On the origin of families of quarks and leptons and their mass matrices with the approach unifying spins and charges Prediction for the observable fourth family?

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Collaborators in this project, which has started almost 15 years ago: A. Borštnik Bračič, G. Bregar, M. Breskvar, D. Lukman, H.B. Nielsen, Jože Vrabec, others

- *Phys. Lett.* **B 292**, 25-29 (1992),
- *Modern Phys. Lett.* **A 10**, 587-595 (1995),
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  Phys. Rev. D **62** (04010-14) (2000),
- *Phys. Lett.* **B 633** 771-775 (2006), hep-th/0311037, hep-th/0509101, with H.B.N.
- hep-ph/0401043, hep-ph/0401055, hep-ph/0301029,
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  hep-ph/0606159, with M.B., D.L..

We are trying to answer the open questions of the Standard model connected with the appearance of families and the Yukawa couplings within the **Approach unifying spins and charges**:

1. Why do only the left handed spinors carry the weak charge, while the right handed are weak chargeless?
2. Where do families of quarks and leptons come from?
3. What does determine the strength of the Yukawa couplings and accordingly the weak scale?

Approach unifying spins and charges offers the answers to these questions:

1. The representation of one Weyl spinor of the group SO(1,13), manifests the left handed weak charged quarks and leptons and the right handed weak chargeless quarks and leptons.
2. It is a part of the simple starting Lagrange density which plays the role of the Higgs field of the Standard model. There are two kinds of the Clifford algebra objects One kind takes care of the spin and the charges. The other kind generates families.

Two kinds of the Clifford algebra objects: γ^a, $\tilde{\gamma}^a$

$$\{\gamma^a, \gamma^b\}_+ = 2\eta^{ab} = \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+, \quad \{\gamma^a, \tilde{\gamma}^b\}_+ = 0,$$
$$\tilde{\gamma}^a B := i(-)^{n_B} B \gamma^a,$$

$$S^{ab} := (i/4)(\gamma^a\gamma^b - \gamma^b\gamma^a),$$
$$\tilde{S}^{ab} := (i/4)(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a),$$
$$\{S^{ab}, \tilde{S}^{cd}\}_- = 0.$$

Action in $d = (1+13)$

$$
\begin{aligned}
S &= \int d^d x \, \mathcal{L} \\
\mathcal{L} &= \frac{1}{2}(E\bar{\psi}\gamma^a p_{0a}\psi) + h.c. \\
p_{0a} &= f^{\alpha}{}_a p_{0\alpha}, \quad p_{0\alpha} = p_\alpha - \frac{1}{2}S^{ab}\omega_{ab\alpha} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{ab\alpha}
\end{aligned}
$$

$$
\mathcal{L} = \bar{\psi}\gamma^m (p_m - \sum_{A,i} g^A \tau^{Ai} A^{Ai}_m)\psi + \sum_{s=7,8} \bar{\psi}\gamma^s p_{0s}\,\psi + \text{the rest}
$$

$$
\tau^{Ai} = \sum_{a,b} c^{Ai}{}_{ab}\, S^{ab},
$$

$$
\{\tau^{Ai}, \tau^{Bj}\}_- = i\delta^{AB} f^{Aijk}\tau^{Ak}
$$

$$
\begin{array}{lllll}
A = 1 & U(1) \text{ hyper charge} & i = \{1\} & \text{usual not. } Y, \\
A = 2 & SU(2) \text{ weak charge} & i = \{1,2,3\} & \text{usual not. } \tau^{i}, \\
A = 3 & SU(3) \text{ colour charge} & i = \{1,\cdots,8\} & \text{usual not. } \lambda^{i}/2,
\end{array}
$$

Yukawa couplings

$$
\begin{aligned}
-\mathcal{L}_Y &= \psi^\dagger \gamma^0 \gamma^s p_{0s} \psi \\
&= \psi^\dagger\, \gamma^0 \{ \stackrel{78}{(+)}\; p_{0+} + \stackrel{78}{(-)}\; p_{0-} \} \psi,
\end{aligned}
$$

$$
\begin{aligned}
p_{0\pm} &= (p_7 \mp i\, p_8) - \\
&\quad \frac{1}{2} S^{ab} \omega_{ab\pm} - \frac{1}{2} \tilde{S}^{ab} \tilde{\omega}_{ab\pm}, \\
\omega_{ab\pm} &= \omega_{ab7} \mp i\, \omega_{ab8}, \\
\tilde{\omega}_{ab\pm} &= \tilde{\omega}_{ab7} \mp i\, \tilde{\omega}_{ab8}
\end{aligned}
$$

We put $p_7 = p_8 = 0$.

