String-like electrostatic interaction from QED with infinite magnetic field

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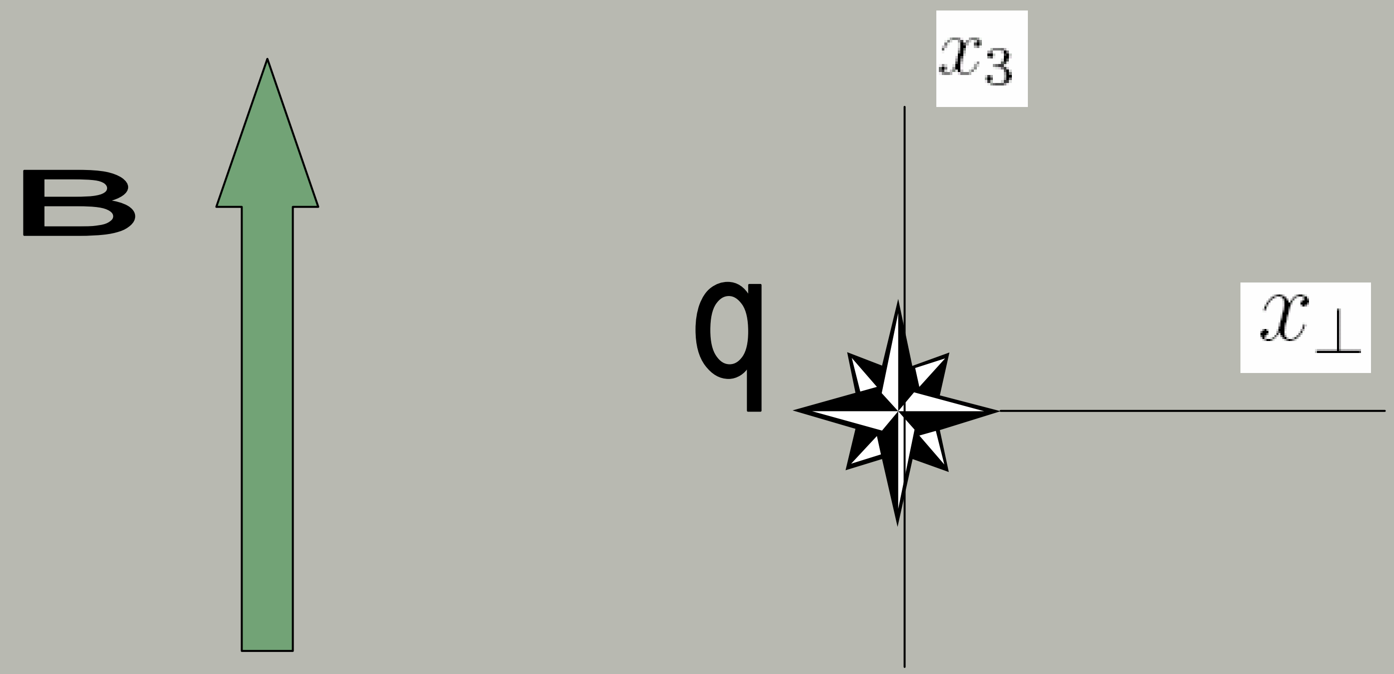
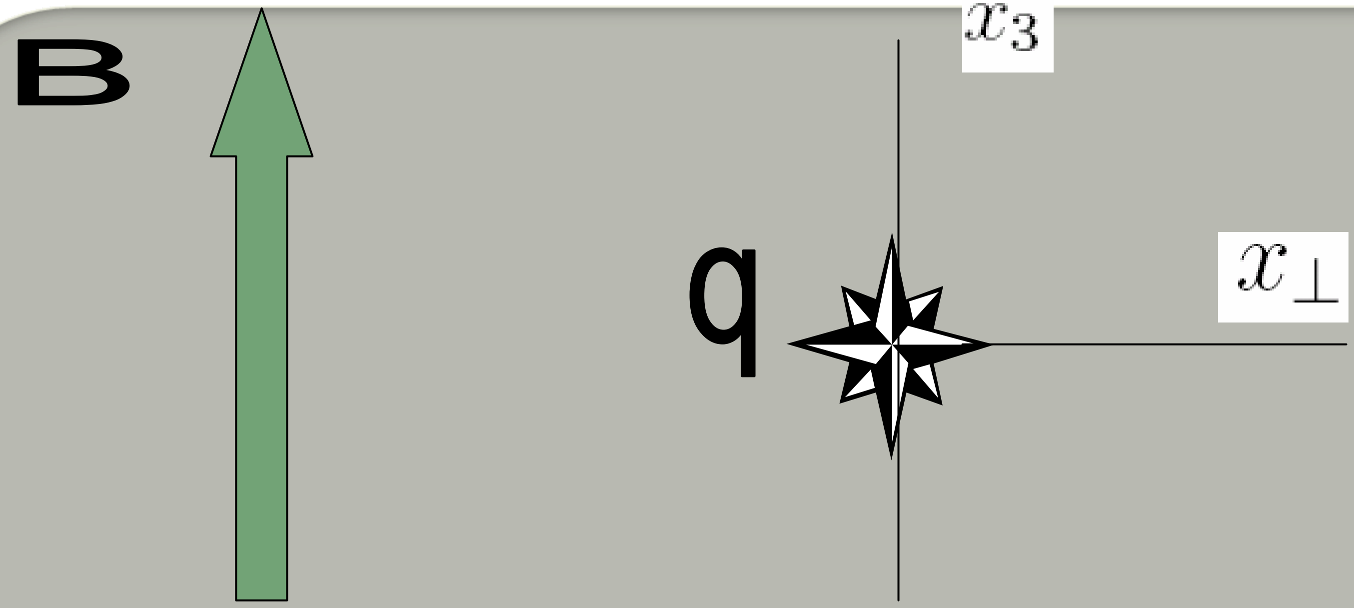
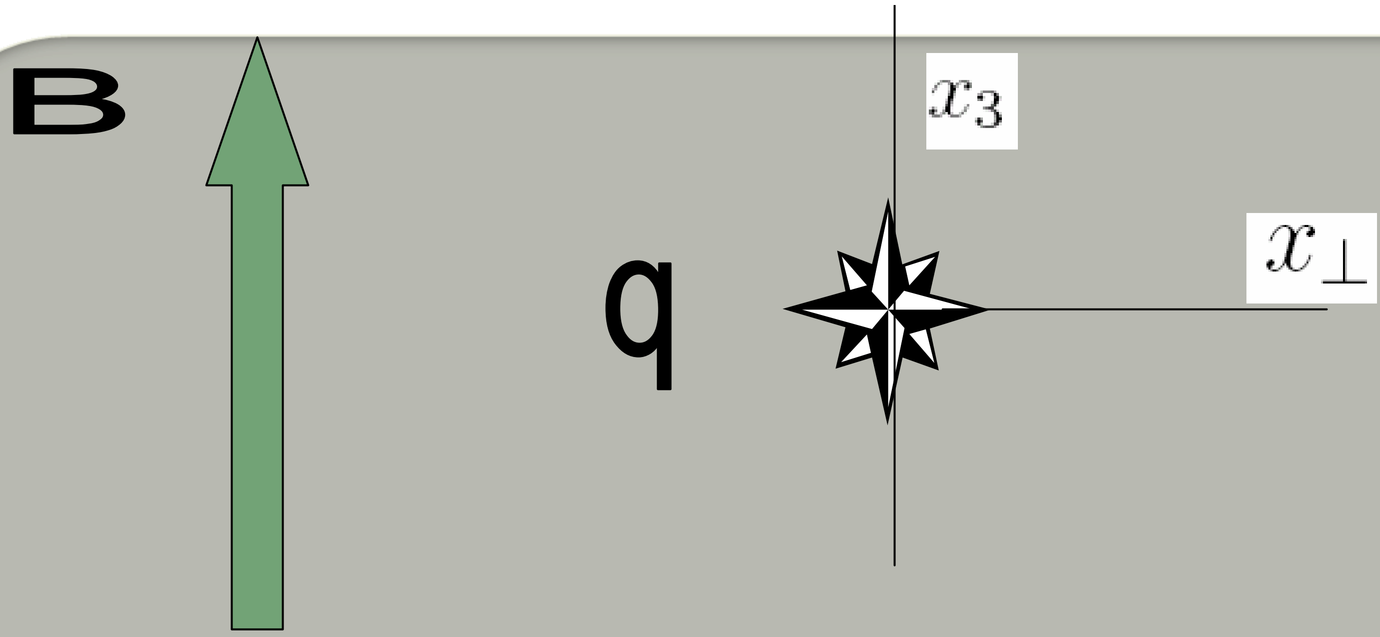


