

# Some puzzles of B-decays.

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(based on papers with M.I.Vysotski)

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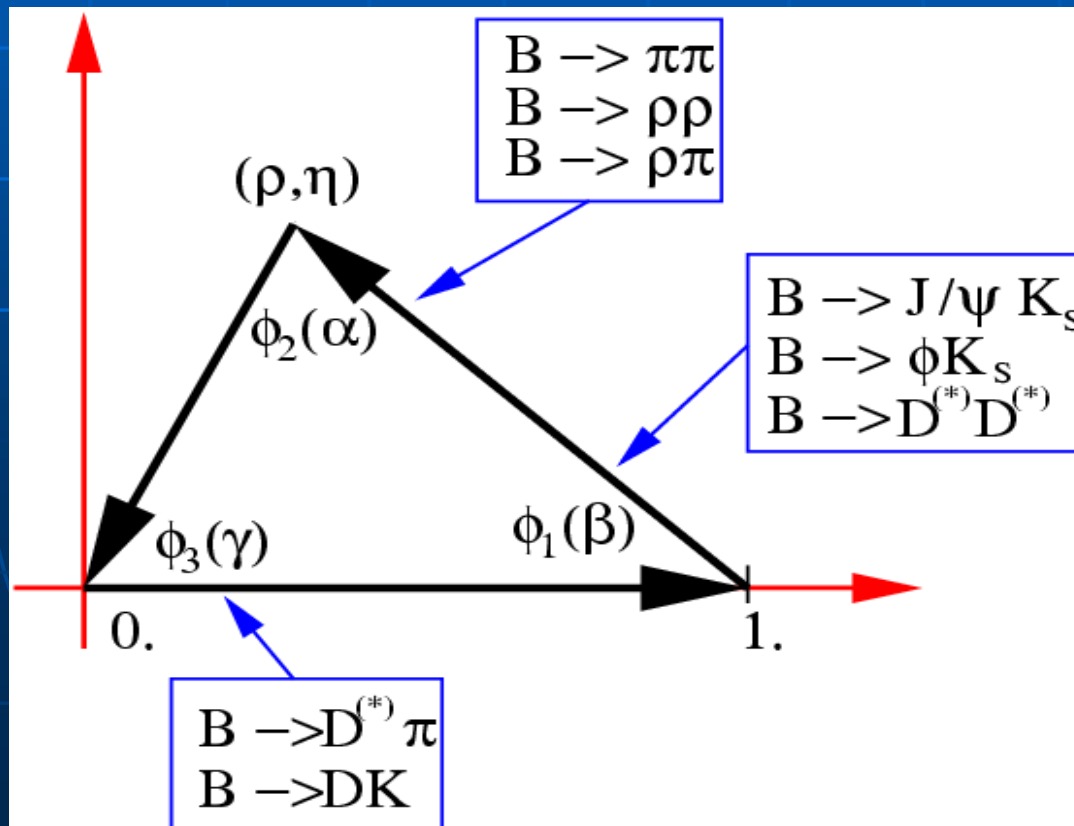
# Introduction.

Importance of theoretical understanding of dynamics in hadronic B-decays.

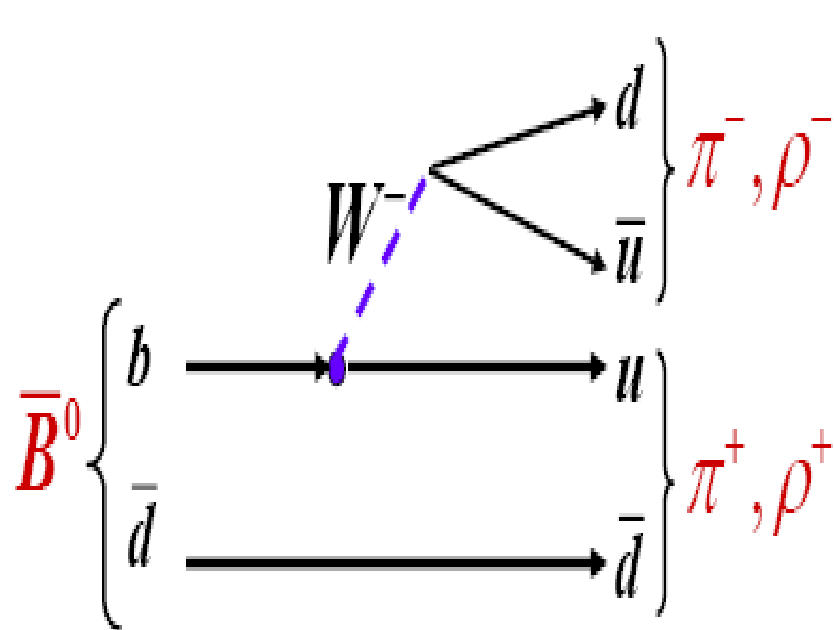
- a) For extraction of CKM parameters from B-decays.
- b) A good laboratory to study large distance aspects of QCD.

