ANALYTIC APPROACH TO CONSTRUCTING EFFECTIVE THEORY OF STRONG INTERACTIONS AND ITS APPLICATION TO PION-NUCLEON SCATTERING

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Contents

- The modern status of πN -interaction; QCD and the effective field theory (EFT) of strong interactions
- The new analytic relativistic approach to constructing effective hadron-hadron interaction operators, based on principals of unitarity and analyticity and methods for solving inverse quantum scattering problem
- Application of analytical approach to πN -scattering
- Conclusions

Introduction

The pion-nucleon dynamics is one of the most fundamental problem in nuclear and particle physics. It is now widely believed that QCD is basic theory of strong interactions. On this basis all hadron-hadron interactions are completely determined by the underlying quark–gluon dynamics. However, due to the formidable mathematical problems, raised by the non-perturbative character of QCD at low and intermediate energies, we are still far from a quantitative understanding of hadron-hadron interactions from this point of view.

Effective Field Theory

The path-integral method together with the idea of spontaneous chiral symmetry breaking leads to Effective Field Theory (EFT) of strong interactions. The EFT formulates the theory of hadron-hadron interactions in terms of meson and baryon (anti-baryon) degrees of freedom. The Lagrangian of EFT is highly nonlinear and has rather complicated form. Therefore in practice the decomposition of this Lagrangian in power series of particle momentum and pion mass factor is used. In this approximation the theory refers to as Chiral Perturbation Theory (***Weinberg S.*** // Nucl. Phys. B. 1991. V. 363. P. 3).

Conceptual difficulties of ChPT

ChPT suffers from some inconsistencies.

- In this approach different regularization methods (for example, a dimensional regularization and a regularization based on introducing cutoff factors in loop integrals (S.R. Beane, et al. Nucl. Phys.,1998, v. A 632, p. 445), or so called infrared regularization (IR) scheme (K. Torikoshi and P.J.Ellis, Phys. Rev. C, 2003, v. 67, 015208)) lead to different predictions for transition amplitudes

Conceptual difficulties of ChPT

- The procedure of expanding the ChPT Lagrangian destroys the correct analytic structure of dynamical cuts nearest to the physical region for amplitudes of hadron-hadron scattering (for example, *NN*-scattering) (R. Higa, et al. Phys. Rev. C, v. 69, 2004, 034009)
- ChPT can be applied to describing strong interactions only at rather low energies.

Analytic approach

Recently the approach to constructing effective interaction operators between strongly interacting composite particles has been proposed (*A.N. Safronov et al.,* Yad. Fiz., 2006, v. 69, p. 408) on the basis of analytic S-matrix theory and methods for solving the inverse quantum scattering problem. We define effective potential (or potential matrix in multi-channel case) as local operator in the partial-wave quasipotential equation (Lippmann-Schwinger type equation), such that it generates an on-mass-shell scattering amplitude which has required discontinuities at dynamical cuts.

In present work a manifestly Poincare-invariant approach to constructing effective potential functions (that is dispersion integrals along left-hand (dynamical) cuts) for pion-nucleon scattering is developed with allowance for inelasticity effects. Hadron exchange mechanisms in *t* and *u* channels and also contact interactions, predicted by effective Lagrangian of chiral perturbation theory, were taken into account for constructing pion-nucleon potential functions in *S*- and *P*-wave states at low energies. Coupling constants of effective Lagrangian were extracted from analysis of available experimental data.

General definitions for πN -scattering

$$S_l = 1 + 2i\rho_l(\nu)T_l(\nu) \qquad \rho_l(\nu) = \nu^{l+1/2} \qquad k^2 = \nu$$

$$f_l(\nu) = \nu^l T_l(\nu) \qquad S_l(\nu) = \eta_l(\nu)\exp[2i\delta_l(\nu)]$$

$$f_{l\pm}^I = \frac{1}{2}\int_{-1}^{1} dz[\, f_1^I P_l(z) + f_2^I P_{l\pm1}(z)]$$

$$f_1^I = \frac{(w+m)^2 - \mu^2}{16\pi w^2}\left[A^I + (w-m)B^I\right]$$

$$f_2^I = -\frac{(w-m)^2 - \mu^2}{16\pi w^2}\left[A^I - (w+m)B^I\right]$$

$$w = \sqrt{\mu^2 + \nu} + \sqrt{m^2 + \nu}$$

k is relative momentum of colliding particles

m is nucleon mass, μ is pion mass

Analytical structure of partial-wave *S*-matrix

As follows from a **principle of analyticity**, partial-wave ***S***-matrix in a complex $\nu = k^2$ -plane has 1) the **poles** corresponding to one particle states (the nucleon pole in P_{11} -state) , 2) the **unitary** (right-hand) cut $0 \le \nu < \infty$, 3) "**inelastic**" cut above the threshold of creation of particles $\nu > \nu_\pi$ and 4) the **dynamic** (left-hand) cut $-\infty < \nu < \nu_L$ caused by **exchange processes** in ***t***- and ***u***-channels of scattering. The nearest to physical region point ν_L of dynamic cut is determined by nucleon exchange mechanisms in ***u***-channel

$$\nu_L = -\frac{\mu^2(4m^2 - \mu^2)}{4(m^2 + 2\mu^2)}$$

Spectral representation of reduced partial-wave amplitude

$$T_l(\nu) = L_l(\nu) + R_l(\nu) + \sum_i \frac{g_{li}^2}{\nu_{li} - \nu}$$

where

$$R_l(\nu) = \frac{1}{\pi}\int_0^{\infty} \frac{|T_l(\nu')|^2}{\nu' - \nu}\sigma_l(\nu')\rho_l(\nu')d\nu'$$

is right-hand cut contribution. It takes into account *s* -channel loop diagrams. $\sigma_l(\nu)$ is Froissart inelasticity parameter (function of energy), that is ratio of total partial-wave cross section to elastic one.

$$L_l(\nu) = \frac{1}{\pi}\int_{-\infty}^{\nu_L} \frac{\operatorname{Im} T_l(\nu')}{\nu' - \nu}d\nu'$$

is potential function, that is determined by discontinuity along left-hand cut.

Pole term give contribution only in P_{11} channel

Potential $L_l(\nu)$ function plays a role of the interaction operator in the *N/D* equations

$$T_l(\nu) = \frac{N_l(\nu)}{D_l(\nu)}$$

$$N_l(\nu) = L_l(\nu) + \frac{1}{\pi}\int_0^\infty \frac{L_l(\nu') - L_l(\nu)}{\nu' - \nu} N_l(\nu')\sigma_l(\nu')\rho_l(\nu')d\nu'$$