B
q
x_3
$x_\perp$



Characteristic magnetic field $B_0 = \frac{m^2}{e} = 4.4 \times 10^{13}$ Gauss

Magnetic field measured by $b \equiv \frac{B}{B_0} \gg 1 \to \infty$



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Electron Compton half-length $\lambda_\mathrm{C}/2 = \frac{1}{2m}$

Larmour length $L_\mathrm{B} = (eB)^{-1/2} = \frac{1}{m\sqrt{b}}$

The hierarchy of two scales: $L_\mathrm{B} \ll \lambda_\mathrm{C}$

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Static potential of a point charge q

$$A_0(\mathbf{x}) = \frac{q}{(2\pi)^3} \int \frac{\mathrm{e}^{-\mathrm{i}\mathbf{kx}} \mathrm{d}^3 k}{\mathbf{k}^2 - \kappa_2(0, k_3^2, k_\perp^2)}, \quad A_{1,2,3}(\mathbf{x}) = 0.$$

κ_2 is one (out of three) eigenvalues of polarization operator at zero frequency $k_0 = 0$

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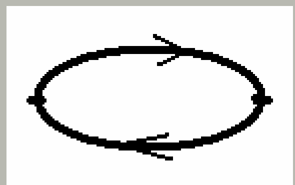
In one-loop

Batalin & A. Sh., 1971 (b>>1)

$$\kappa_2(0, k_3^2, k_\perp^2) = -\frac{2\alpha b m^2}{\pi}\exp\left(-\frac{k_\perp^2}{2m^2 b}\right) T\left(\frac{k_3^2}{4m^2}\right),$$

where T is a known function,

$$T(0) = 0, \; T(\infty) = 1$$

κ_2  grows linearly with magn. field

Loskutov & Skobelev, 1975; Melrose & Stoneham, 1976; A.Sh., 1976

$$A_0(\mathbf{x}) = \frac{q}{(2\pi)^3}\int \frac{\mathrm{e}^{-\mathrm{i}\mathbf{kx}}\mathrm{d}^3k}{\mathbf{k}^2 - \kappa_2(0, k_3^2, k_\perp^2)}, \quad A_{1,2,3}(\mathbf{x}) = 0.$$

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$$T(0) = 0,\ T(\infty) = 1$$

Short- and long-range parts

$$A_0(\mathbf{x}) = A_{\text{s.r.}}(\mathbf{x}) + A_{\text{l.r.}}(\mathbf{x})$$

$$A_{\text{s.r.}}(\mathbf{x}) \equiv A_0(\mathbf{x})\ |_{T=1}$$

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$$A_{\text{s.r.}}(\mathbf{x}) \equiv A_0(\mathbf{x}) \,|_{T=1}$$

For $L_{\text{B}} < |\mathbf{x}| \ll \lambda_{\text{C}}$

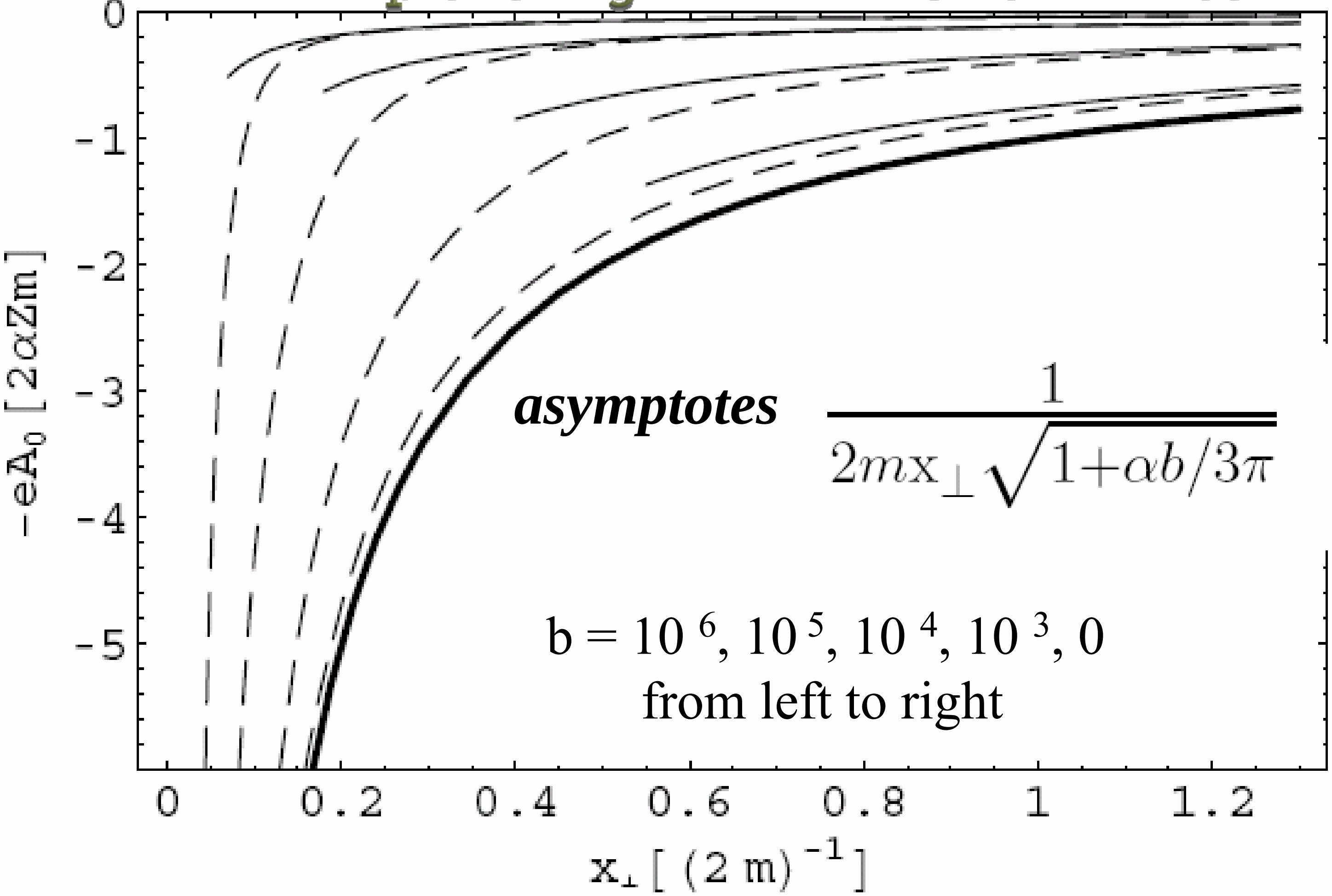
$$A_{\text{s.r.}}(\mathbf{x}) \simeq \frac{q}{4\pi} \frac{\exp\{-\left(\frac{2\alpha b}{\pi}\right)^{\frac{1}{2}} m|\mathbf{x}|\}}{|\mathbf{x}|}$$

