Decay constants and Masses of Heavy-Light mesons.

The ratios f_{D_s}/f_D and f_{B_s}/f_B as a test of the running mass m_s at low scale

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4. The ratios $f_{D_s}/f_D, f_{B_s}/f_B$ as a test of $m_s(\mu)$ at low μ.

Exp. data

$$D(D_s) \to \mu\nu_\mu \qquad (CLEO)$$

The best accuracy is reached.

$$F_D = 222(20)\ MeV, \quad f_{D_s} = 283(23)\ MeV$$

$$\eta_D(\exp) = \frac{f_{D_s}}{f_D} = 1.27(14) \qquad (2005)$$

$$B^+ \to \tau^+ \nu_\tau \quad (2005)$$

$$f_B = 160^{+50}_{-80}\ MeV \quad (BaBar,\ 2006)$$

$$f_B = 229 \pm 70\ MeV \quad (Belle, 2006)$$

$$\Gamma(D_s^+ \to l^+ \nu_l) = \frac{G_F^2}{8\pi} f_{D_s}^2 m_l^2 M_{D_s} |V_{cs}|^2 \left(1 - \frac{m_l^2}{M_{D_s}^2}\right)^2$$

$$f_{B_s}(lattice) = 266(10)\ MeV$$

$$\frac{f_{B_s}}{f_B} = 1.20(4) \quad (lattice, \quad n_f = 2+1)$$

We have calculated the masses of HL mesons, hyperfine splittings, $f_P(1S), f_P(2S), f_V(1S)$.

1. The running mass $m_s(\mu_1)$ at a low scale, $\mu_1 \approx 0.5$ GeV, can be extracted from the values η_D and η_B if they are known with high accuracy, $\lesssim 5\%$.

2. The values η_D and $\eta_{\rm B}$, can be obtained only if the running mass $m_s(\mu_1)$ is inside the range $170 - 210$ MeV.

3. In the chiral limit, $m_d = m_u = 0$, as well as for $m_d = 8$ MeV, for $m_s(\mu_1) = 180$ MeV the ratios η_D and η_B are

$$\eta_D = 1.25, \quad \eta_B = 1.19\,. \tag{1}$$

4. The value $m_s(0.5\text{ GeV})$ satisfies the relation

$$\frac{m_s(0.5\ \mathrm{GeV})}{m_s(2\ \mathrm{GeV})} \approx 1.97\,. \tag{2}$$

In our analysis we use the analytical expression for the leptonic decay constant in the pseudoscalar (P) channel, with the use of the path-integral representation for the correlator $G_{\rm P}$ of the currents $j_{\rm P}(x)$:

$G_{\rm P}(x) = \langle j_{\rm P}(x) j_{\rm P}(0)\rangle_{\rm vac}$:

$$J_{\rm P} = \int G_{\rm P}(x) d^3x = 2N_c \sum_n \frac{\langle Y_{\rm P}\rangle_n |\varphi_n(0)|^2}{\omega_{qn}\omega_{Qn}} e^{-M_n t}, \qquad (3)$$

where M_n and $\varphi_n(r)$ are the eigenvalue and eigenfunction (e.f.) of the relativistic string Hamiltonian, while $\omega_{qn}(\omega_{Qn})$ is the average kinetic energy of a quark $q(Q)$ for a given nS state:

$$\omega_{qn} = \langle \sqrt{m_q^2 + \mathbf{p}^2}\rangle_{nS}, \quad \omega_{Qn} = \langle \sqrt{m_Q^2 + \mathbf{p}^2}\rangle_{nS}. \qquad (4)$$

