(n)},$$

$\theta_l J_{(l)}^n$ are quark components of J^n,

$\theta_1 = \frac{2}{3},\ \theta_2 = \frac{2}{3},\ \theta_3 = -\frac{1}{3}$ for protons,

$\theta_1 = -\frac{1}{3}, \theta_2 = -\frac{1}{3}, \theta_3 = \frac{2}{3}$ for neutrons.

Quark currents

Conservation equations for quark currents

$$\partial_n(\theta_l J^n_{(l)}) = 0, \quad J^n_{(l)} = e_p \overline{\Psi}\Gamma^n p_l \Psi,$$

$$p_{1,2} = \tfrac{1}{2}\begin{pmatrix} 1\,0\,1 \\ 0\,1\,1 \\ 1\,1\,0 \end{pmatrix} \pm \varepsilon \begin{pmatrix} 0\,1\,1 \\ 1\,0\,1 \\ 1\,1\,0 \end{pmatrix}, \qquad p_3 = \begin{pmatrix} 0\,1\,0 \\ 1\,0\,0 \\ 0\,0\,1 \end{pmatrix}$$

$$\int \theta_l J^0_{(l)} d^3x = \theta_l e_p .$$

Three normalization conditions

$$\int \Psi^{+} p \Psi d^{3}x = 1, \quad p = \tfrac{1}{2}\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix},$$

$$\int \Psi^{+} u \Psi d^{3}x = 0, \quad u = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix},$$

$$\int \Psi^{+} \Psi d^{3}x = 3.$$

Generalized Pauli Equation

Representation of the wave function:

$$\Psi = \exp(-i\alpha mc^2 t/\hbar)\varphi.$$

Equations for nucleons:

$$i\hbar\partial/\partial t - \Phi\alpha - e_p A_0\beta)\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix} = c(\sigma\mathbf{P})\begin{pmatrix}\varphi_{\mathbf{3}}\\ \varphi_4\end{pmatrix},$$

$$[(2mc^2 + \Phi)\alpha + i\hbar\partial/\partial t - e_p A_0\beta)\begin{pmatrix}\varphi_3\\ \varphi_4\end{pmatrix} = c(\sigma\mathbf{P})\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}.$$

Generalized Pauli Equations

3-dimensional vectors σ and $\mathbf{P}$ have the form

$$\sigma = (\sigma_1, \sigma_2, \sigma_3), \quad \mathbf{P} = (P_1, P_2, P_3),$$

σ_k are Pauli matrices, $P_k = -i\hbar\partial_k + e_p \beta A_k / c.$

Generalized correlation:

$$(\sigma\mathbf{P})(\sigma\mathbf{P}) = P^2 - e_p \hbar \beta (\sigma\mathbf{H}) / c,$$

$\mathbf{H}$ is the vector of magnetic field strength,

$P^2 = |\mathbf{P}|^2.$

Generalized Pauli Equation

Nonrelativistic case

$$\begin{pmatrix} \varphi_3 \\ \varphi_4 \end{pmatrix} = \frac{1}{2mc + \Phi / c} \alpha (\sigma \mathbf{P}) \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}.$$

Equation for φ_1 and φ_2 :

$$(i\hbar \partial / \partial t - e_p A_0 \beta) f = \Phi \alpha f +$$

$$+ \frac{\alpha (P^2 - e_p \hbar \beta (\sigma \mathbf{H}) / c)}{2m[1 + \Phi / (2mc^2)]} f, \quad f = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}.$$

Energy E of a nucleon

$$E = \int f^{+} \left[(mc^2 + \Phi)\alpha + \frac{\alpha(P^2 - e_p \hbar \beta(\sigma \mathbf{H})/c}{2m[1 + \Phi/(2mc^2)]} \right] f d^3 x.$$

Consequence of the normalization conditions

$$\int f^{+} \alpha f d^3 x = 1.$$

Energy of a nucleon

The nucleon energy when $|\Phi| << 2mc^2$:

$$E = mc^2 + \overline{\Phi}_0 +$$

$$+\frac{1}{2m}\int f^{+}[1-\Phi/(2mc^2)][\alpha P^2 - e_p\hbar\alpha\beta(\sigma\mathbf{H})/c]fd^3x.$$

$$\overline{\Phi}_0 = \int f^{+}\Phi\alpha f d^3x,$$

$\overline{\Phi}_0$ is the mean value of the nucleon nuclear potential.

Magnetic moments of nucleons

Magnetic part E_{mag} of the nucleon energy:

$$E_{mag} = -\frac{e_p \hbar}{2mc} \int f^+ [(1 - \Phi/(2mc^2)] \alpha\beta (\sigma \mathbf{H})\, f d^3 x.$$

Magnetic moment μ of a nucleon:

$$\mu = \frac{e_p \hbar}{2mc} [1 - \overline{\Phi}/(2mc^2)] \int f^+ \alpha\beta \; f d^3 x,$$

$\overline{\Phi}$ is some mean value of Φ.

Magnetic moments of nucleons

Magnetic moment μ_p of a proton:

$$\beta_p = \alpha, \quad \alpha\beta_p = \alpha^2 = 1,$$

$$\mu_p = \frac{3e_p\hbar}{2mc}[1 - \overline{\Phi}_p/(2mc^2)].$$

Magnetic moment μ_n of a neutron:

$$\alpha\beta_n = -(2/3)1 + (1/3)u,$$

$$\mu_n = -\frac{e_p\hbar}{mc}[1 - \overline{\Phi}_n/(2mc^2)].$$

Nucleon’s mass in its nuclear field

Energy of a nucleon at rest:

$$E = mc^2 + \overline{\Phi}_0,$$

where m is its mass when the nuclear field

is absent.

$$m_p c^2 = m_1 c^2 + \overline{\Phi}_{0p}, \; m_n c^2 = m_2 c^2 + \overline{\Phi}_{0n},$$

m_p, m_n are the experimental masses of the proton and neutron.

Magnetic moments of nucleons

Theoretical values:

$$\mu_p = \ 3[1+\overline{\Phi}_1/(2m_pc^2)]\mu_0$$

$$\mu_n = -2[1+\overline{\Phi}_2/(2m_pc^2)]\mu_0,$$

$$\mu_0 = e_p\hbar/(2m_pc),$$

$$\overline{\Phi}_1 = 2\overline{\Phi}_{0p} - \overline{\Phi}_p, \quad \overline{\Phi}_2 = 2\overline{\Phi}_{0n} - \overline{\Phi}_n.$$

Experimental values:

$$\mu_p^{\text{exp}} = 2.79\mu_0, \quad \mu_n^{\text{exp}} = -1.91\mu_0.$$

Conclusions

1) The found theoretical values of the anomalous magnetic moments of nucleons are close to their experimental values.

2) The obtained formulas for the current densities of nucleons correspond to two u-quarks and one d-quark for the proton and to one u-quark and two d-quarks for the neutron.