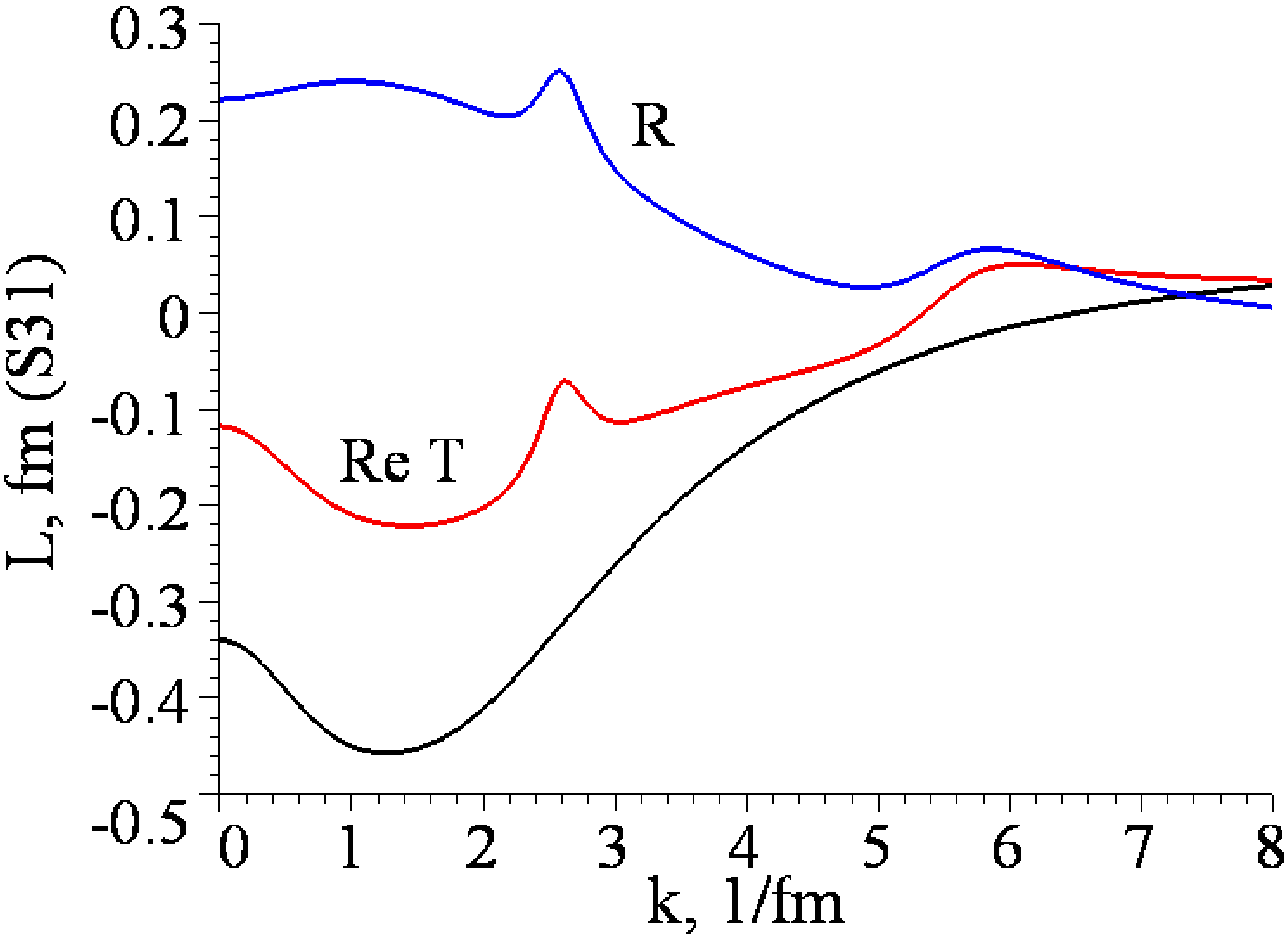
$$D_l(\nu) = 1 - \frac{1}{\pi}\int_0^\infty \frac{N_l(\nu')}{\nu' - \nu}\sigma_l(\nu')\rho_l(\nu')d\nu'$$

Model independence of discontinuity along dynamic cut

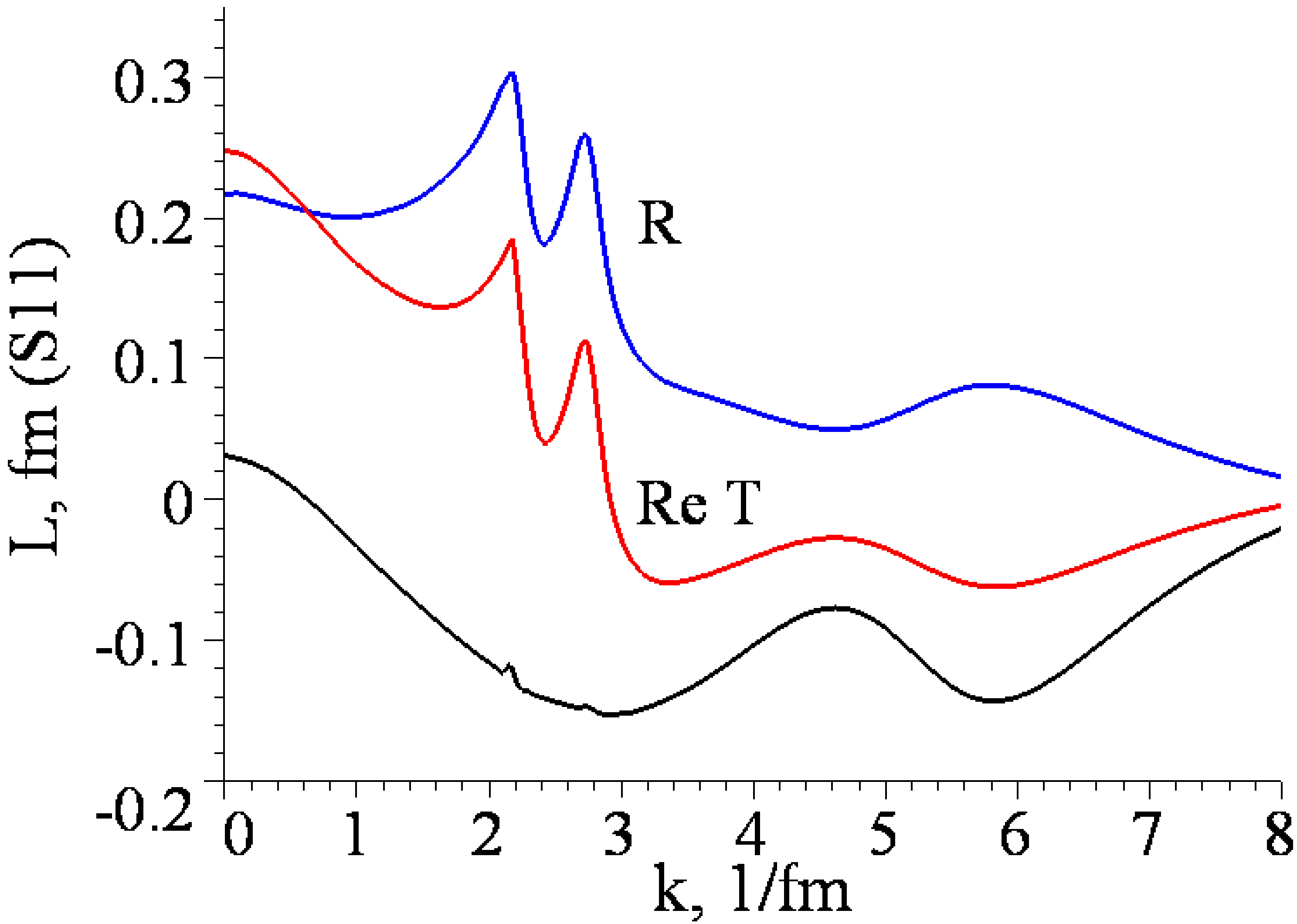
The discontinuities of the partial-wave *S*-matrix along dynamic cuts are determined by model-independent quantities – renormalized coupling constants and on-mass-shell amplitudes of elementary sub-processes (Cutkosky cutting rules, Cutkosky R.E. //J. Math. Phys. 1960. v. 1, p. 429). Therefore the structure at least the nearest to physical region cuts of the partial-wave scattering amplitudes can be determined in model-independent manner.

We have extracted the information on potential functions in *S*- and *P*-wave channels of scattering, using the data of the energy-dependent phase shift analysis of pion-nucleon scattering (on-line computer code SAID).