# Unitarity triangle

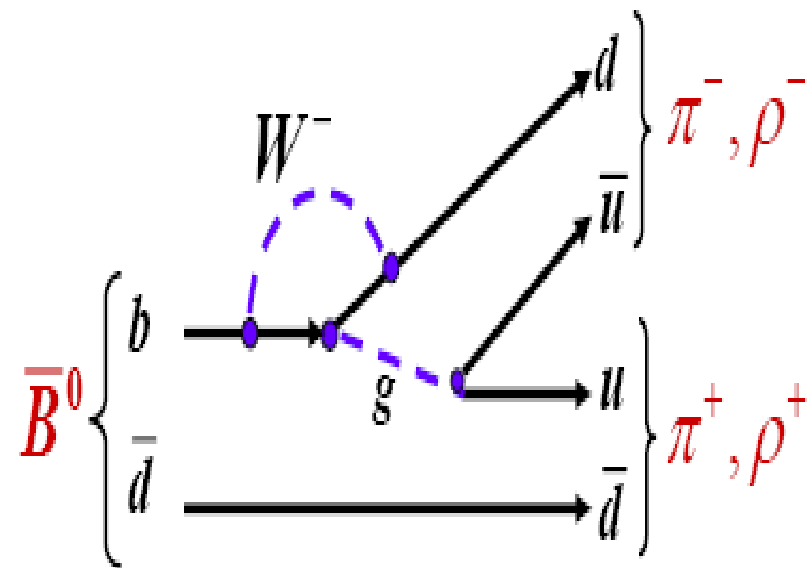
$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$



# The diagrams for B decays



Tree



Penguin

# The $\rho\rho - \pi\pi$ puzzle.

A large difference in  $+-/00$  branching ratios for  $\pi\pi$  and  $\rho\rho$  -decays.

Table 1

Mode	Br( $10^{-6}$ )	Mode	Br( $10^{-6}$ )
$B_d \rightarrow \pi^+\pi^-$	$5.2 \pm 0.2$	$B_d \rightarrow \rho^+\rho^-$	$23.1 \pm 3.3$
$B_d \rightarrow \pi^0\pi^0$	$1.3 \pm 0.2$	$B_d \rightarrow \rho^0\rho^0$	$1.16 \pm 0.46$
$B_u \rightarrow \pi^+\pi^0$	$5.7 \pm 0.4$	$B_u \rightarrow \rho^+\rho^0$	$18.2 \pm 3.0$

$C$ -averaged branching ratios of  $B \rightarrow \pi\pi$  and  $B \rightarrow \rho\rho$  decays.

$$R_\rho \equiv \frac{\text{Br}(B_d \rightarrow \rho^+\rho^-)}{\text{Br}(B_d \rightarrow \rho^0\rho^0)} \approx 20, \quad R_\pi \equiv \frac{\text{Br}(B_d \rightarrow \pi^+\pi^-)}{\text{Br}(B_d \rightarrow \pi^0\pi^0)} \approx 4$$

# Matrix elements for $B \rightarrow \pi\pi$ ( $\rho\rho$ ) decays.

$$M_{\bar{B}_d \rightarrow \pi^+\pi^-} = \frac{G_F}{\sqrt{2}} |V_{ub}V_{ud}^*| m_B^2 f_\pi f_+(0) \left\{ e^{-i\gamma} \frac{1}{2\sqrt{3}} A_2 e^{i\delta_2^\pi} + e^{-i\gamma} \frac{1}{\sqrt{6}} A_0 e^{i\delta_0^\pi} + \left| \frac{V_{td}^* V_{tb}}{V_{ub} V_{ud}^*} \right| e^{i\beta} P e^{i(\delta_P^\pi + \bar{\delta}_0^\pi)} \right\},$$

$$M_{\bar{B}_d \rightarrow \pi^0\pi^0} = \frac{G_F}{\sqrt{2}} |V_{ub}V_{ud}^*| m_B^2 f_\pi f_+(0) \left\{ e^{-i\gamma} \frac{1}{\sqrt{3}} A_2 e^{i\delta_2^\pi} - e^{-i\gamma} \frac{1}{\sqrt{6}} A_0 e^{i\delta_0^\pi} - \left| \frac{V_{td}^* V_{tb}}{V_{ub} V_{ud}^*} \right| e^{i\beta} P e^{i(\delta_P^\pi + \bar{\delta}_0^\pi)} \right\},$$

$$M_{\bar{B}_u \rightarrow \pi^-\pi^0} = \frac{G_F}{\sqrt{2}} |V_{ub}V_{ud}^*| m_B^2 f_\pi f_+(0) \left\{ \frac{\sqrt{3}}{2\sqrt{2}} e^{-i\gamma} A_2 e^{i\delta_2^\pi} \right\},$$

# Structure of the matrix elements

$$\delta_0^\pi, \delta_2^\pi \quad \delta_P^\pi + \delta_0^\pi$$

are the phases due to final state interactions for tree ( $I=0,2$ ) and penguin diagrams correspondingly.