Effective photon mass squared $\dfrac{2\alpha}{\pi L_{\text{B}}^2} T(\infty)$ Kuznetsov, *et al*. 2002

For $|\mathbf{x}| > \lambda_{\text{C}}$

$$A_0(x_3, x_\perp) \simeq \frac{1}{4\pi} \frac{q}{\sqrt{(x'_\perp)^2 + x_3^2}}, \quad x'_\perp = x_\perp \left(1 + \frac{\alpha b}{3\pi}\right)^{1/2}$$

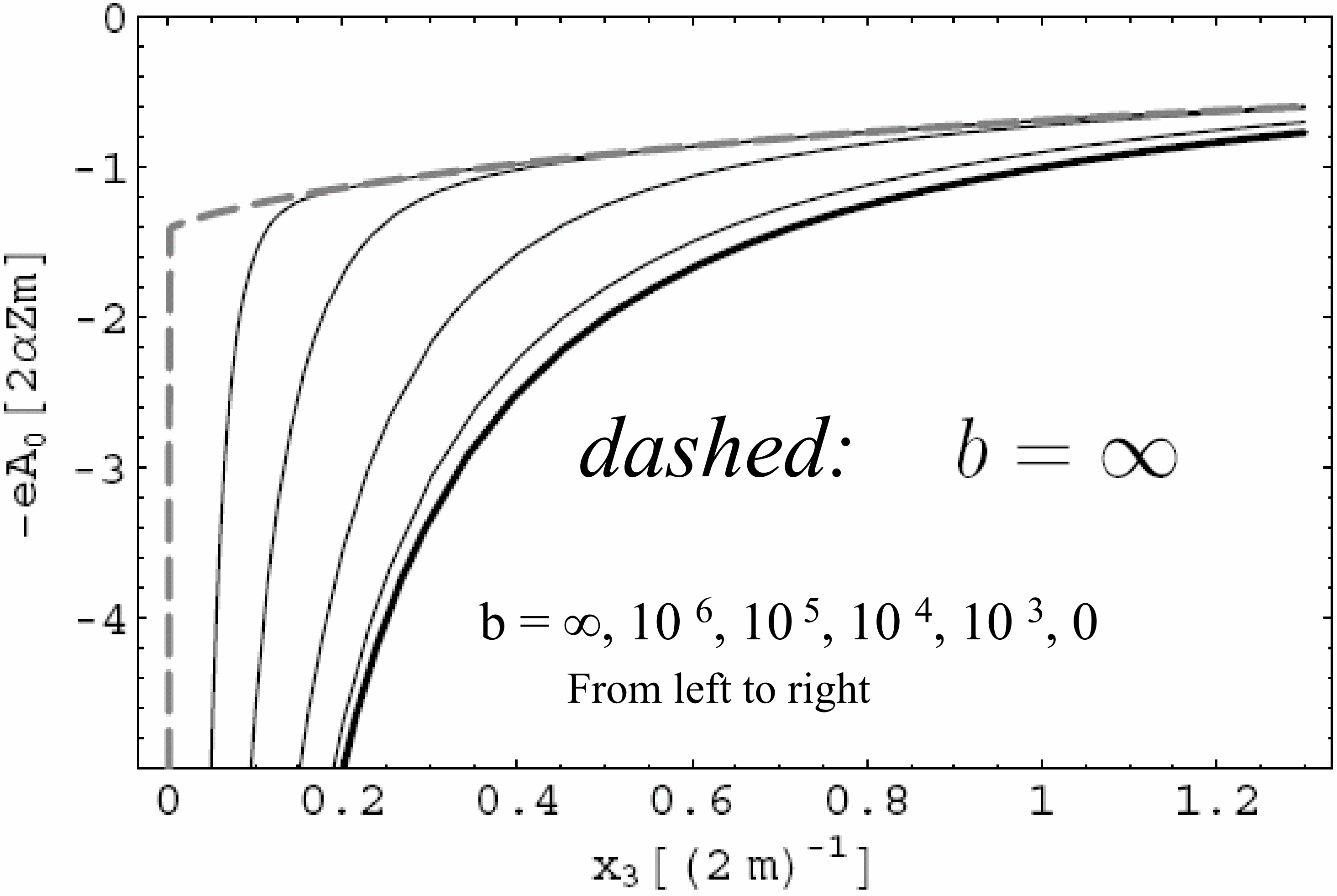
Electron potential energy in the field of a point charge *q=eZ* plotted against transverse distance at $x_3 = 0$

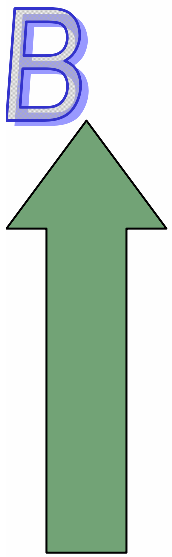


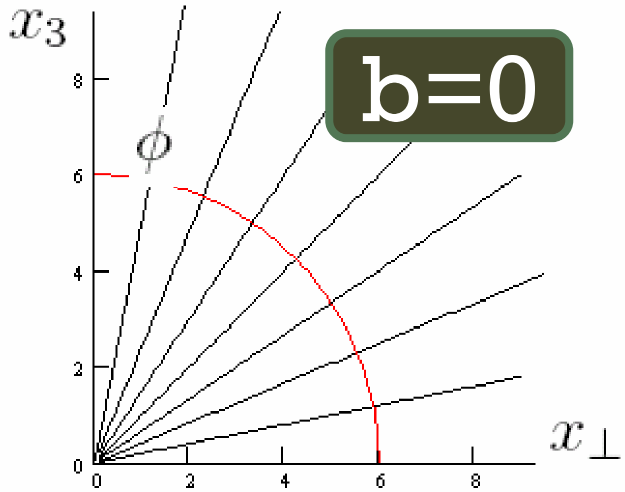
A unit along ordinate values *Z* 7.46 keV