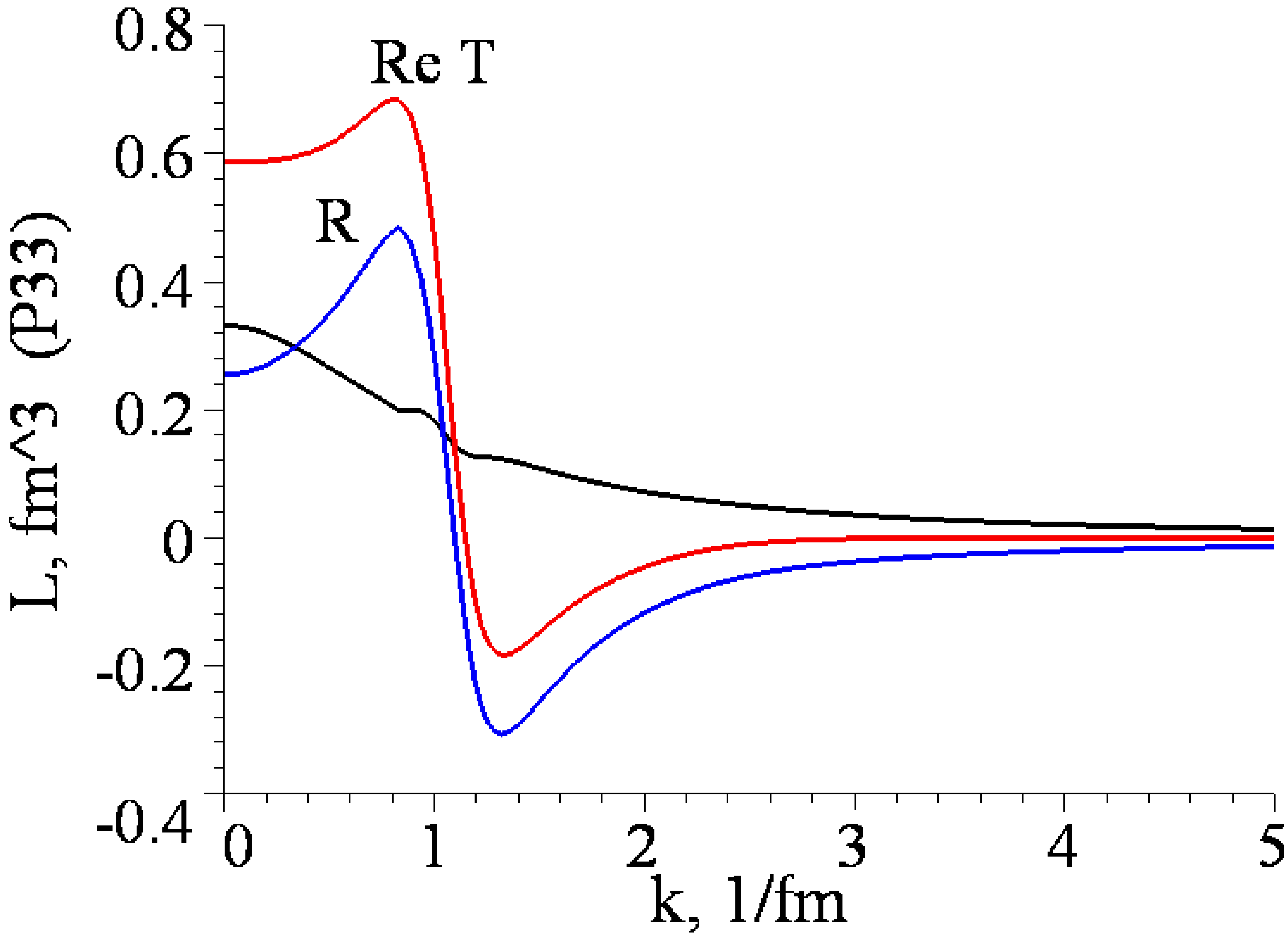
On the other hand, the potential functions were calculated at low energies taking into account the dynamic (left-hand) cuts nearest to physical region, and the contact interactions generated by effective Lagrangian of chiral perturbation theory. Conceptually contact interactions take into account contributions of distant dynamical singularities. The information on coupling constants of ChPT effective Lagrangian was extracted from this analysis.



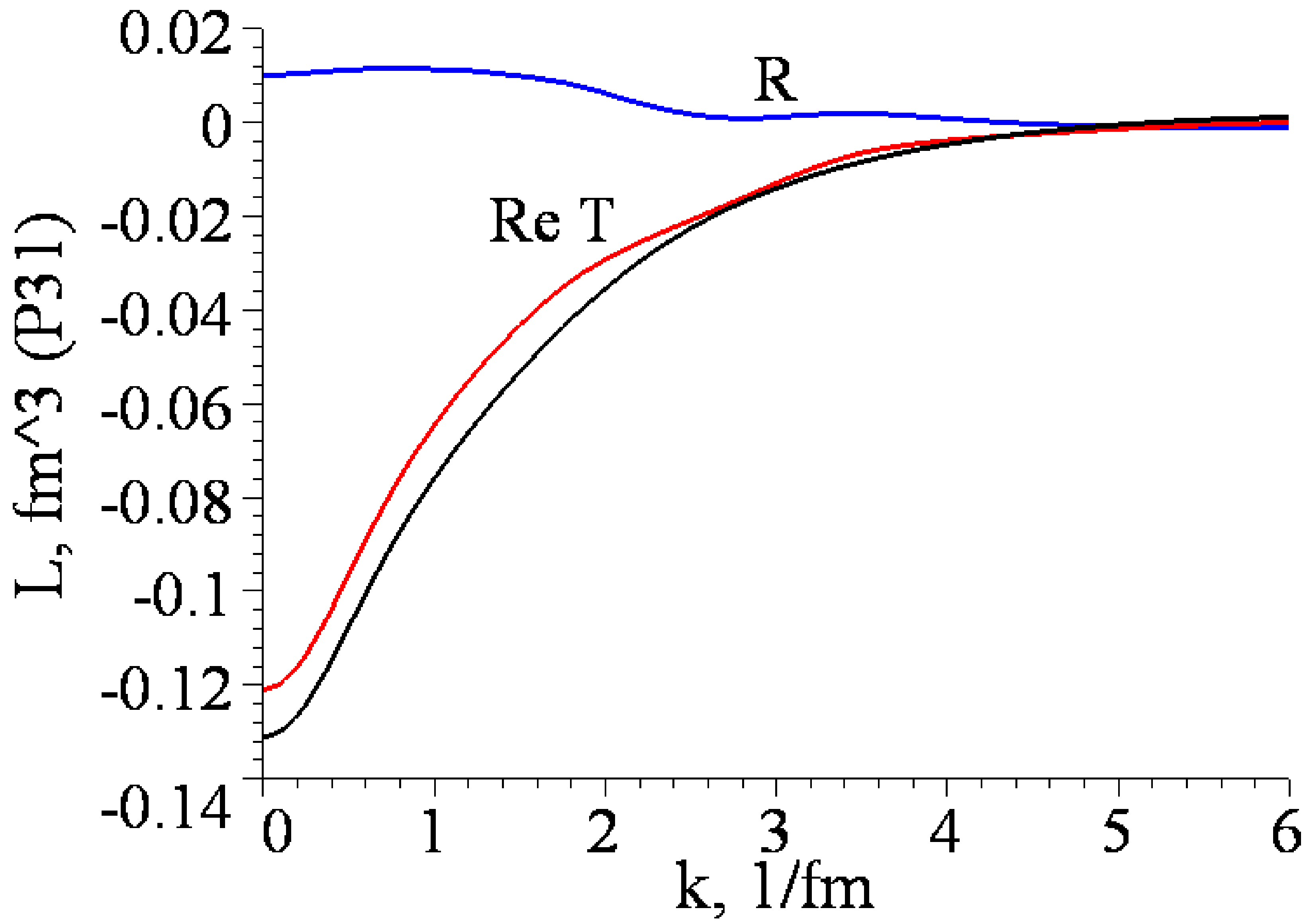
Potential function (black curve), real part **T** (red curve) and real part right-hand cut contribution (blue curve) for S_{31} - state.