$\beta$  and  $\gamma$  are CKM phases.

P-contribution to these decays is rather small ( $P/T \sim 0.1$ ), but it is important for CP-violation.



# Determination of penguin contribution.

The values of P can be determined, using SU(3)-symmetry, from  $B_u \rightarrow K^0 \pi^+$ ,  $K^{0*} \rho^+$ -decays, where penguin diagrams give dominant contributions. **M.Gronau, J.Rosner**

$$\text{Br}(B_d \rightarrow \rho^+ \rho^-)_P = \left( \frac{f_\rho}{f_{K^*}} \right)^2 \left[ \lambda \sqrt{\eta^2 + (1 - \rho)^2} \right]^2 \frac{\tau_{B_d}}{\tau_{B_u}} \text{Br}(K^{0*} \rho^+) \approx 0.34 \cdot 10^{-6},$$

$$\text{Br}(B_d \rightarrow \pi^+ \pi^-)_P = \left( \frac{f_\pi}{f_K} \right)^2 \left[ \lambda \sqrt{\eta^2 + (1 - \rho)^2} \right]^2 \frac{\tau_{B_d}}{\tau_{B_u}} \text{Br}(K^0 \pi^+) \approx 0.59 \cdot 10^{-6}$$

# Isospin analysis

Neglecting by P-term

$$\cos(\delta_0^\pi - \delta_2^\pi) = \frac{\sqrt{3}}{4} \frac{B_{+-} - 2B_{00} + \frac{2}{3} \frac{\tau_0}{\tau_+} B_{+0}}{\sqrt{\frac{\tau_0}{\tau_+} B_{+0}} \sqrt{B_{+-} + B_{00} - \frac{2}{3} \frac{\tau_0}{\tau_+} B_{+0}}},$$

From data on  $B \rightarrow \pi\pi$  decays we get

$$|\delta_0^\pi - \delta_2^\pi| = 48^\circ.$$

With account of P-term

$$|\delta_0^\pi - \delta_2^\pi| = 37^\circ \pm 10^\circ.$$

- Large deviation from factorization in phases.

For  $B \rightarrow \rho\rho$  decays FSI-phases are smaller:

$$|\delta_0^\rho - \delta_2^\rho| = 20^{+8^\circ}_{-20^\circ} ,$$

Below the model will be presented, which explains the pattern of FSI-phases in  $B \rightarrow \pi\pi$  ,  $B \rightarrow \rho\rho$  decays.

## How to calculate FSI?

For single channel case from unitarity follows Migdal-Watson theorem:

the phase of the matrix element for the decay  $X \rightarrow ab$  is equal to the phase of the elastic scattering amplitude  $\delta_{ab}$ .

Generalization to isospin.

$$\delta_1(K \rightarrow \pi\pi) = \delta_1(\pi\pi)$$

$$\text{At } M_K \quad \delta_0(\pi\pi) = (35 \pm 3)^\circ, \quad \delta_2(\pi\pi) = (-7 \pm 2)^\circ$$

## Application to heavy (D,B)-mesons.

For heavy mesons there are many open channels and application of unitarity is not straightforward.

Different ideas about FSI for heavy quark decays.

a) Effects of FSI should decrease with the mass of heavy quark  $M_Q$ .

Arguments (J.D.Bjorken).

b) FSI do not decrease with  $M_Q$ .  
(at least for two-body final states)