Electron potential energy plotted against longitudinal distance at $X_\perp = 0$

$q = eZ$



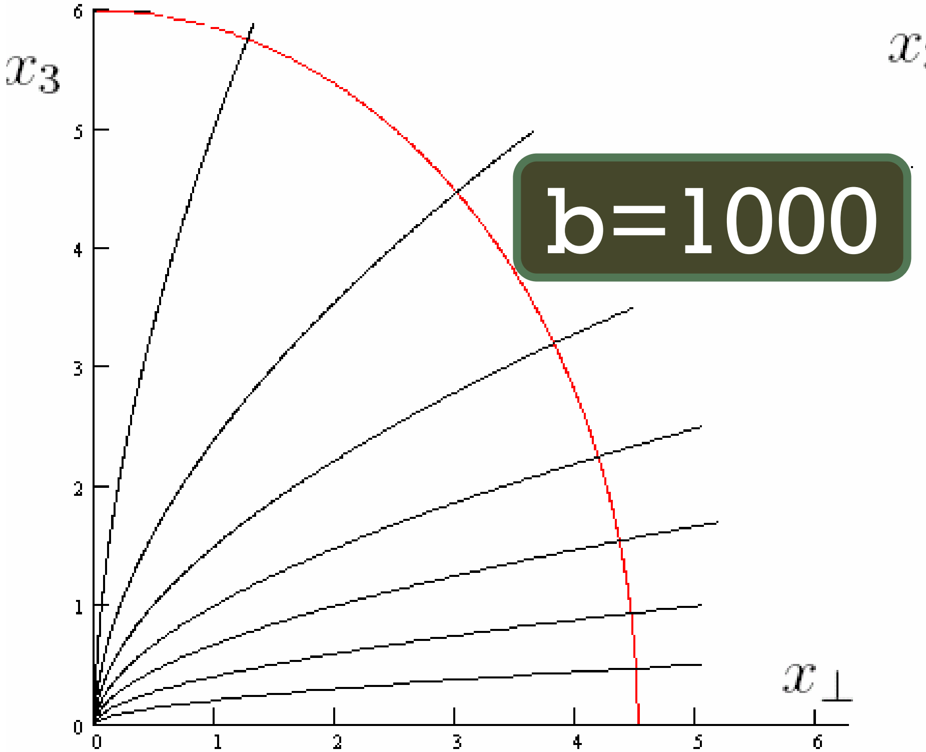


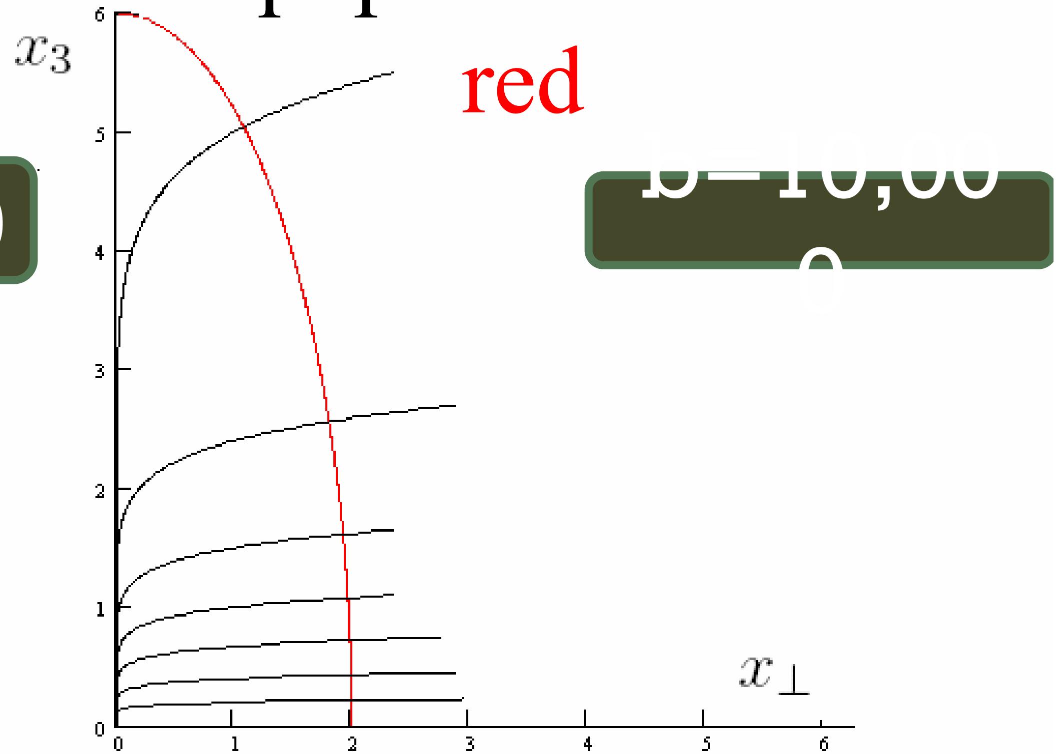


Electric lines of force: black

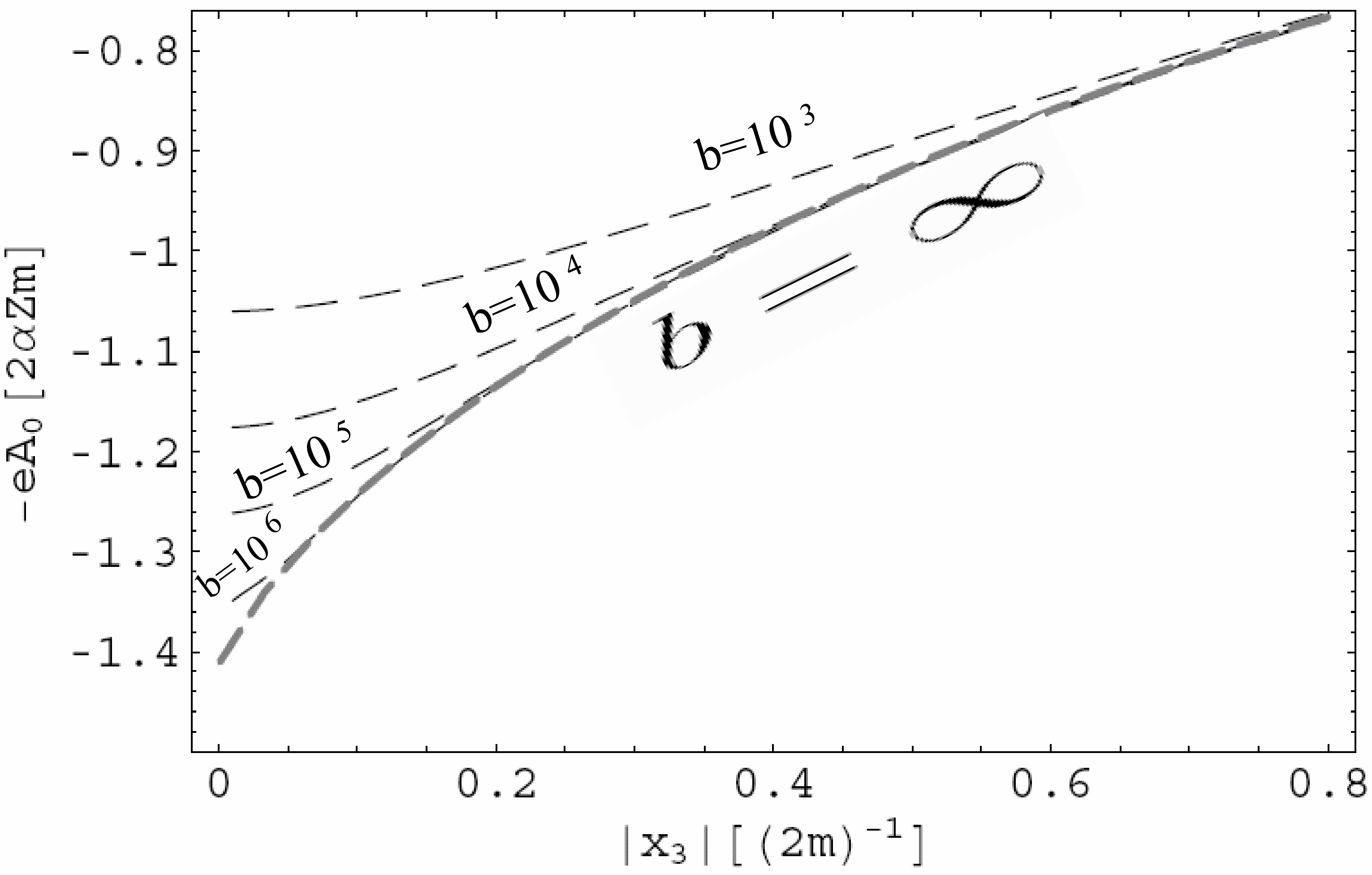
$$x_3 = \tan\phi \; x_\perp^{\left(\frac{1}{1+\frac{\alpha b}{3\pi}}\right)}$$

Equipotential surface: red

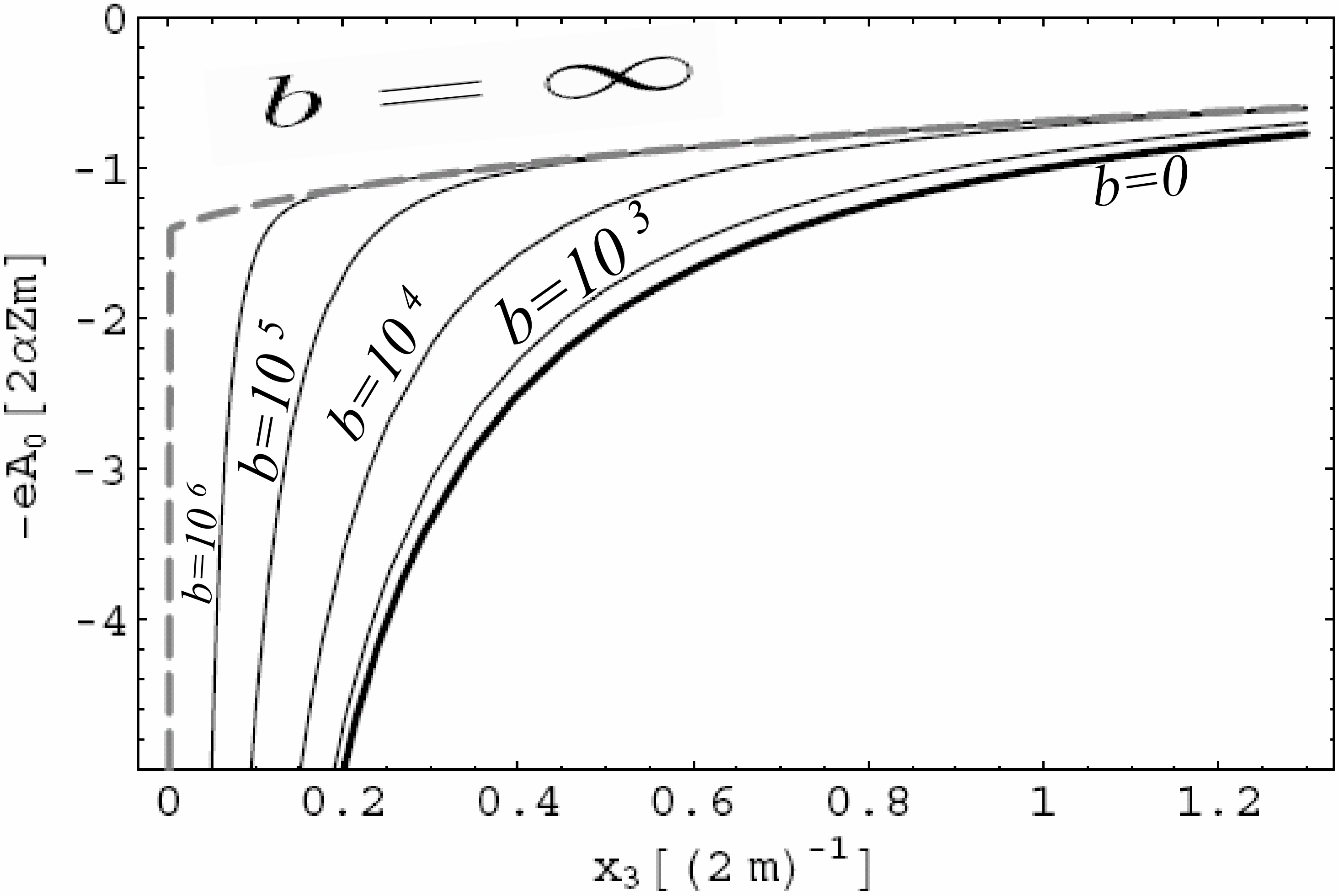




Long-range part of the potential plotted against the longitudinal distance at $X_\perp = 0$



Electron potential energy plotted against longitudinal distance at $X_\perp = 0$



Potential on the string
(infinite magnetic field)

Long-range part on the axis $X_\perp = 0$

near the charge $2x_3 m \ll 1$

$$A_{\mathrm{l.r.}}(x_3,0)|_{b=\infty} = \frac{qm}{2\pi}[1.4152 + 0.495 \cdot 2mx_3(\ln(2mx_3) - 0.77)]$$