Our technique to represent spinors and work elegantly with them

- *J. of Math. Phys.* **43**, 5782-5803 (2002), hep-th/0111257,
- *J. of Math. Phys.* **44** 4817-4827 (2003), hep-th/0303224, both with H.B. Nielsen.

$$\begin{aligned}
\overset{ab}{(\pm i)}: &= \frac{1}{2}(\gamma^a \mp \gamma^b),\; \overset{ab}{[\pm i]}:= \frac{1}{2}(1 \pm \gamma^a\gamma^b) \\
&\text{for } \eta^{aa}\eta^{bb} = -1, \\
\overset{ab}{(\pm)}: &= \frac{1}{2}(\gamma^a \pm i\gamma^b),\; \overset{ab}{[\pm]}:= \frac{1}{2}(1 \pm i\gamma^a\gamma^b), \\
&\text{for } \eta^{aa}\eta^{bb} = 1.
\end{aligned}$$

$$S^{ab} \overset{ab}{(k)} = \frac{k}{2} \overset{ab}{(k)}, \quad S^{ab} \overset{ab}{[k]} = \frac{k}{2} \overset{ab}{[k]},$$
$$\tilde{S}^{ab} \overset{ab}{(k)} = \frac{k}{2} \overset{ab}{(k)}, \quad \tilde{S}^{ab} \overset{ab}{[k]} = -\frac{k}{2} \overset{ab}{[k]}\,.$$

$$\gamma^a \overset{ab}{(k)} = \eta^{aa} \overset{ab}{[-k]}, \quad \gamma^b \overset{ab}{(k)} = -ik \overset{ab}{[-k]},$$
$$\gamma^a \overset{ab}{[k]} = \overset{ab}{(-k)}, \quad \gamma^b \overset{ab}{[k]} = -ik\eta^{aa} \overset{ab}{(-k)}$$

$$\tilde{\gamma}^a \overset{ab}{(k)} = -i\eta^{aa} \overset{ab}{[k]}, \quad \tilde{\gamma}^b \overset{ab}{(k)} = -k \overset{ab}{[k]},$$
$$\tilde{\gamma}^a \overset{ab}{[k]} = i \overset{ab}{(k)}, \quad \tilde{\gamma}^b \overset{ab}{[k]} = -k\eta^{aa} \overset{ab}{(k)}\,.$$

γ^a transform $\overset{ab}{(k)}$ into $\overset{ab}{[-k]}$, $\tilde{\gamma}^a$ transform $\overset{ab}{(k)}$ into $\overset{ab}{[k]}$.

$$
\begin{aligned}
\stackrel{ab}{(k)}\stackrel{ab}{(k)} &= 0, \; \stackrel{ab}{(k)}\stackrel{ab}{(-k)} = \eta^{aa} \stackrel{ab}{[k]}, \; \stackrel{ab}{[k]}\stackrel{ab}{[k]} = \stackrel{ab}{[k]}, \\
\stackrel{ab}{[k]}\stackrel{ab}{[-k]} &= 0, \stackrel{ab}{(k)}\stackrel{ab}{[k]} = 0, \quad \stackrel{ab}{[k]}\stackrel{ab}{(k)} = \stackrel{ab}{(k)}, \\
\stackrel{ab}{(k)}\stackrel{ab}{[-k]} &= \stackrel{ab}{(k)}, \quad \stackrel{ab}{[k]}\stackrel{ab}{(-k)} = 0.
\end{aligned}
$$