# Classification of FSI in $1/N$ -expansion.

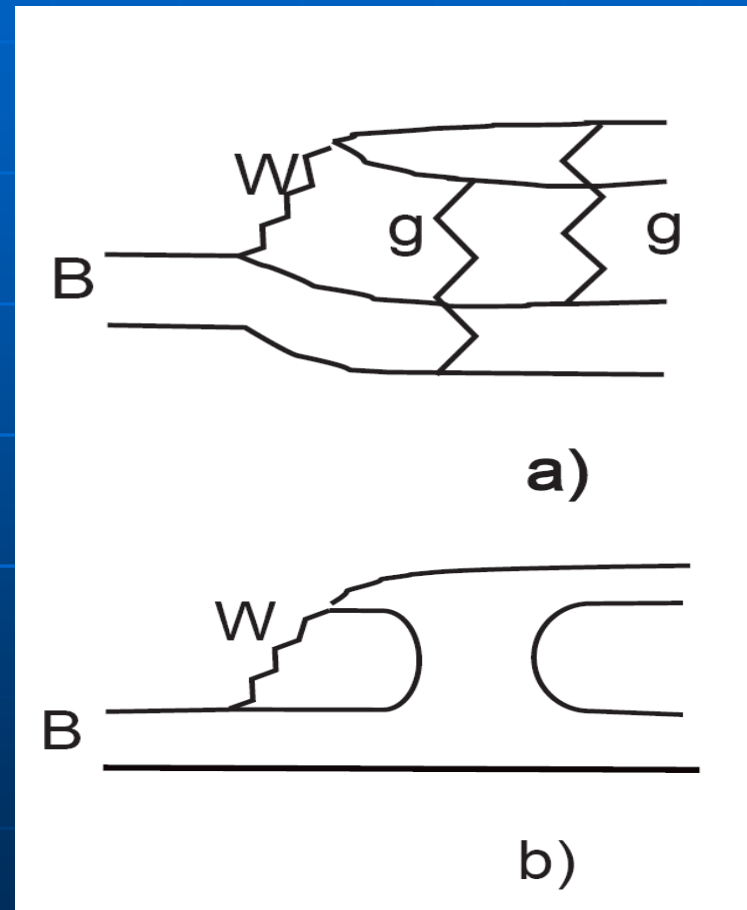
In  $1/N$ -expansion the following diagrams with FSI are possible:

The diagram a)  $\sim 1/N^2$  and does not decrease with  $MQ$  (pomeron), while the diagram b)  $\sim 1/N$  and decreases as  $1/MQ$  (reggeon).

A.Kaidalov(1989)

Similar conclusions:

J.P.Donoghue et al.(1996)



# Experimental results on FSI phases in D, B-decays

Data on  $D \rightarrow \pi\pi$  branching ratios lead to:

$$|\delta_2 - \delta_0| = (86^\circ \pm 4^\circ)$$

In B-decays:

From  $B \rightarrow D\pi$  decays FSI difference between  
 $I=1/2$  and  $I=3/2$  amplitudes  $\delta_{D\pi} = 30^\circ \pm 7^\circ$

From analysis of  $B \rightarrow \pi\pi$  decays:

$$|\delta_2 - \delta_0| = (37^\circ \pm 10^\circ)$$

Large FSI phases!

However small phases in  $B \rightarrow \rho\rho$   
( $\pi\pi, \rho\rho$ -puzzle).

# Unitarity based approaches.

Formal solution of unitarity:

$$M_{X \rightarrow ab} = \sum_m M_0(X \rightarrow m) S^{1/2}_{m,ab}$$

Difficult to apply for realistic multi-channel S-matrix.

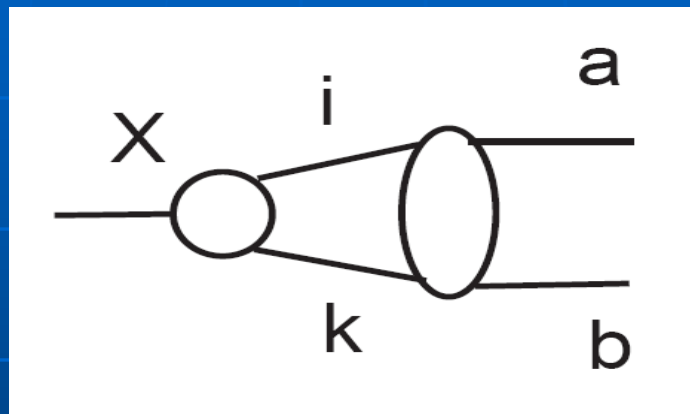
Some recent work along these lines:

A. Deandrea et al.,  
L. Wolfenstein, F. Wu



# Method of calculations.

We use Feynman diagrams approach, which is often applied to high-energy hadronic interactions.



Amplitudes for the transitions  $ab \rightarrow ik$  with large masses of the states  $i, k$  should be strongly suppressed (as powers of  $1/M^2 i(k)$  ).

It is possible to prove that  $M^2 i(k) \sim \Lambda MB$ .

The states with  $M_i \sim 1 \text{ GeV}$  are taken into account.

## Method of calculations (cont).

Transforming  $\int d^4k \rightarrow \int d^2k_t dM_i^2 dM_k^2$

We obtain

$$M_I(B \rightarrow ab) = \sum M_I^0(B \rightarrow ik) (\delta_{ia} \delta_{kb} + i T_I(ik \rightarrow ab))$$

$T_I(ik \rightarrow ab)$  is  $J=0$  projection of the corresponding scattering amplitude.

