Phase transitions in dense quark matter in a constant curvature gravitational field

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Phase transitions in dense quark matter

- Chiral symmetry breaking (χSB) — quark condensate $< \bar{q}q > \neq 0$ (low baryon density and low temperature)
- Color superconductivity (CSC) — diquark condensate $< qq > \neq 0$ (high baryon density and low temperature)

As diquark condensate $< qq >$ appears in the color anti-triplet channel, the color symmetry $SU_c(3)$ should be spontaneously broken inside the CSC phase.

Motivation

- We study the influence of strong gravitational field on phase transitions in dense quark matter with quark and diquark condensates.
- We also study the influence of finite temperature and chemical potential on the phase transitions.

We use the effective Nambu-Jona-Losinio - type model.
To study the influence of gravity we take the static Einstein Universe.

The extended NJL model in curved space-time

The extended Nambu–Jona-Lasinio model with two flavors of quarks:

$$\mathcal{L} = \bar{q}\left[i\gamma^{\mu}\nabla_{\mu} + \mu\gamma^{0}\right]q + \frac{G_1}{2N_c}\left[(\bar{q}q)^2 + (\bar{q}i\gamma^5\vec{\tau}q)^2\right] + \\ + \frac{G_2}{N_c}\left[i\bar{q}_c\varepsilon\epsilon^b\gamma^5 q\right]\left[i\bar{q}\varepsilon\epsilon^b\gamma^5 q_c\right], \quad (1)$$

where μ is the quark chemical potential.
The Lagrangian is invariant under the chiral $SU(2)_L \times SU(2)_R$ and color $SU_c(3)$ groups. This model may be considered as low energy limit of QCD.

Bosonization procedure

Bosonization procedure leads to the linearized version of the model:

$$\tilde{\mathcal{L}} = \bar{q}\left[i\gamma^\mu\nabla_\mu + \mu\gamma^0\right]q - \bar{q}\left(\sigma + i\gamma^5\vec{\tau}\vec{\pi}\right)q - \frac{3}{2G_1}(\sigma^2 + \vec{\pi}^2) - \\ - \frac{3}{G_2}\Delta^{*b}\Delta^b - \Delta^{*b}\left[i\bar{q}^t C\varepsilon\epsilon^b\gamma^5 q\right] - \Delta^{*b}\left[i\bar{q}\varepsilon\epsilon^b\gamma^5 C\bar{q}^t\right]. \quad (2)$$

The Lagrangians (1) and (2) are equivalent on equations of motion:

$$\Delta^b = -\frac{G_2}{3}\, iq^t C\varepsilon\epsilon^b\gamma^5 q, \qquad \sigma = -\frac{G_1}{3}\,\bar{q}q, \qquad \vec{\pi} = -\frac{G_1}{3}\,\bar{q}i\gamma^5\vec{\tau}q.$$

Collective boson fields:

- σ and $\vec{\pi}$ — color singlets.
- Δ^b — color anti-triplet.

Symmetry breaking:

- If $\langle\sigma\rangle \neq 0 \rightarrow$ chiral symmetry is dynamically broken.
- If $\langle\Delta^b\rangle \neq 0 \rightarrow$ the color and electromagnetic symmetries are broken.

Effective action

The effective action for boson fields:

$$iS_{\rm eff}(\sigma,\vec{\pi},\Delta^b,\Delta^{*b}) = \ln\int[dq][d\bar{q}]\exp\{\, i\int d^Dx\sqrt{-g}\,\tilde{\mathcal{L}}\} =$$
$$= -N_c\int d^Dx\sqrt{-g}\left[\frac{\sigma^2+\vec{\pi}^2}{2G_1}+\frac{\Delta^b\Delta^{*b}}{G_2}\right]+\tilde{S}_q, \quad (3)$$

where $\tilde{S}_q$ is a quark contribution.
The mean field approximation:

$$\sigma = -\frac{G_1}{3}\langle\bar{q}q\rangle, \quad \vec{\pi} = -\frac{G_1}{3}\langle\bar{q}i\gamma^5\vec{\tau}q\rangle, \quad \Delta^b = -\frac{G_2}{3}\langle iq^t C\varepsilon\epsilon^b\gamma^5 q\rangle.$$

The ground state

We may choose the ground state of our model in the simplest form:

$$\langle \Delta^1 \rangle = \langle \Delta^2 \rangle = \langle \vec{\pi} \rangle = 0$$

$$\langle \sigma \rangle \neq 0 \qquad \langle \Delta^3 \rangle \neq 0$$

Evidently, this choice breaks color symmetry to the residual color group $SU_c(2)$.