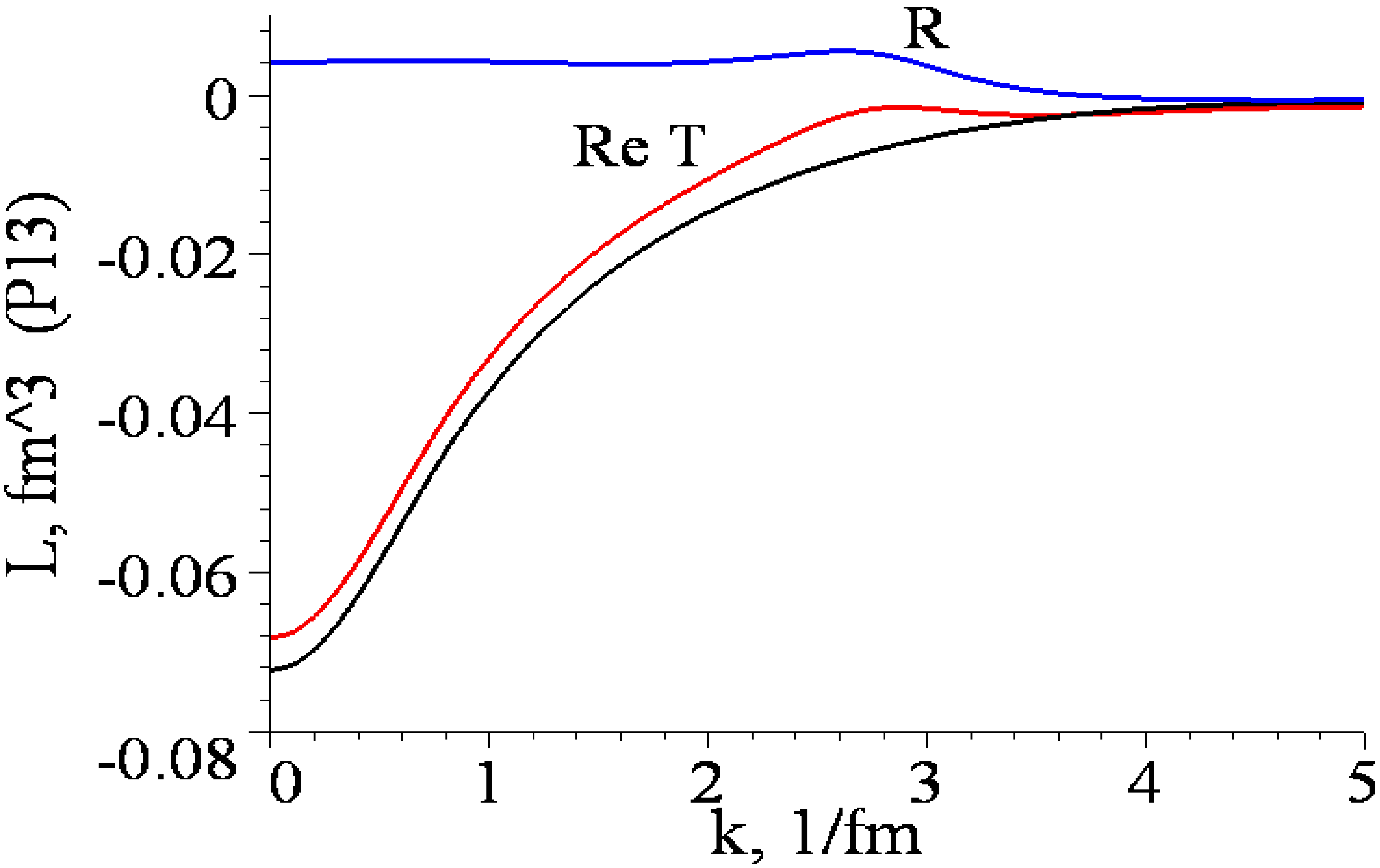
Potential function (black curve), real part T (red curve) and real part right-hand cut contribution (blue curve) for S_{11}- state.



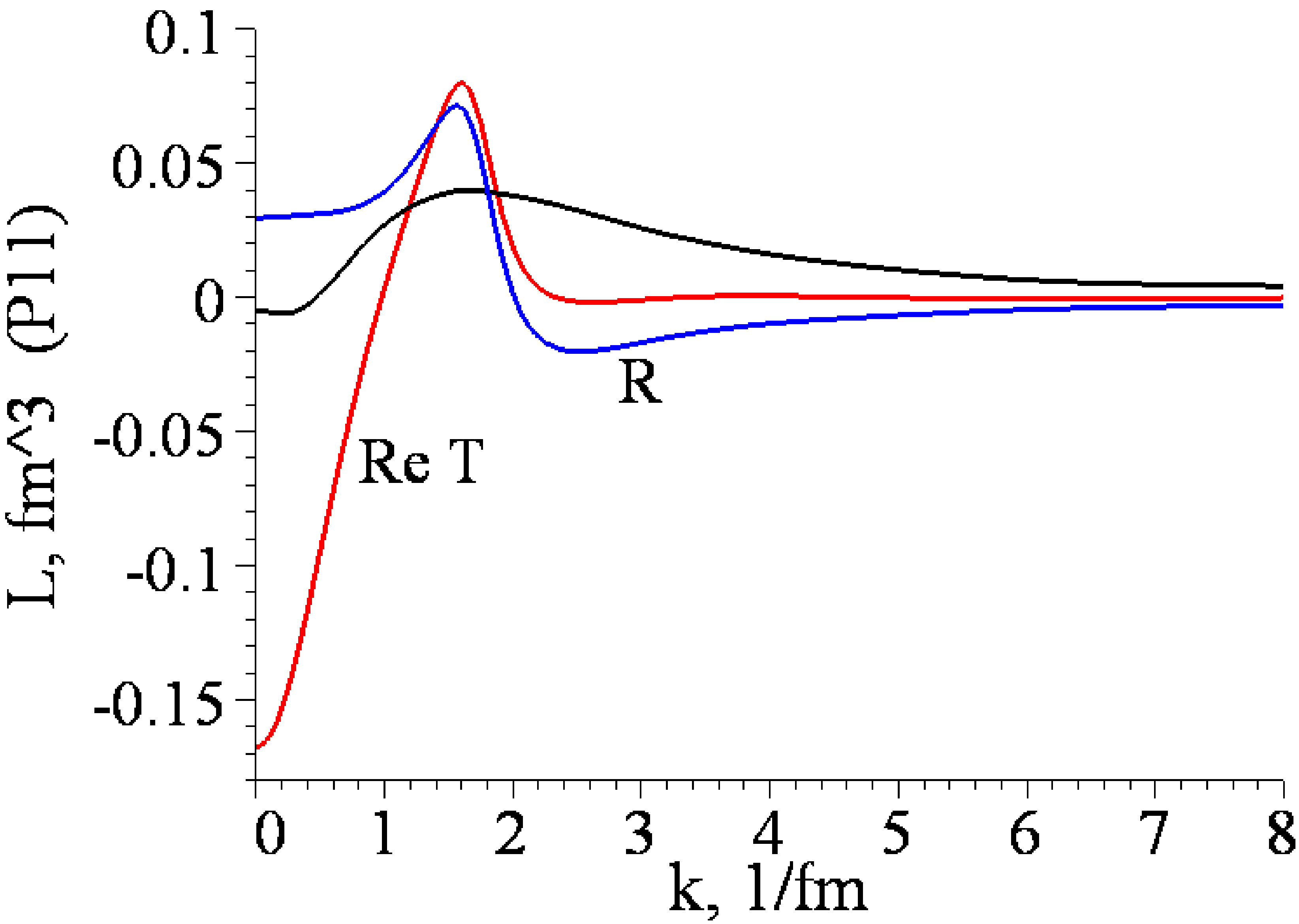
Potential function (black curve), real part **T** (red curve) and real part right-hand cut contribution (blue curve) for P_{33} - state.



Potential function (black curve), real part T (red curve) and real part right-hand cut contribution (blue curve) for P_{31} - state.



Potential function (black curve), real part **T** (red curve) and real part right-hand cut contribution (blue curve) for P_{13}- state.