Confinement of massless electrons

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Confinement of massless electrons

Short-range part on the axis $X_\perp = 0$

$$A_{s.r.}(x_3, 0)|_{b=\infty} = 2.180\frac{q}{2\pi}\delta(x_3)$$

Radiative shift of ground level in H-like atom

One-dimensional Schroedinger equation

$$-\frac{1}{2m}\frac{\mathrm{d}^2\Psi(x_3)}{\mathrm{d}x_3^2} - eA_0(x_3, x_\perp = 0)\Psi(x_3) = E\Psi(x_3), \qquad |x_3| > L_\mathrm{B}$$

Unbounded ground state energy with Coulomb potential

$$b \to \infty$$

$$E_0 = -2Z^2\alpha^2 m \ln^2 \frac{\sqrt{b}}{2\alpha}$$

Loudon & Elliott, 1959

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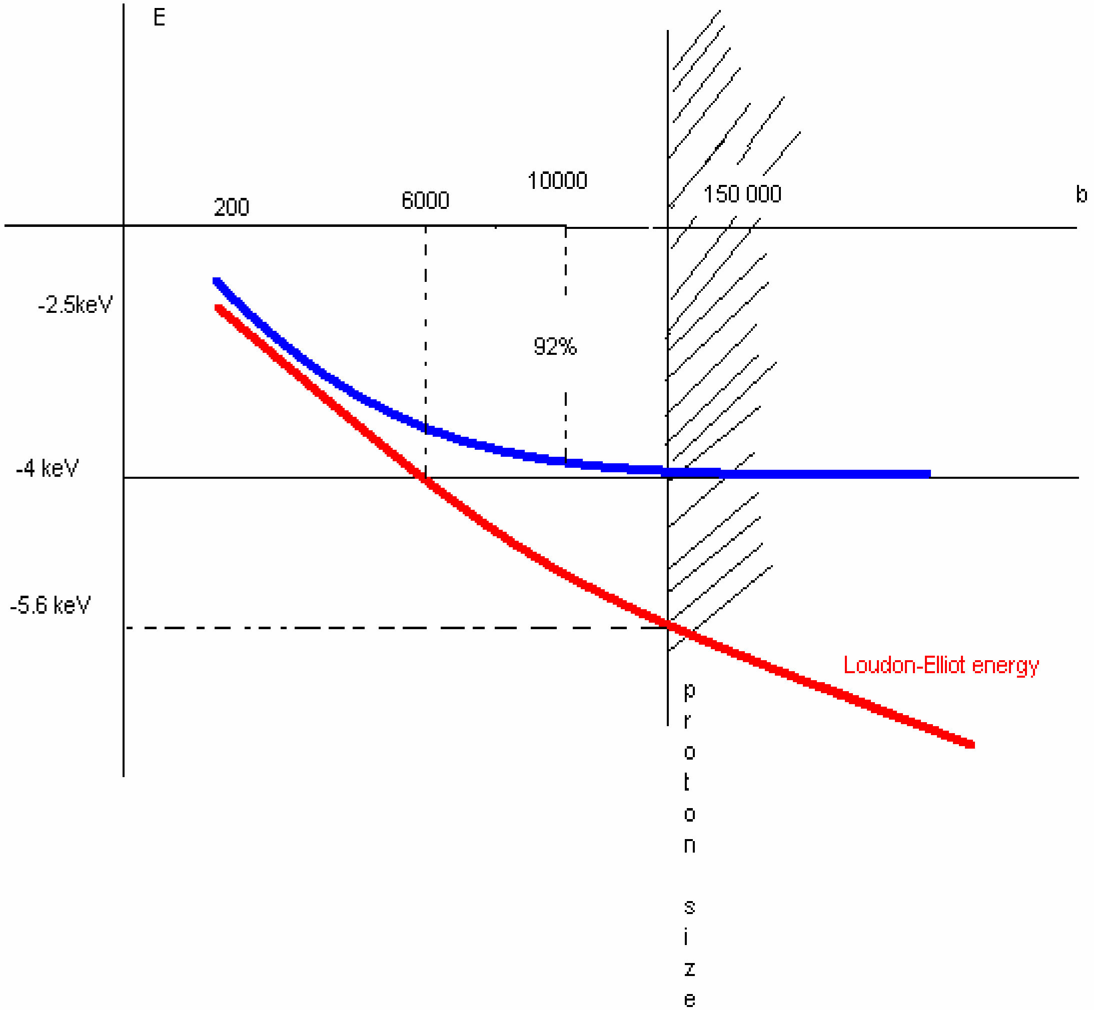
Finite ground state energy with modified (delta-function) potential

$b \to \infty$

$$E_{\mathrm{lim}} = -2mZ^2\alpha^2\ 73.6 = -Z^2 \times 4.0\ \mathrm{keV}$$

Hydrogen ground state energy (sketch)



Conclusions

In magnetic fields $B>>B_0$, the electrostatic field of a point charge :

~ **has short-range Yukawa form in Larmour scale, with photon mass determined by inverse Larmour length**

~ **has an anisotropic long-range form in Compton scale, decreasing across the field faster than along**