Note that for real  $T_I(ik \rightarrow ab)$  this formula gives the same result as unitarity condition. However at high energies amplitudes have substantial imaginary parts.

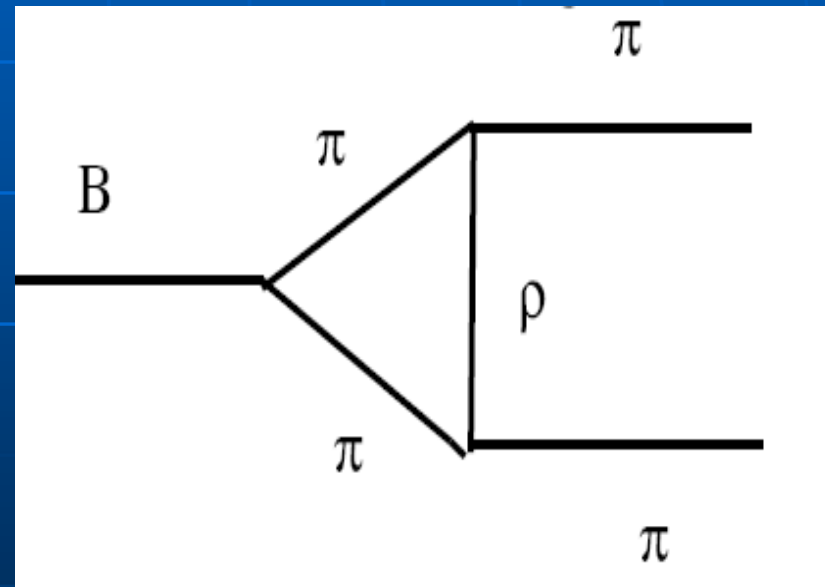
## Method of calculations (cont).

There are many papers on this subject, which use two-body intermediate states for calculations of effects due to FSI.

For example: H-Y. Cheng, C-K. Chua and A.Soni

The diagrams of the following type are used:

Triangle diagram



## Reggeization of t-channel exchanges.

For exchange by an elementary vector meson in the t-channel the partial wave amplitudes do not decrease as energy increases.

However it is well known from phenomenology of high-energy binary reactions that  $\rho$ -exchange should be reggeized.

In this case its contribution to the FSI decreases

$$\text{as } \exp(-(1-\alpha_\rho(0))\ln(s)) \sim 1/s^{1/2} \sim 1/MQ$$

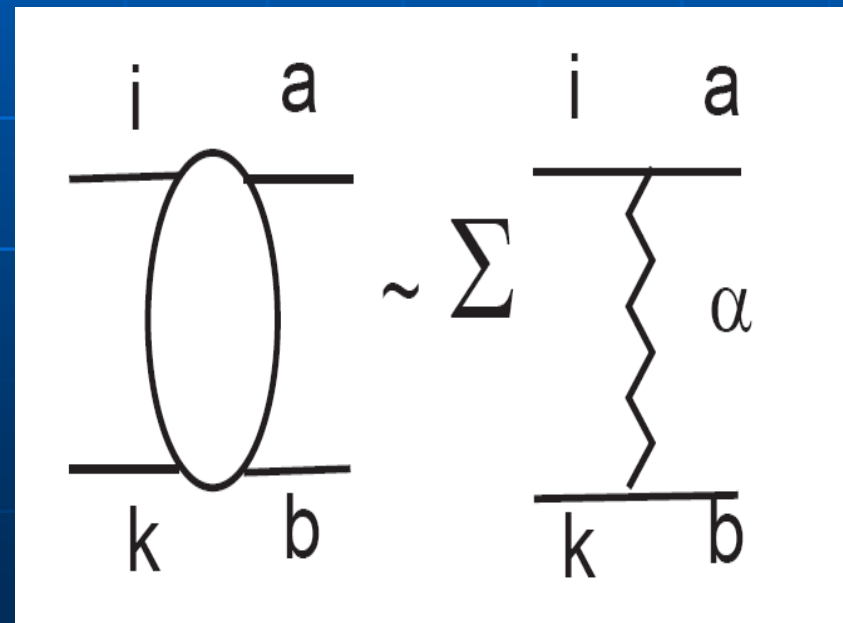
$$\text{for } \alpha_\rho(0) = 0.5$$

# Reggeization of t-channel exchanges.

Situation is even more drastically changed for  $D^*$ -trajectory with  $\alpha_{D^*}(0) \approx -0.8$ .

We approximate high-energy scattering amplitudes by exchanges of Regge poles.

Amplitudes at high energies in Regge model.