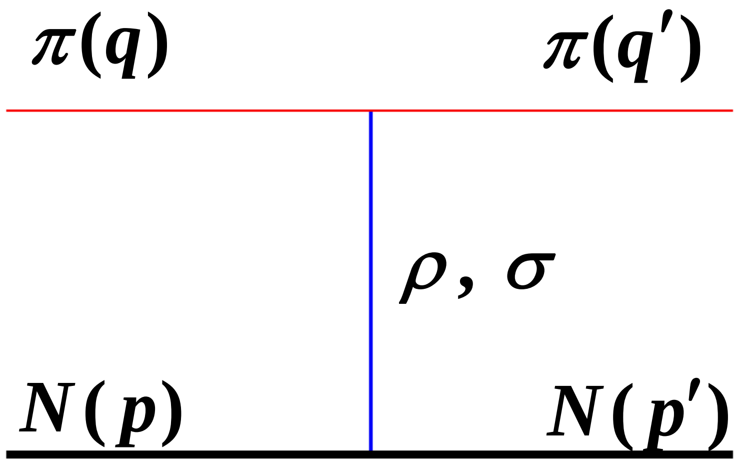


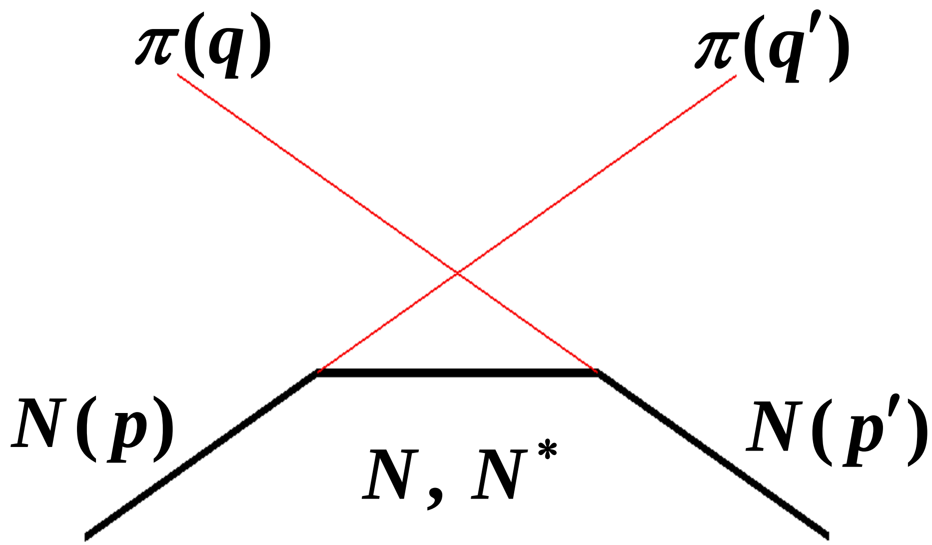
Potential function (black curve), real part **T** (red curve) and real part right-hand cut contribution (blue curve) for P_{11}- state.

Exchange mechanisms and contact interactions predicted by ChPT

To calculate the potential functions for ***S***- and ***P***-wave πN scattering we take into account the exchange mechanisms (ρ, σ in ***t***-channel and ***N*** , N^* in ***u**-channel*) and also the contact interactions, predicted by ChPT (L_2, L_3, L_4 -contributions). L_3 -contribution is determined by $\beta_\pi, \kappa_\pi, \kappa_1, \kappa_2$ -coupling constants ($c_1 = \kappa_2 / m$, $c_2 = \kappa_1 / 2m$, $c_3 = 2\beta_\pi / m$, $c_4 = \kappa_\pi / m$) and L_4 is determined by $\lambda_2, \lambda_3, \lambda_4, \lambda_5$ (***Torikoshi K.*** *et al.* Phys. Rev. C, v. 67, 2003, 015208). $\lambda_1 = -0.844$ as follows from relation $\lambda_1 = \frac{g_A m^2}{4\mu^2}\left(1 - \frac{f_\pi^2 g_{\pi NN}^2}{m^2 g_A^2}\right)$

Exchange mechanisms in πN-scattering





where N^* is one of baryon resonances :

$\Delta(1232)\ P_{33},\ \ N(1440)\ P_{11},\ \ N(1535)\ S_{11},$

$\Delta(1600)\ P_{33},\ \ N(1620)\ S_{31},\ N(1650)\ S_{11},$

$N(1720)\ P_{13},\ \ \Delta(1910)\ P_{31},$

$$g_A = 1.26,\ \ f_\pi = 92.4\,\text{MeV},\ \ g_\rho = 19.70,$$

$$g_\sigma = 16.62,\ \ \kappa_\rho = 5,\ \ m_\sigma = 700\,\text{MeV},$$

$$g_{\pi NN} = 13.169\ \ (g^2_{\pi NN}/4\pi = 13.8),$$

where $g_\rho = g_{\pi\pi\rho}\, g^{(V)}_{\rho NN}$, $g_\sigma = g_{\pi\pi\sigma}\, g_{\sigma NN}$,

$$\kappa_\rho = g^{(T)}_{\rho NN} / g^{(V)}_{\rho NN}$$

Vertex coupling constants of virtual dissociation (synthesis) of baryon resonances ($N^* \leftrightarrow \pi + N$) were calculated on the basic of information about partial width of these resonances.

We believe that contact interactions is generated by distant left-hand singularities of scattering amplitudes and so they give contribution to potential functions $L_l(\nu)$. Contributions of ChPT Lagrangians of third and fourth order (L_3 and L_4) to invariant functions are determined by expressions

$$A_C^{(+)} = \frac{2}{mf_\pi^2}\left[\beta_\pi(2\mu^2 - t) - 2\kappa_2\mu^2 + \lambda_4\varepsilon^2\right] \qquad A_C^{(-)} = -\frac{2\kappa_\pi}{mf_\pi^2}\,\varepsilon$$

$$B_C^{(+)} = \frac{1}{mf_\pi^2}\left(\kappa_1 - 2\lambda_4\right)\varepsilon$$

$$B_C^{(-)} = \frac{1}{2f_\pi^2}\left(1 + 4\kappa_\pi\right)$$

$$-\frac{1}{2m^2 f_\pi^2}\left(\lambda_2 t + 8\lambda_3\mu^2 - 2\lambda_5\varepsilon^2\right) \qquad \varepsilon = (s-u)/4m$$

Variants of calculation

Variant A: (i) ρ, σ -exchange mechanisms in t-channel; (ii) N, N^*-exchange mechanism in u-channel, where N^* is one of the baryon resonances

$\Delta(1232)\ P_{33},\ N(1440)\ P_{11},\ N(1535)\ S_{11}, \Delta(1600)\ P_{33},$