In Eq. (4) $m_q(m_Q)$ is the pole mass of the lighter (heavier) quark in a heavy-light (HL) meson. The matrix element (m.e.) $\langle Y_{\rm P}\rangle_n$ refers the to P channel (with exception of the π and K mesons):

$$\langle Y_{\rm P}\rangle_n = m_q m_Q + \omega_{qn}\omega_{Qn} - \langle \mathbf{p}^2\rangle_{nS}. \qquad (5)$$

$$
\begin{aligned}
j_\Gamma(x) &= \bar{\psi}_1(x)\Gamma\psi_2(x), \\
\Gamma &= t^a \otimes (1, \gamma_5, \gamma_\mu, i\gamma_\mu\gamma_5) \text{ for S, P, V, and A channels}, \\
t^a &= \frac{\lambda^a}{2}, \quad tr(t^a t^b) = \frac{1}{2}\delta_{ab},
\end{aligned} \tag{6}
$$

$$
G_\Gamma(x) \equiv \langle j_\Gamma(x) j_\Gamma(0)\rangle_v
$$

$$
= 4N_c \int \hat{Y}_\Gamma (D^3 z)_{\mathbf{x}0} (D^3 \bar{z})_{\mathbf{x}0} (D\omega_1)(D\omega_2) \exp(-K_1 - K_2) W_\sigma. \tag{7}
$$

$$
4\hat{Y}_\Gamma = tr_L(m_1 - \hat{D}_1)\Gamma(m_2 - \hat{D}_2)\Gamma \to tr_L(m_1 + \omega_1 \hat{\dot{z}})\Gamma(m_2 - \omega_2 \hat{\dot{\bar{z}}})\Gamma, \tag{8}
$$

$$\begin{aligned}
\langle Y_{\rm V}\rangle &= m_1m_2+\langle\bar\omega_1\rangle\langle\bar\omega_2\rangle+\frac{1}{3}\langle\mathbf{p}^2\rangle,\\
\langle Y_{\rm P}\rangle &= m_1m_2+\langle\bar\omega_1\rangle\langle\bar\omega_2\rangle-\langle\mathbf{p}^2\rangle=\langle Y_{A_4}\rangle\\
\langle Y_S\rangle &= m_1m_2-\langle\bar\omega_1\rangle\langle\bar\omega_2\rangle-\langle\mathbf{p}^2\rangle,\\
\langle Y_{A_i}\rangle &= m_1m_2-\langle\bar\omega_1\rangle\langle\bar\omega_2\rangle-\frac{\langle\mathbf{p}^2\rangle}{3}.
\end{aligned} \tag{9}$$

On the other hand, for the integral $J_{\rm P}$ (3) one can use the conventional spectral decomposition:

$$J_{\rm P}=\int G_{\rm P}(x)d^3x=\sum_n\frac{1}{2M_n}(f_{\rm P}^n)^2e^{-M_nT}. \tag{10}$$

Then from Eqs. (10) and (3) one obtains that

$$(f_{\rm P}^n)^2=\frac{2N_c\langle Y_{\rm P}\rangle_n|\varphi_n(0)|^2}{\omega_{qn}\omega_{Qn}M_n}. \tag{11}$$

Table 1: The average energies $\bar{\omega}_i(nS) = \left\langle \sqrt{m_i^2 + \mathbf{p}^2} \right\rangle_{nS}$ $(i = 1, 2)$, the reduced mass $\omega_{\rm r}$, and the excitation energy $\varepsilon_n(\omega_{\rm r})$ (in MeV) for the $1S$ and $2S$ heavy-light mesons.

Meson	D	D_s	B	B_s	B_c
$\bar{\omega}_1(1S)$	507	559	587	639	1662
$\bar{\omega}_2(1S)$	1509	1515	4827	4830	4869
$\omega_{\rm r}(1S)$	379	408	523	564	1238
$\varepsilon_1(\omega_{\rm r})$	541	534	432	406	149
$\bar{\omega}_1(2S)$	643	692	741	789	1732
$\bar{\omega}_2(2S)$	1585	1590	4862	4865	4898
$\omega_{\rm r}(2S)$	457	482	643	679	1279
$\varepsilon_2(\omega_{\rm r})$	1164	1124	985	959	687

Table 2: The spin-averaged masses $M_{\rm cog}(1S)$ (in MeV)$^{a)}$.