## Applications to $B \rightarrow \pi\pi$ and $B \rightarrow \rho\rho$ decays.

For  $B \rightarrow \pi\pi$  and  $B \rightarrow \rho\rho$  decays the  $\pi\pi$ ,  $\rho\rho$  and  $\pi A_1$  intermediate states were used.

In the amplitudes of  $\pi\pi \rightarrow \pi\pi$   $P$ ,  $f$  and  $\rho$  - exchanges have been taken into account.

In the amplitudes of  $\pi\pi \rightarrow \rho\rho$   $\pi$ -exchange gives the main contribution to the longitudinally polarized rho.

In the amplitudes of  $\pi\pi \rightarrow \pi A_1$   $\rho$  -exchange contributes.

# Applications to $B \rightarrow \pi\pi$ and $B \rightarrow \rho\rho$ decays.

The pion exchange in contribution of  $\rho\rho$  intermediate state (neglected by other authors  $\sim 1/M^2q$ ) plays an important role in the resolution of  $\pi\pi$ - $\rho\rho$  puzzle.

The pomeron and f-exchanges do not contribute to the phase difference of amplitudes with  $I=0$  and  $I=2$  and it decreases as  $\sim 1/Mq$  for  $Mq \rightarrow \infty$ .

Vertices of reggeons with pions were taken from analysis of  $\pi N$ ,  $NN$ -scattering and Regge factorization.

## Results.

Branching  $B \rightarrow \rho^+\rho^- \approx 5$  times larger than the one for  $B \rightarrow \pi^+\pi^-$  and  $\rho\rho$  – intermediate state is very important in  $B \rightarrow \pi\pi$  decays, while  $\pi\pi$  – intermediate state plays a minor role in  $B \rightarrow \rho\rho$  decays.

Final result is:

$$B \rightarrow \pi\pi: \delta_0 = 30^\circ \quad ; \quad \delta_0 - \delta_2 = 40^\circ (\pm 15^\circ)$$
$$\delta_2 = -10^\circ$$

$$B \rightarrow \rho\rho: \delta_0 = 11^\circ \quad ; \quad \delta_0 - \delta_2 = 15^\circ (\pm 5^\circ)$$
$$\delta_2 = -4^\circ$$



# CP-violation in B-decays

Time-dependent CP asymmetry

$$A_{CP}(\Delta t) = S \sin(\Delta m \Delta t) - C \cos(\Delta m \Delta t)$$



Mixing induced CPV



Direct CPV

$$C_{\pi\pi} \equiv \frac{1 - |\lambda_{\pi\pi}|^2}{1 + |\lambda_{\pi\pi}|^2}, \quad S_{\pi\pi} \equiv \frac{2\text{Im}(\lambda_{\pi\pi})}{1 + |\lambda_{\pi\pi}|^2}, \quad \lambda_{\pi\pi} \equiv e^{-2i\beta} \frac{M_{\bar{B} \rightarrow \pi\pi}}{M_{B \rightarrow \pi\pi}},$$

# Direct CPV in $B \rightarrow \pi\pi$

CPV-parameter  $C$  is sensitive to a magnitude of penguin contribution and phases.

$$C_{+-} = -\frac{\tilde{P}}{\sqrt{3}} \sin \alpha [\sqrt{2} A_0 \sin(\delta_0 - \tilde{\delta}_0 - \delta_P) + A_2 \sin(\delta_2 - \tilde{\delta}_0 - \delta_P)] /$$
$$/ \left[ \frac{A_0^2}{6} + \frac{A_2^2}{12} + \frac{A_0 A_2}{3\sqrt{2}} \cos(\delta_0 - \delta_2) - \sqrt{\frac{2}{3}} A_0 \tilde{P} \cos \alpha \cos(\delta_0 - \tilde{\delta}_0 - \delta_P) - \right.$$
$$\left. - \frac{A_2 \tilde{P}}{\sqrt{3}} \cos \alpha \cos(\delta_2 - \tilde{\delta}_0 - \delta_P) + \tilde{P}^2 \right], \quad \tilde{P} \equiv \left| \frac{V_{td}^* V_{tb}}{V_{ub} V_{ud}^*} \right| P .$$

## Direct CPV in $B \rightarrow \pi\pi$

The ratios of amplitudes was determined above:

$$A_0/A_2 = 0.8 \pm 0.09, \quad P/A_2 = 0.092 \pm 0.02$$

Phases  $\delta_0$  and  $\delta_2$  were calculated.

What about phases of penguin contribution?

In PQCD the phase  $\delta_P$  is  $\sim 10^\circ$  and positive.