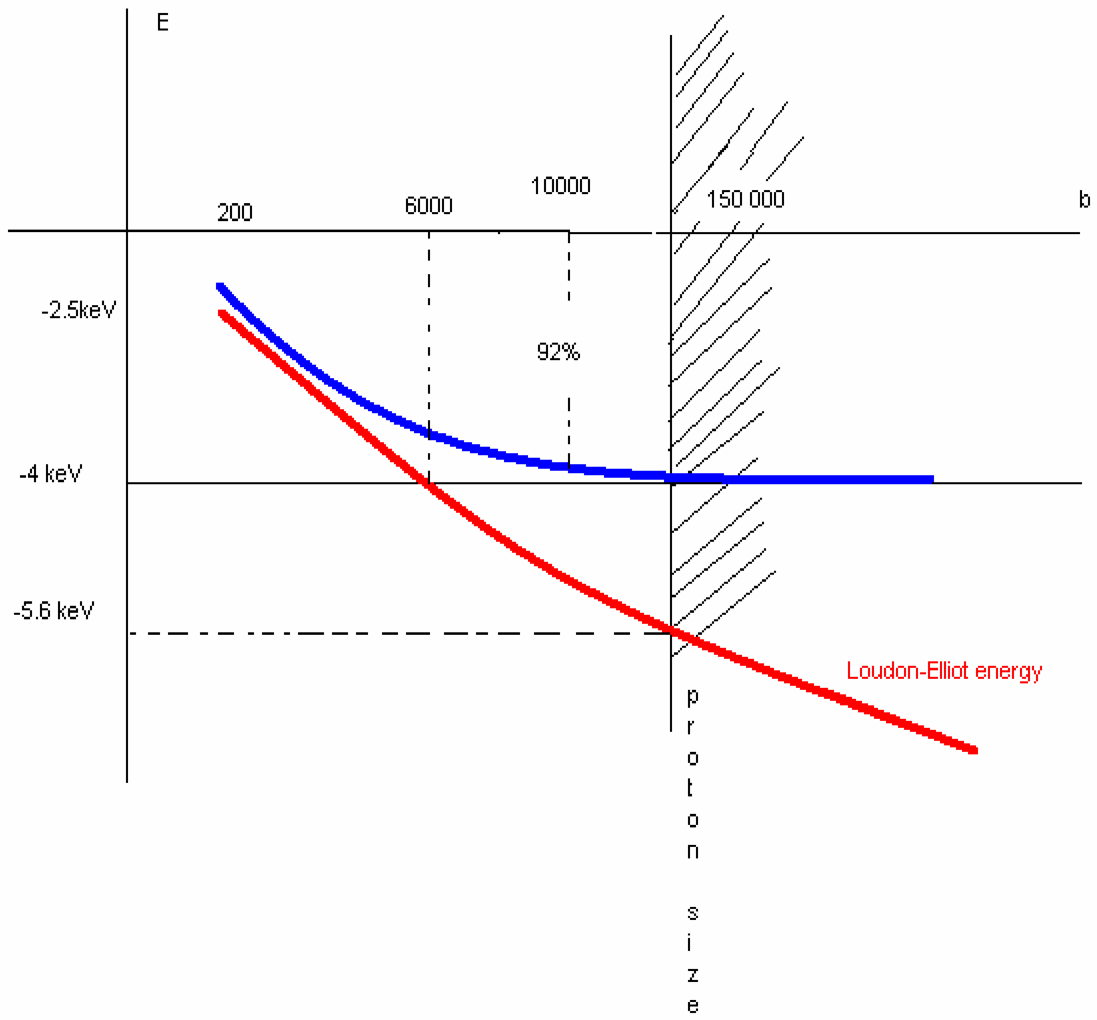
~ **in the infinite-*B* limit, is concentrated in a string along *B* – *a sign of dimensional reduction in photonic sector***

~ **String potential leads to "confinement" within Compton length and has delta-function singularity in the charge**