$N(1620)\ S_{31}, N(1650)\ S_{11}, N(1720)\ P_{13},\ \Delta(1910)\ P_{31}$

(iii) contact interactions, generated by ChPT Lagrangians L_2, L_3, L_4

Variant B: the same but in (ii) only N and $\Delta(1232)\ P_{33}$

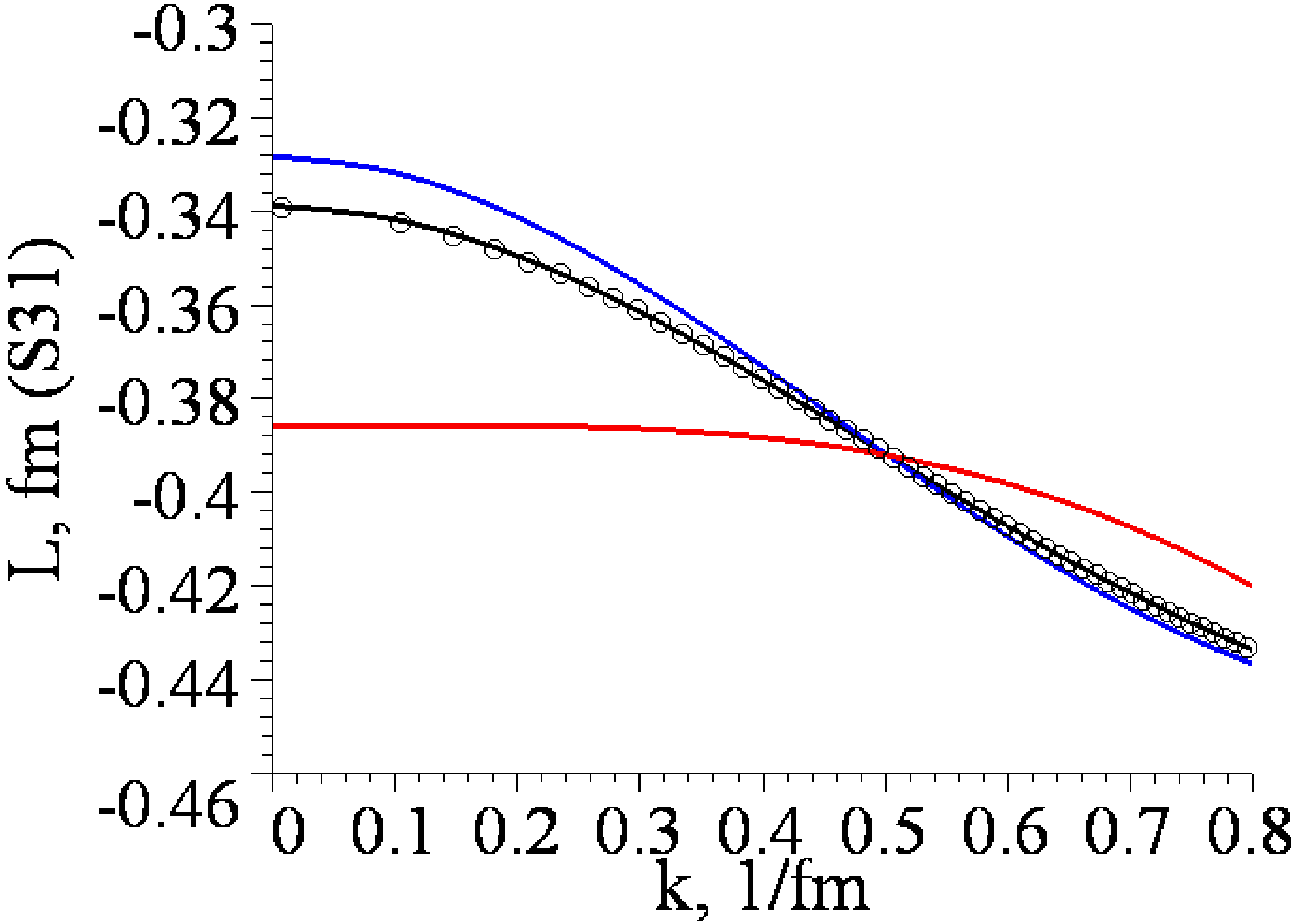
Variant C: only N and (iii)

Coupling constants of contact intecactions

variant	β_π	κ_π	κ_1	κ_2
A	-0.67	-0.43	5.56	-0.61
B	-0.68	-0.54	5.01	0.18
C	-0.43	1.44	-0.42	2.58

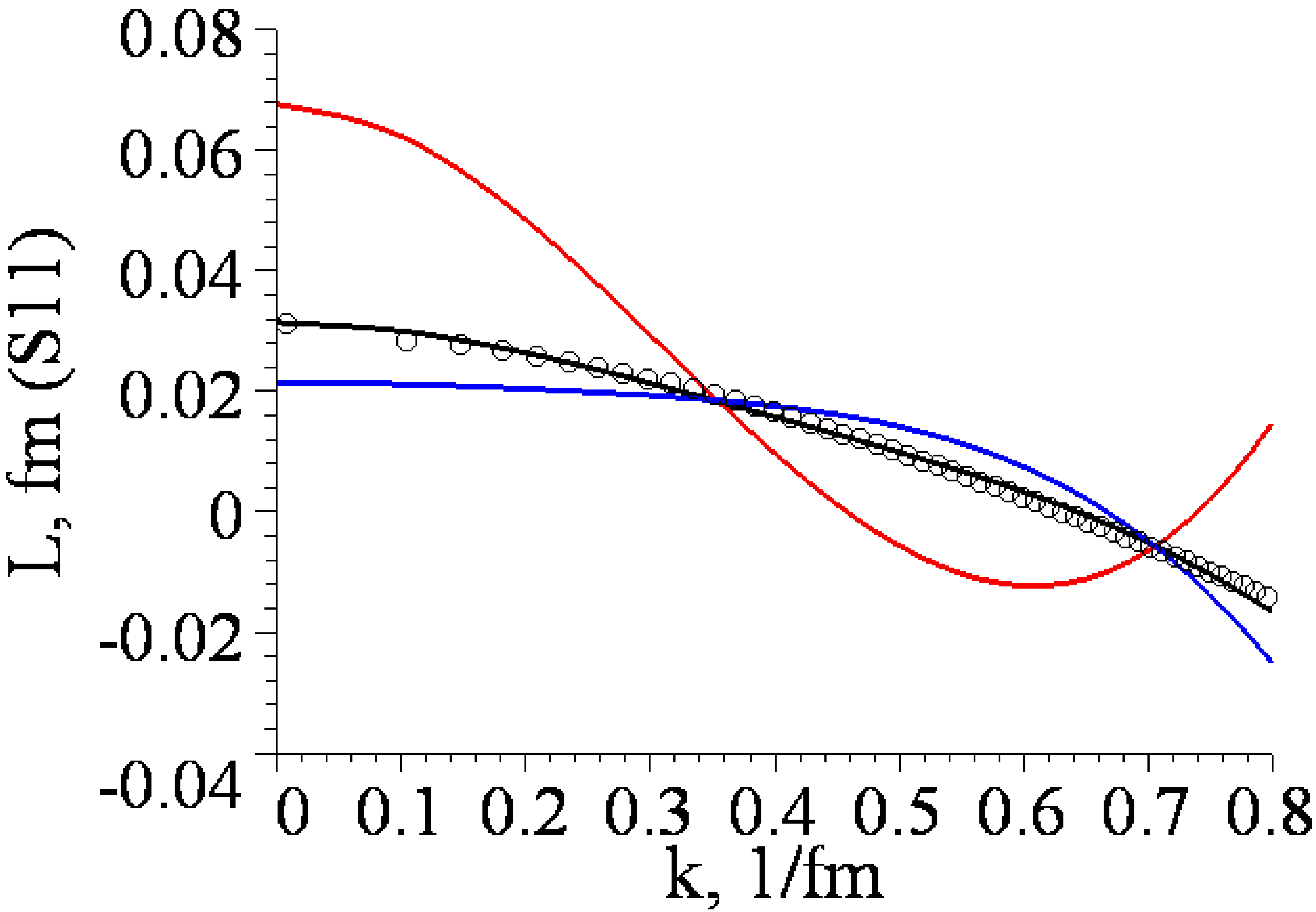
variant	λ_2	λ_3	λ_4	λ_5
A	2.21	-4.90	6.14	-6.59
B	3.03	-10.85	5.85	-11.37
C	-8.31	13.08	-3.80	19.77