Effective potential

The effective potential:

$$V_{\text{eff}} \equiv -\frac{S_{\text{eff}}}{\int d^D x \sqrt{-g}} = \frac{3\sigma^2}{2G_1} + \frac{3\Delta^b \Delta^{*b}}{G_2} + \tilde{V}, \tag{4}$$

where

$$\tilde{V} = -\frac{\tilde{S}_q}{v}, \quad v = \int d^D x \sqrt{-g}.$$

Performing the integration over quark fields one can obtain the quark contribution to the effective action:

$$\begin{aligned} i\tilde{S}_q(\sigma, \Delta) &= \ln \det \left[(i\hat{\nabla} - \sigma + \mu\gamma^0) \right] + \\ &+ \ln \det{}^{1/2} \left[4|\Delta|^2 + (-i\hat{\nabla} - \sigma + \mu\gamma^0)(i\hat{\nabla} - \sigma + \mu\gamma^0) \right]. \end{aligned}$$

Static Einstein Universe

The line element in the static D-dimensional Einstein Universe:

$$ds^2 = dt^2 - a^2(d\theta^2 + \sin^2\theta d\Omega_{D-2}), \tag{5}$$

where a is radius of the Universe. The global topology is $\mathbb{R} \otimes \mathbb{S}^{D-1}$.
The scalar curvature:

$$R = \frac{(D-1)(D-2)}{a^2}. \tag{6}$$

The volume of the Universe:

$$V(a) = \frac{2\pi^{D/2} a^{D-1}}{\Gamma(\frac{D}{2})}. \tag{7}$$

Evaluation of determinants in the static Einstein Universe

One may introduce the Hamiltonian operators for massive and massless particles:

$$\begin{aligned} \hat{\mathcal{H}} &= \vec{\alpha}\,\hat{\vec{p}} + \sigma\gamma^0, \\ \hat{H} &= \vec{\alpha}\,\hat{\vec{p}}, \end{aligned} \tag{8}$$

where $\alpha^k = \gamma^0\gamma^k$, and $(\hat{p})_k = -i\nabla_k,\ k = 1\ldots D-1$.
The quark contribution to the effective action:

$$2i\tilde{S}_q(\sigma,\Delta) = \ln\det\left[\hat{\mathcal{H}}^2 - (\hat{p}_0 - \mu)^2\right] + \ln\det\left[4|\Delta|^2 + (\hat{\mathcal{H}} - \mu)^2 - \hat{p}_0^2\right]. \tag{9}$$

The second operator has the energy gap $|\Delta|$ in the spectrum, which leads to the color superconductivity.

The eigenvalues in the static Einstein Universe

The eigenvalues of Hamiltonian operators:

$$\begin{aligned} \hat{H}\psi_n &= \pm\omega_n\psi_n, \quad \omega_n = \frac{1}{a}\left(n+\frac{D-1}{2}\right), \quad n=0,1,2\ldots \\ \hat{\mathcal{H}}\psi_n &= \pm E_n\psi_n, \quad E_n = \sqrt{\omega_n^2+\sigma^2}. \end{aligned} \tag{10}$$

The degeneracies of ω_n and E_n are equal to

$$d_n = \frac{2^{[(D+1)/2]}\Gamma(D+n-1)}{n!\Gamma(D-1)}, \tag{11}$$

where $[x]$ is the integer part of x.

Thermodynamic potential and gap equations

The effective potential at finite temperature or thermodynamic potential (TDP):

$$\Omega_{\text{eff}}(\sigma,\Delta) = N_c\left(\frac{\sigma^2}{2G_1}+\frac{|\Delta|^2}{G_2}\right) - \\ -\frac{N_f}{V}(N_c-2)\sum_{n=0}^{\infty} d_n\left\{E_n + T\ln\left(1+e^{-\beta(E_n\pm\mu)}\right)\right\} - \\ -\frac{N_f}{V}\sum_{n=0}^{\infty} d_n\left\{\sqrt{(E_n\pm\mu)^2+4|\Delta|^2} + 2T\ln\left(1+e^{-\beta\sqrt{(E_n\pm\mu)^2+4|\Delta|^2}}\right)\right\}. \quad (12)$$

The global minimum point of TDP is determined from the gap equations:

$$\frac{\partial\Omega_{\text{eff}}}{\partial|\Delta|}=0, \quad \frac{\partial\Omega_{\text{eff}}}{\partial\sigma}=0.$$

that define values of condensates $\langle\sigma\rangle$ and $\langle\Delta^3\rangle$.

Regularization

The TDP is divergent at high energy (large n). Soft cutoff: $\exp\left(-\omega_n/\Lambda\right)$. In flat space-time Λ can be determined from the pion mass or the pion decay constant. In curved space-time we have no such experiments. Qualitative discussion. To obtain the dimensionless TDP we divide all quantities by an appropriate power of the cutoff parameter:

$$\Omega^{\mathrm{reg}}(\sigma, \Delta) = N_c \left(\frac{\sigma^2}{2G_1} + \frac{|\Delta|^2}{G_2} \right) -$$
$$- \frac{N_f}{V}(N_c - 2) \sum_{n=0}^{\infty} e^{-\omega_n} d_n \left\{ E_n + T \ln\left(1 + e^{-\beta(E_n \pm \mu)}\right) \right\} -$$
$$- \frac{N_f}{V} \sum_{n=0}^{\infty} e^{-\omega_n} d_n \Bigg\{ \sqrt{(E_n \pm \mu)^2 + 4|\Delta|^2} +$$
$$+ 2T \ln\left(1 + e^{-\beta\sqrt{(E_n \pm \mu)^2 + 4|\Delta|^2}}\right) \Bigg\}.$$