The sign of the phase for contribution of  $D\bar{D}$  intermediate state in Regge model depends on the intercept of  $D^*$ -trajectory.

$$M \sim \exp(-i\pi\alpha(0))$$

For linear  $D^*$ -trajectory with

$$\alpha' = 0.5 \text{ GeV}^{-2}, \quad \alpha_{D^*}(0) = -0.8$$

and  $\bar{\delta}_P$  is negative:  $\bar{\delta}_P \sim -10^\circ$

In leading log PQCD calculation

$\alpha_{D^*}(0) \geq 0$  and  $\bar{\delta}_P$  is positive.

Thus the sign of  $\bar{\delta}_P$  gives an important information on dynamics of high-energy interactions.

If  $\delta_P$  is positive it is possible to obtain the lower bound for  $C_{+-}$ :  $C_{+-} > -0.18$

Belle and BABAR give different results for this quantity:  $C_{+-}(\text{Belle}) = -0.55(0.09)$ ,  
 $C_{+-}(\text{BaBar}) = -0.21(0.09)$

Using  $d \leftrightarrow s$  symmetry and  $C$  from  $B \rightarrow K\pi$  decay one obtains

$$\begin{aligned}
 C_{+-} &= \left(\frac{f_\pi}{f_K}\right)^2 A_{CP}(K^+\pi^-) \frac{\Gamma(B \rightarrow K^+\pi^-) \sin(\beta + \gamma)}{\Gamma(B \rightarrow \pi^+\pi^-) \sin(\gamma)} \left| \frac{V_{td}}{V_{ts}\lambda} \right| = \\
 &= 1.2^{(-2)} (-0.093 \pm 0.015) \frac{19.8 \sin 82^\circ}{5.2 \sin 60^\circ} 0.87 = -0.24 \pm 0.04,
 \end{aligned}$$

If negative  $\delta_P$  is allowed, then it is possible to obtain  $C_{+-}$  closer to Belle result.

For  $B \rightarrow \pi^0 \pi^0$  decay we have

$$C_{00} \approx -1.06 [0.8 \sin(\delta_0 - \tilde{\delta}_0 - \delta_P) - 1.4 \sin(\delta_2 - \tilde{\delta}_0 - \delta_P)] = -0.50 \pm 0.08$$

Very large  $|C_{00}|$  is predicted. It is not sensitive to  $\delta_P$ . Present experimental error is too big:  $C_{00} = -0.36 \pm 0.32$

# Polarization of vector mesons in B decays.

Tree diagrams with light quarks lead to longitudinal polarizations of vector mesons in the final states. This prediction agrees with experiment for  $\rho\rho$  channel, but is violated for  $K^*\rho$  and  $\phi K^*$  channels.

$$f_L = \Gamma_L / \Gamma$$

$B \rightarrow K^{*0} \rho_+$	$0.48 \pm 0.08$
$B \rightarrow K^{*0} \rho^0$	$0.57 \pm 0.12$
$B \rightarrow \phi K^{*+}$	$0.50 \pm 0.07$
$B \rightarrow \phi K^{*0}$	$0.491 \pm 0.032$

# Polarization of vector mesons

Note that for all these channels there are no tree diagrams.

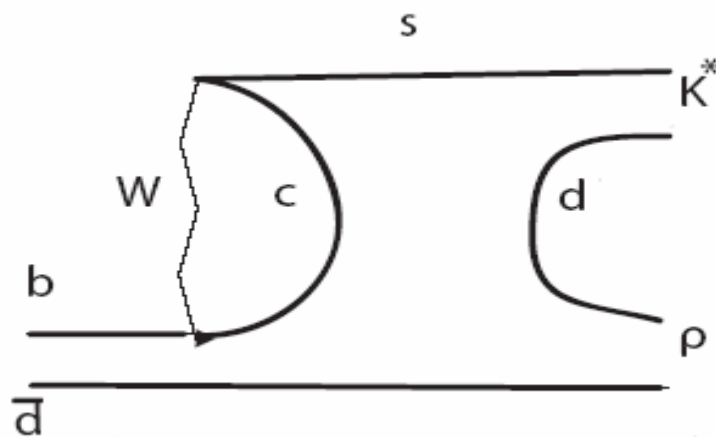
However the rule is very general for hard processes (helicity conservation for light quarks). Corrections  $\sim m_V^2/m_b^2$

Soft FSI can lead to deviations from this rule, because in a rescattering process  $ik \rightarrow ab$  large spin flip amplitudes are possible. This is especially important when the intermediate particles  $(i,k)$  are pseudoscalar mesons and  $a,b$  are vector mesons: for leading Regge-exchanges with natural spin-parity only transverse polarization of vector mesons is allowed.