Meson multiplet	$M_0(1S)$	$\Delta_{\rm SE}$	$M_{\rm cog}(1S)$	$M_{\rm cog}(\exp)^{b)}$
$D - D^*$	2139	-164	1975	1974.8(4)
$D_s - D_s^*$	2177	-105	2072	2076.0(6)
$B - B^*$	5433	-120	5313	5313.5(6)
$B_s - B_s^*$	5468	-72	5396	5400.7(32)
$B_c - B_c^*$	6332	-17	6315	$-$

$M(B_c^*)$ (predicted) $= 6329(7)$ MeV

Table 3: The masses (in MeV) of the ground states (for $T_g = 0.3$ fm).

	$D^{\pm}$	$D^{*\pm}$	D_s	D_s^*
this paper	1869.1	2010.3	1966.8	2107.1
Exper.	1869.3	2010.0	1968.2	2112.0
	± 0.4	± 0.4	± 0.4	± 0.6
	B	B^*	B_s	B_s^*
this paper	5279.3	5324.2	5362.0	5407.4
Exper.	5279.0	5325.0	5367.7	5411.7
	± 0.5	± 0.6	± 1.8	± 3.2

Table 4: The decay constants $f_{\rm P}$ (in MeV).

	D	D_s	B	B_s
Simonov	206	252	174	$-$
Svetic	230(25)	248(27)	196(29)	216(32)
Ebert	234	268	189	218
lattice quenched	235(22)	256(28)	196(38)	206(10)
lattice $n_f = 2+1$	201(20)	249(20)	216(35)	266(10)
this paper	210(10)	260(10)	182(8)	216(8)
experiment	222.6(20)$^{a)}$	280(23)	160^{+50}_{-80} $^{b)}$ 229(70)$^{c)}$	

$^{a)}$ CLEO; $^{b)}$ BaBar; $^{c)}$ Belle data

Table 5: The ratios f_{D_s}/f_D, f_{B_s}/f_B, and f_{D_s}/f_{B_s}.

	f_{D_s}/f_D	f_{B_s}/f_B	$f_{D_s}/f_{B_s}{}^{a)}$
RPM Ebert	1.15	1.15	1.23
BS Cvetic	1.08(1)	1.10(1)	1.15(1)
lattice (2+1) unquenched	1.24(8)	1.20(4)	1.01(8)
this work	1.24(4)	1.19(3)	1.20(4)
experiment	1.27(14)		

$$\mathbf{m_s}(\mu_1) \qquad \mu_1 \sim \sqrt{\langle \mathbf{p}^2 \rangle} \sim 0.5 - 0.6 GeV$$

$$R_D \approx R_{D_s} = 0.55(1) \text{ fm}, \quad R_B \approx R_{B_s} = 0.50(1) \text{ fm}, \tag{12}$$

It is of interest to notice that for HL mesons the ratios

$$\xi_D = \xi_{D_s} = \frac{|R_{1D}(0)|^2}{\omega_q \omega_c} = 0.345(3) \tag{13}$$

$$\xi_B = \xi_{B_s} = \frac{|R_{1B}(0)|^2}{\omega_q \omega_b} = 0.146(2). \tag{14}$$

It is important that the equalities $\xi_D = \xi_{D_s}$ and $\xi_{\rm B} = \xi_{B_s}$ practically do not depend on the details of the interaction in HL mesons. Therefore, in the ratios $\eta_D(\eta_B)$ the factors given in Eq. (13), $\xi_D(\xi_B)$ cancel and one obtains

$$\eta^2_{D(B)} = \left(\frac{m_s m_{c(b)}}{\langle Y_{\rm P} \rangle_{D(B)}} + \frac{\omega_s \omega_{c(b)} - \langle \mathbf{p}^2 \rangle_{D_s(B_s)}}{\langle Y_{\rm P} \rangle_{D(B)}} \right) \frac{M_{D(B)}}{M_{D_s(B_s)}}. \tag{15}$$

In Eq. (15) the second term is close to 1.1, while the first term, proportional to m_s, is not small, giving about 30-60% for different m_s.