Phase transitions

We will use numerical calculations to find the global minimum of regularized TDP

$$\Omega^{\mathrm{reg}}(\sigma, \Delta) - \Omega^{\mathrm{reg}}(0, 0).$$

We fix the constant G_2 as in flat space-time:

$$G_2 = \frac{3}{8} G_1.$$

We also choose the coupling constant G_1 in such a way that the chiral and/or color symmetries were completely broken: $G_1 = 10$ (strong coupling). Consider the case $D = 4$.

Zero temperature

The behavior of the condensates (σ_0, Δ_0) at the global minimum point as a functions of μ at $R = 3$ and $T = 0$, for $G_1 = 10$:

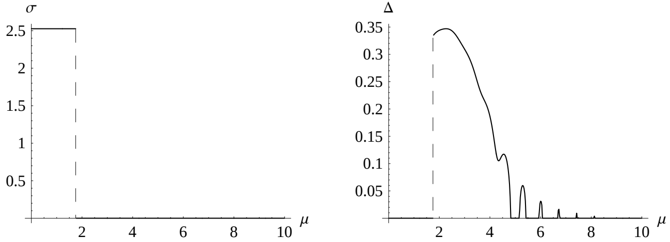

Figure: Condensates σ_0 and Δ_0 as functions of μ for $R = 3$, $G_1 = 10$ (all quantities are given in units of Λ).

When the chemical potential exceeds the critical value μ_c, the chiral symmetry is restored while the color symmetry is broken.

Fermion number density $n = -\partial\Omega^{\rm reg}/\partial\mu$ at zero temperature.

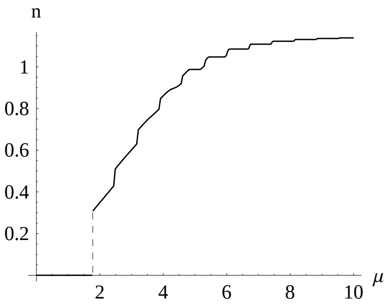

Figure: The fermion number density at R=3, T=0, $G_1 = 10$.

Since the first derivative of thermodynamic potential is discontinuous chemical potential leads to the first order phase transition.

Phase portrait at T=0:

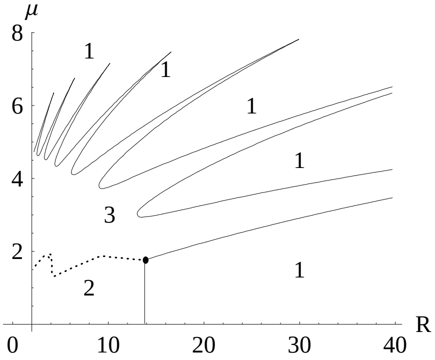

Figure: The phase portrait at T=0 for $G_1 = 10$. Dotted (solid) lines denote first (second) order phase transitions. The bold point denotes a tricritical point. The numbers 1,2 and 3 designate the symmetric, chiral symmetry breaking and superconducting phases, respectively.

- As in flat space-time there is no mixed phase where both condensates are nonzero.
- One can see oscillations of the phase curve. Possible explanation: discreteness of the fermion energy levels in the compact space (similar effect in the magnetic field H, where fermion Landau levels are also discrete (van Alphen-de Haas magnetic oscillations)).

Phase portraits at finite temperature

The growing temperature leads to the restoration of the chiral and color symmetries. The similarity of $R-\mu$ and $\mu-T$ plots: Curvature R and temperature T play similar roles in restoring the symmetries.

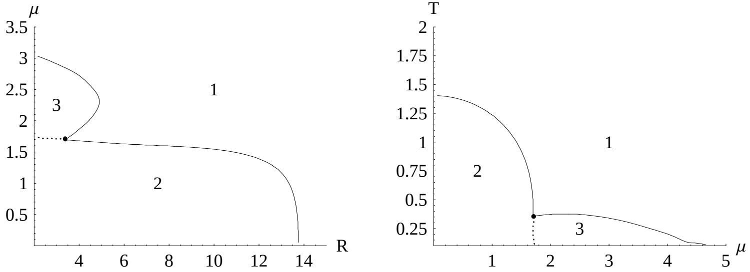

Figure: The phase portraits at T=0.35 (left) and at R=3 (right).

Summary

- In the framework of the NJL model we have derived a nonperturbative expression for the effective potential of the theory in the mean field approximation.
- The influence of gravity was exactly taken into account in the case of the static Einstein Universe.
- The influence of the chemical potential and temperature on the phase transitions was studied.
- The oscillation effect of the phase curves was found, which may be explained by discreteness of the fermion energy levels in the compact space.