Potential function for pion-nucleon scattering in S31-state



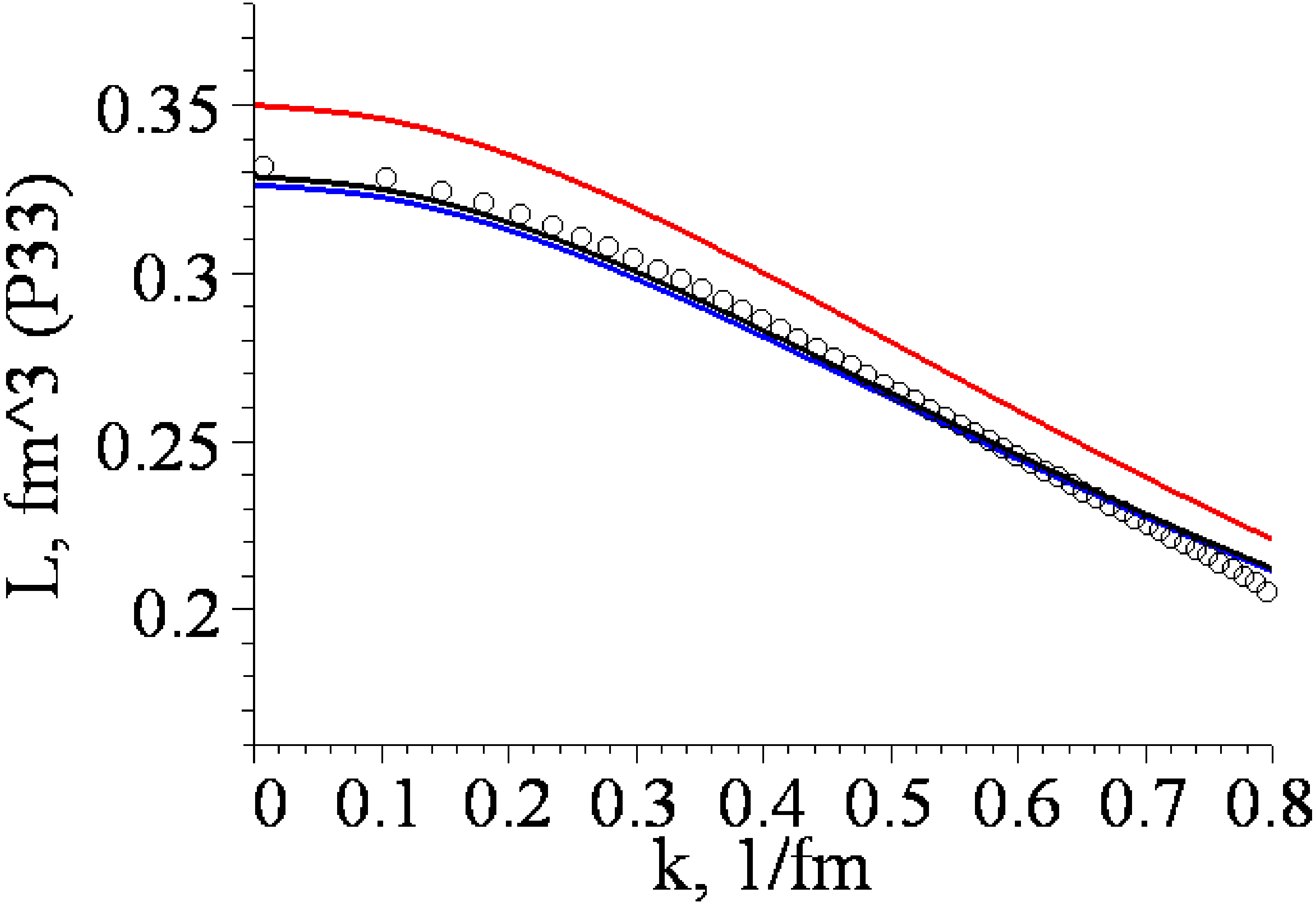
Circles – values extracted from phase-shift analyses. Solid curves – theoretical predictions: black (A), blue (B), red (C)

Potential function for pion-nucleon scattering in S11-state



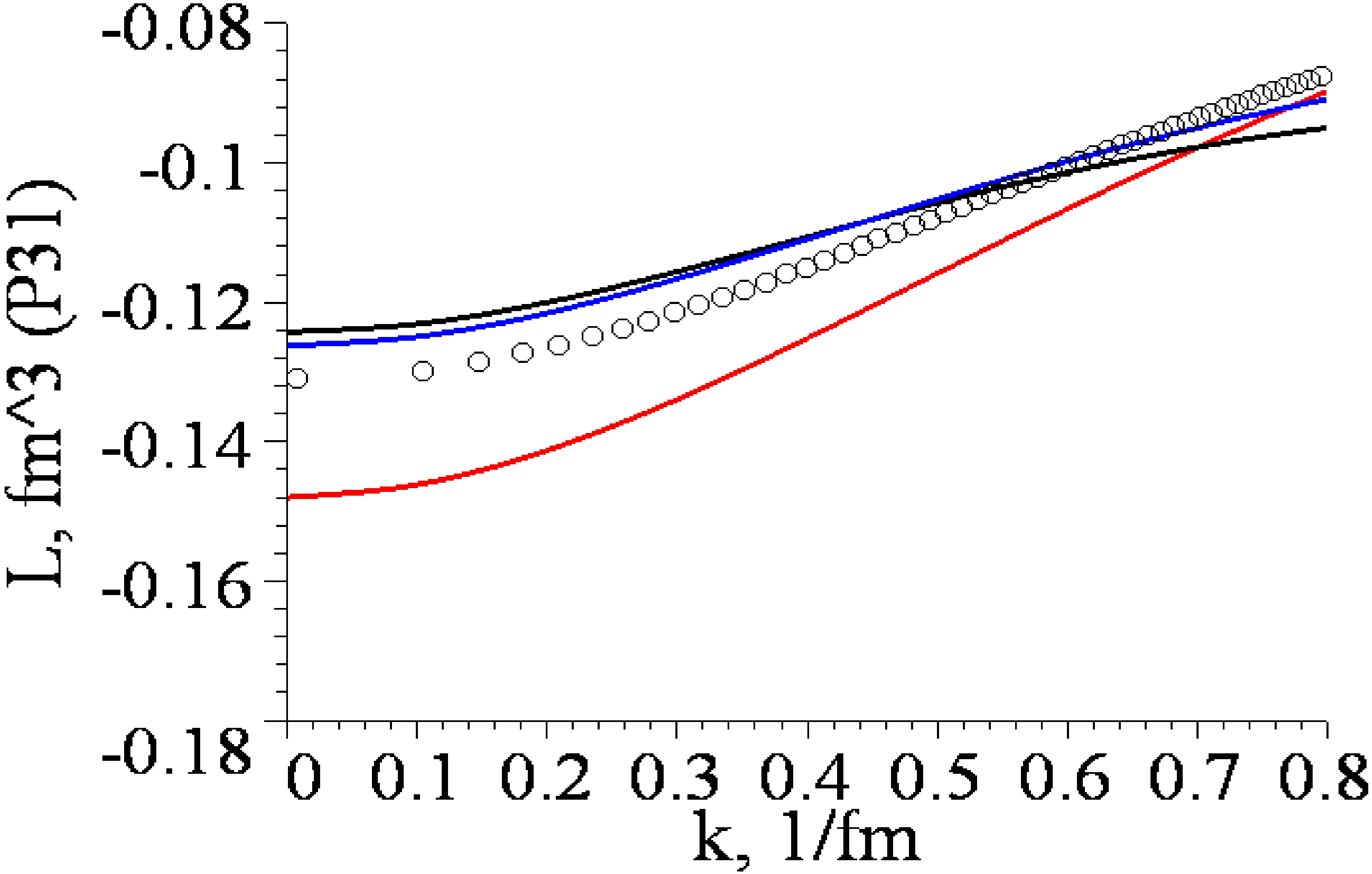
Circles – values extracted from phase-shift analyses. Solid curves – theoretical predictions: black (A), blue (B), red (C)