~ **there is no unlimited growth of binding energy of hydrogen atom**

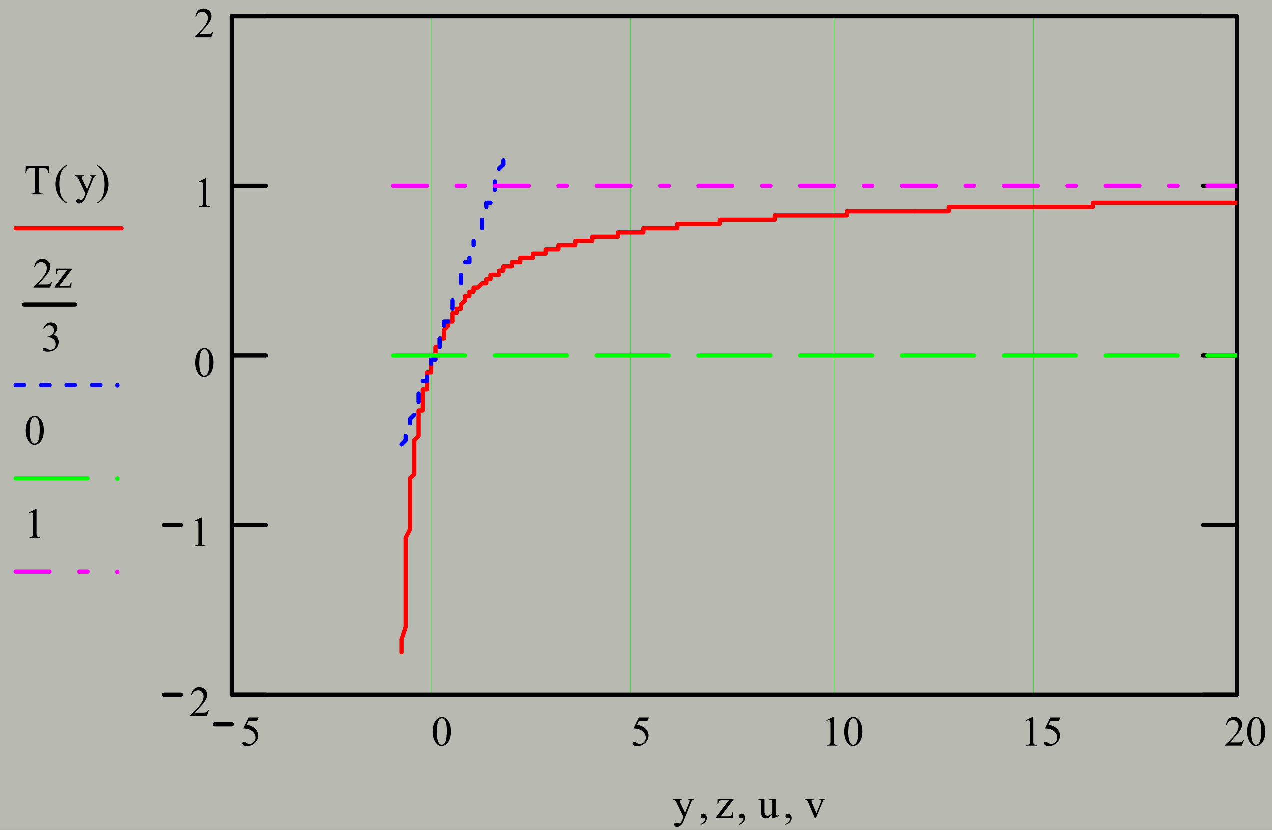
The end

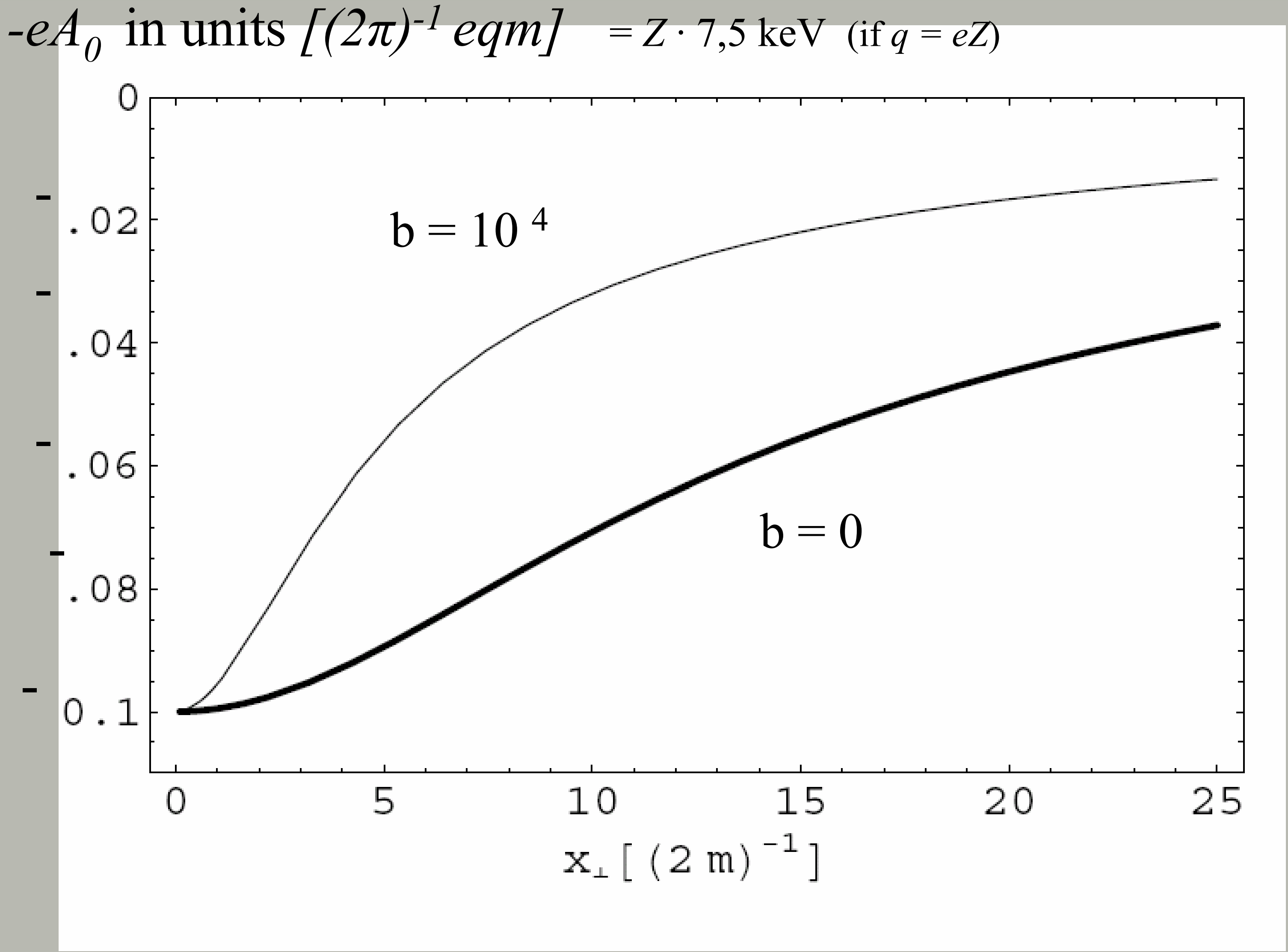
Hydrogen ground state energy (sketch)



T(y)

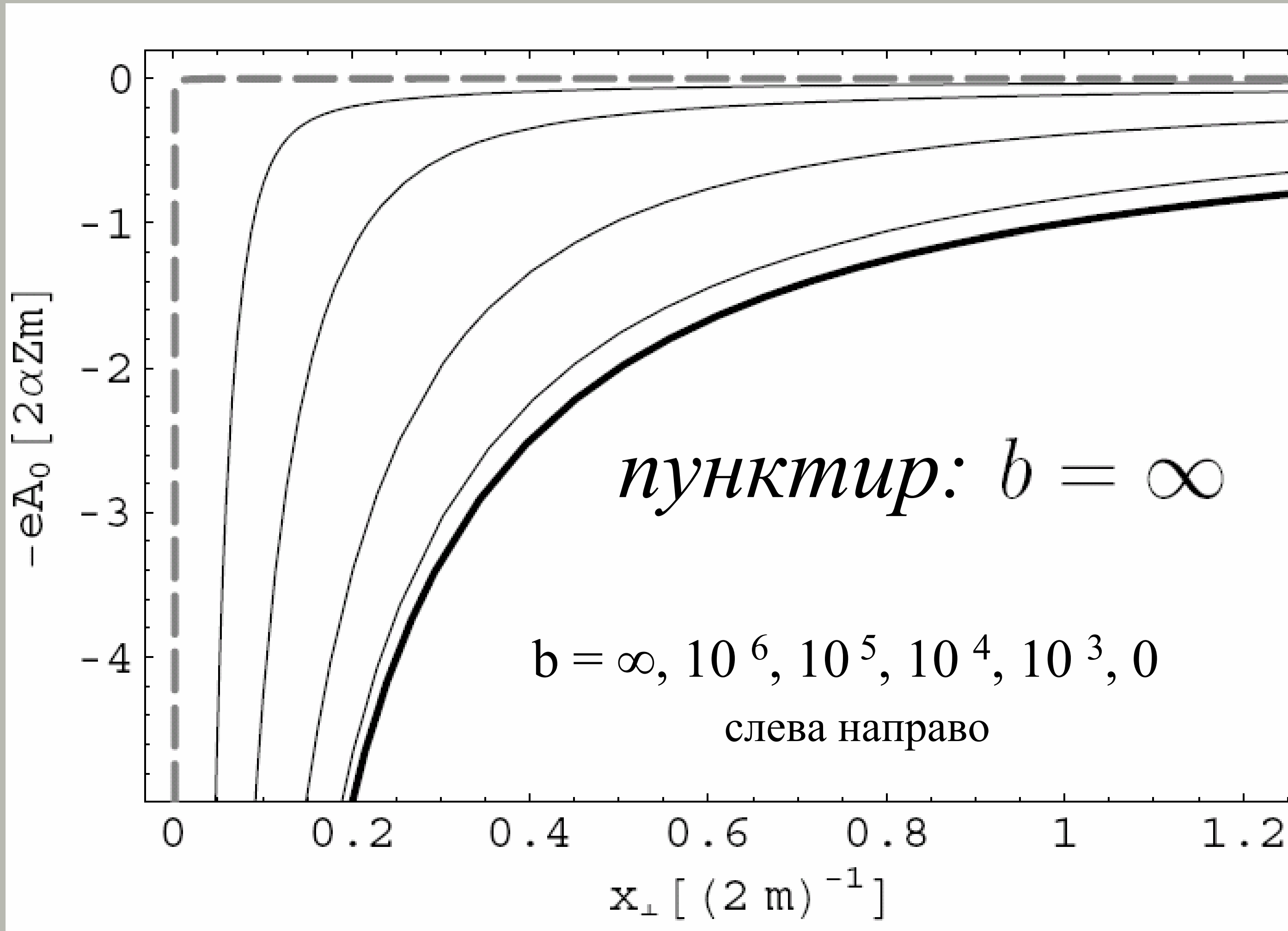
$$T(y) := y \cdot \left[\int_0^1 \frac{1-\eta^2}{1+y\cdot\left(1-\eta^2\right)}\, d\eta \right]$$





Electron potential energy plotted against transverse distance at $x_3 = 10\ (2m)^{-1}$

Короткодействующая часть потенциальной энергии как функция продольного расстояния при $X_\perp = 0$



Потенциальная энергия электрона как функция продольного расстояния $X_\perp$ = 0

(пунктир – ее дальнодействующая часть)