# FSI and polarization

For  $K^*\rho$  and  $\phi K^*$  final state there is a large contribution of  $DD_s$  intermediate state and large transverse polarization is generated (for detailed estimates see [M.Ladisa et. al.](#)).



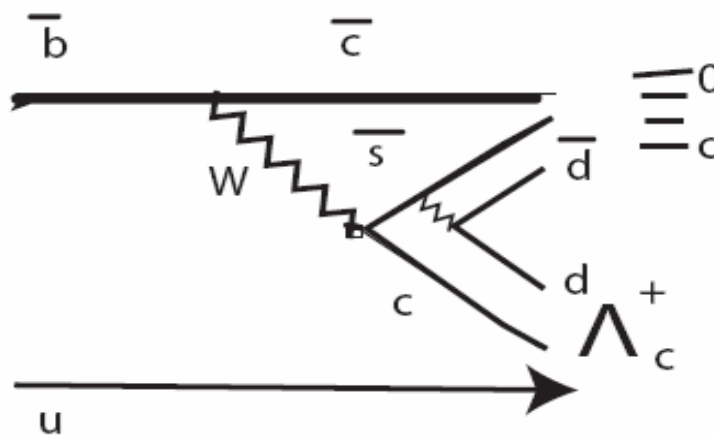
# Charm-anticharm baryons production.

Large probability of  $\Lambda_c \Xi_c$  decays has been observed recently.

$\text{Br}(\Lambda_c \Xi_c) \sim 10^{-3}$ , while

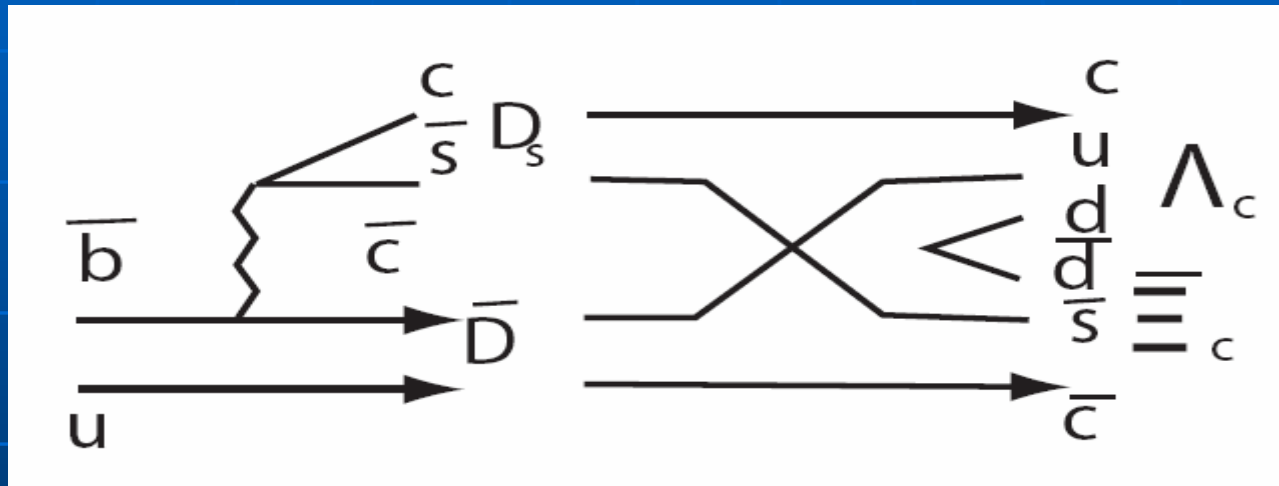
$\text{Br}(\Lambda_c \underline{p}) = (2.19 \pm 0.8) 10^{-5}$

This is strange from PQCD point of view



# FSI in $\Lambda_c \Xi_c$ decays

$\underline{D}D_s$  ( $D^*D_s, \underline{D}D_s^*, \dots$ ) intermediate states can play an important role



Two heavy mesons  $\underline{D}$  and  $D_s$  have rather small momenta ( $p=1.8$  GeV) and light quarks are slow in the B rest frame.

## FSI in $\Lambda_c \Xi_c$ and $\Lambda_c \underline{p}$ decays

Thus all quarks have large projections to wave functions of final baryons.

For  $\Lambda_c \underline{p}$  case the light (u,d) quarks in  $\pi, \rho$ -mesons have large momenta.

The resulting suppression can be estimated in the Regge model (nucleon trajectory in the t-channel) and is  $\sim 10^{-2}$  compared to  $\Lambda_c \Xi_c$ .

# Conclusions.

- FSI play an important role in two-body hadronic decays of heavy mesons.
- Theoretical estimates with account of the lowest intermediate states give a satisfactory agreement with experiment and provide an explanation of some puzzles observed in B decays.