Potential function for pion-nucleon scattering in P33 -state



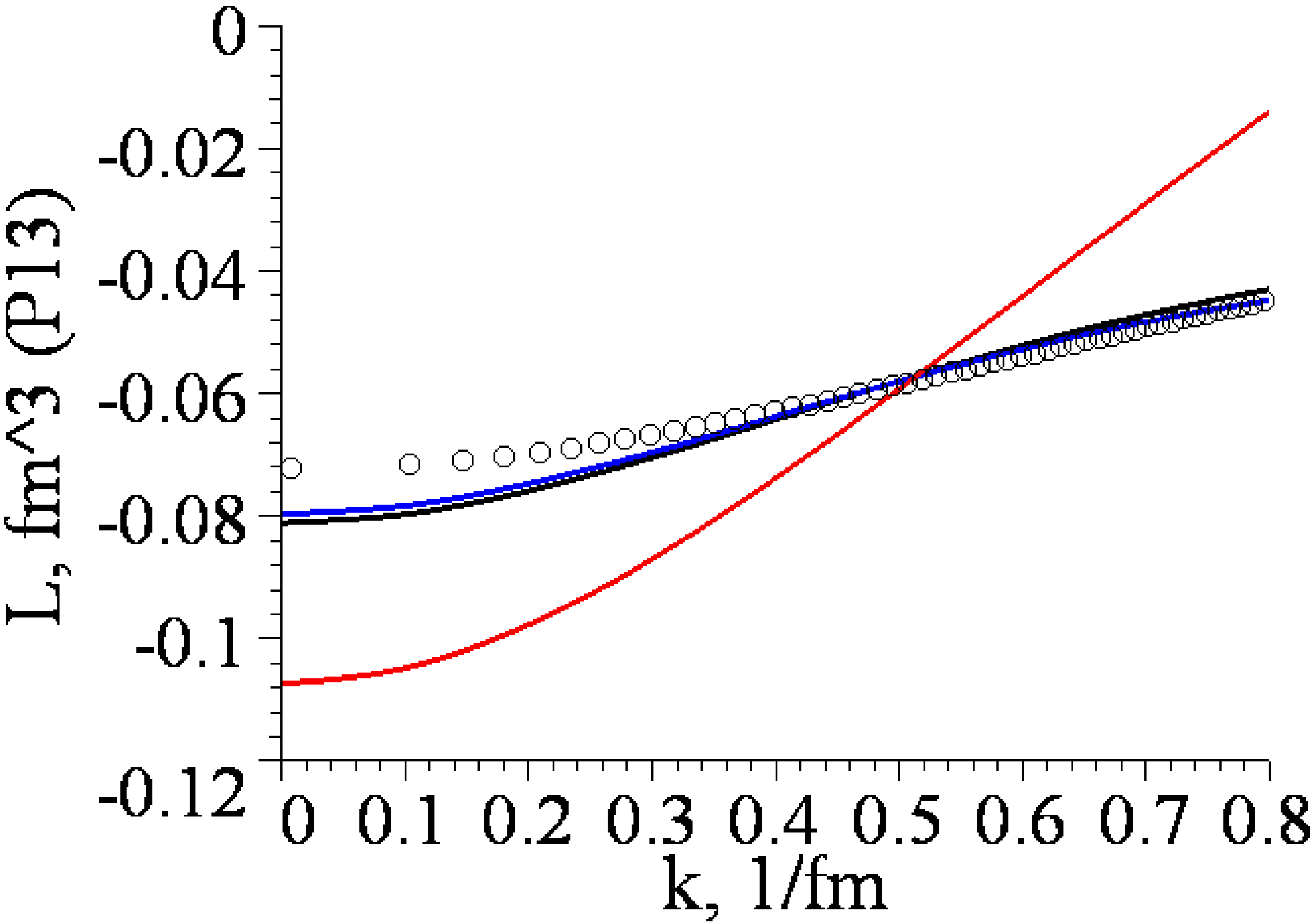
Circles – values extracted from phase-shift analyses. Solid curves – theoretical predictions: black (A), blue (B), red (C)

Potential function for pion-nucleon scattering in P31-state



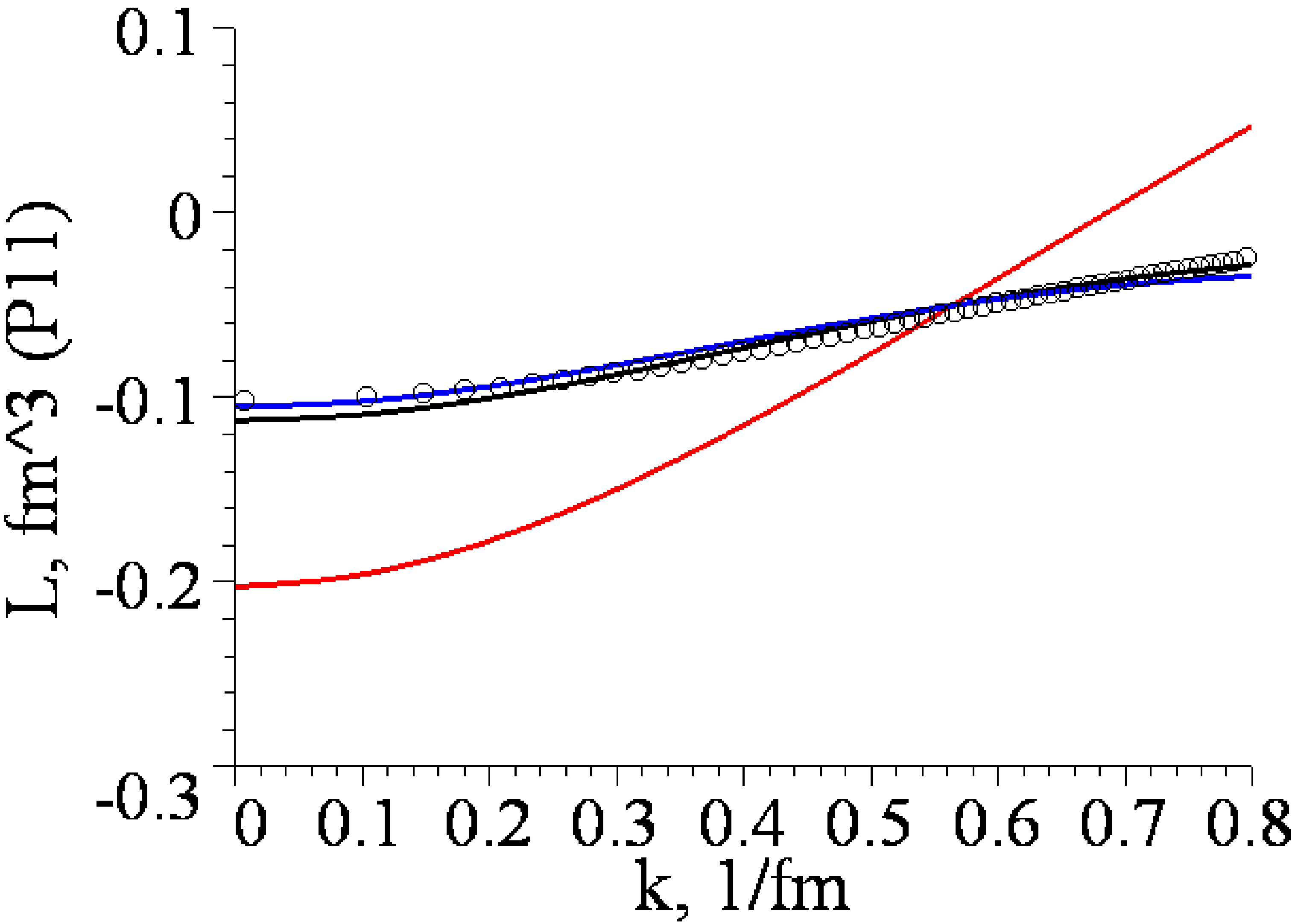
Circles – values extracted from phase-shift analyses. Solid curves – theoretical predictions: black (A), blue (B), red (C)

Potential function for pion-nucleon scattering in P13-state



Circles – values extracted from phase-shift analyses. Solid curves – theoretical predictions: black (A), blue (B), red (C)

Potential function for pion-nucleon scattering in P11-state



Circles – values extracted from phase-shift analyses. Solid curves – theoretical predictions: black (A), blue (B), red (C)

Conclusion

In modern studies of hadron physics pion-nucleon interaction plays an important role because

(i) the pion is unique object associated with the Goldstone boson in theory of spontaneous breaking of chiral symmetry and

(ii) there is extensive experimental data base for checking theoretical predictions.

In present work a manifestly Poincare-invariant analytic approach to constructing effective potential functions for pion-nucleon scattering is developed.

Conclusion

On the one hand, we have extracted the information on potential functions in *S*- and *P*-partial-wave channels, using the data of the energy-dependent phase shift analysis of pion-nucleon scattering.

On the other hand, the potential functions were calculated at low energies taking into account the dynamic (left-hand) cuts nearest to physical region, and the contact interactions generated by effective Lagrangian of chiral perturbation theory. The information on coupling constants of ChPT effective Lagrangian was extracted from this analysis. It has been shown that nearest to physical region dynamical singularities of scattering amplitudes play an important role in understanding low energy pion-nucleon physics.