With an accuracy of $\lesssim 2\%$

$$\begin{aligned} \eta_D^2 &= 2.708 \times m_s(\text{GeV}) + 1.077, \text{ if } m_d = m_u = 0, \\ \eta_{D^+}^2 &= 2.648 \times m_s(\text{GeV}) + 1.054, \text{ if } m_d = 8 \text{ MeV}, \end{aligned} \tag{16}$$

i.e. in the chiral limit

$$\eta_D = 1.14\ (m_s = 85\ \text{MeV}),\ 1.25\ (m_s = 180\ \text{MeV}),$$

$$1.27\ (m_s = 200\ \text{MeV}), \tag{17}$$

and for $m_d = 8$ MeV, $\eta_D = 1.13$, 1.24, and 1.26 for the same values of m_s, so decreasing only by $\sim 1\%$.

For the B and B_s mesons

$$\begin{aligned} \eta_B^2 &= 1.90 \times m_s + 1.075\ (m_d = m = 0); \\ \eta_{B^0}^2 &= 1.871 \times m_s + 1.076\ (m_d = 8\ \text{MeV}), \end{aligned} \tag{18}$$

which practically coincide, and in the chiral limit $(m_d = m_u = 0)$

$$\eta_{\text{B}} = 1.11\ (m_s = 85\ \text{MeV}), \quad 1.19\ (m_s = 180\ \text{MeV})$$

$$1.21 \; (m_s = 200 \text{ MeV}). \tag{19}$$

These values of η_B appear to be only $3-5\%$ smaller that η_D.

Thus for $m_s = 180$ MeV and $m_d = 8$ MeV we have obtained

$$\eta_{D^+} = 1.25(2), \quad \eta_B = 1.19(1), \tag{20}$$

in good agreement with experiment and lattice data see Eq. (1).

To check our choice of $m_s = 180$ MeV, we estimate the ratio $m_s(0.5$ GeV$)/m_s(2$ GeV) using the conventional perturbative (one-loop) formula for the running mass:

$$m(\mu^2) = m_0 \left(\frac{1}{2}\ln\frac{\mu^2}{\Lambda^2}\right)^{-d_m} \left[1 - d_1 \frac{\ln\ln\frac{\mu^2}{\Lambda^2}}{\ln\frac{\mu^2}{\Lambda^2}}\right]. \tag{21}$$

Here m_0 is an integration constant and the constants are

$$d_1 = \frac{8}{\beta_0^3}\left(51 - \frac{19}{3}n_f\right),\; \beta_0 = 11 - \frac{2}{3}n_f,\; d_m = \frac{4}{\beta_0}. \tag{22}$$

Then, from Eq. (21) $m_s(2$ GeV$) = 0.618\; m_0$, m_s(1 GeV) $= 0.7825\; m_0$, and $m_s(0.5\ \text{GeV}) = 1.217\ m_0$ and therefore

$$\frac{m_s(1\ \text{GeV})}{m_s(2\ \text{GeV})} = 1.27, \quad \frac{m_s(0.5\ \text{GeV})}{m_s(2\ \text{GeV})} = 1.97\ . \tag{23}$$

It means that m_s(0.5 GeV) $= 180$ MeV, which we used in our calculations, corresponds to m_s(2 GeV) $=$ 91 MeV which coincides with the conventional value of m_s(2 GeV) $=$ 90± 15 MeV. Our estimate of $m_s(0.5$ GeV) $=$ 180 MeV supports our choice of this value in the relativistic string Hamiltonian, which provides a good description of the HL meson spectra and decay constants, and gives rise to the relatively large values of η_D and η_B .