$$
\begin{aligned}
\stackrel{ab}{(\tilde{k})}\stackrel{ab}{(k)} &= 0, \quad \stackrel{ab}{(-\tilde{k})}\stackrel{ab}{(k)} = -i\eta^{aa} \stackrel{ab}{[k]}, \\
\stackrel{ab}{(\tilde{k})}\stackrel{ab}{[k]} &= i \stackrel{ab}{(k)}, \quad \stackrel{ab}{(\tilde{k})}\stackrel{ab}{[-k]} = 0.
\end{aligned}
$$

$$
\stackrel{ab}{(\tilde{\pm} i)} = \frac{1}{2}(\tilde{\gamma}^a \mp \tilde{\gamma}^b), \quad \stackrel{ab}{(\tilde{\pm} 1)} = \frac{1}{2}(\tilde{\gamma}^a \pm i\tilde{\gamma}^b),
$$

Cartan

$$S^{03}, S^{12}, S^{56}, S^{78}, S^{9\,10}, S^{11\,12}, S^{13\,14}.$$

An eigen state of Cartan ($\Gamma^{(1,13)} = -1$)

$$\stackrel{03}{(+i)}\stackrel{12}{(+)} \mid \stackrel{56}{(+)}\stackrel{78}{(+)} \mid\mid \stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)} |\psi\rangle =$$
$$\frac{1}{2^7}(\gamma^0 - \gamma^3)(\gamma^1 + i\gamma^2)|(\gamma^5 + i\gamma^6)(\gamma^7 + i\gamma^8)||$$
$$(\gamma^9 + i\gamma^{10})(\gamma^{11} - i\gamma^{12})(\gamma^{13} - i\gamma^{14})|\psi\rangle.$$

The eightplet of quarks of a particular color charge ($\tau^{33} = 1/2$, $\tau^{38} = 1/(2\sqrt{3})$, and $\tau^{41} = 1/6$)

i		$\vert^{a}\psi_i >$	$\Gamma^{(1,3)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{21}	Y	Y'
		Octet, $\Gamma^{(1,7)} = 1$, $\Gamma^{(6)} = -1$, of quarks							
1	u_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \mid \stackrel{56}{(+)}\stackrel{78}{(+)} \mid\mid \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{2}{3}$	$-\frac{1}{3}$
2	u_R^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{[-]} \mid \stackrel{56}{(+)}\stackrel{78}{(+)} \mid\mid \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{2}{3}$	$-\frac{1}{3}$
3	d_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \mid \stackrel{56}{[-]}\stackrel{78}{[-]} \mid\mid \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{2}{3}$
4	d_R^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{[-]} \mid \stackrel{56}{[-]}\stackrel{78}{[-]} \mid\mid \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{2}{3}$
5	d_L^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{(+)} \mid \stackrel{56}{[-]}\stackrel{78}{(+)} \mid\mid \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
6	d_L^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{[-]} \mid \stackrel{56}{[-]}\stackrel{78}{(+)} \mid\mid \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
7	u_L^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{(+)} \mid \stackrel{56}{(+)}\stackrel{78}{[-]} \mid\mid \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$
8	u_L^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{[-]} \mid \stackrel{56}{(+)}\stackrel{78}{[-]} \mid\mid \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{6}$

$\gamma^0 \stackrel{78}{(-)}$ transforms u_R of the 1^{st} row into u_L of the 7^{th} row, while $\gamma^0 \stackrel{78}{(+)}$ transforms d_R of the 3^{rd} row into d_L of the 5^{th} row, doing what the Higgs and γ^0 do in the Standard model.

$\tilde{S}^{ab}$ generate families.

$$\tilde{S}^{03} = \frac{i}{2}[\overset{03}{(\tilde{+}i)}\overset{12}{(\tilde{+})} + \overset{03}{(\tilde{-}i)}\overset{12}{(\tilde{+})} + \overset{03}{(\tilde{+}i)}\overset{12}{(\tilde{-})} + \overset{03}{(\tilde{-}i)}\overset{12}{(\tilde{-})}]$$

Both vectors bellow describe a right handed u-quark of the same colour.

$$\overset{03}{(\tilde{-}i)}\overset{12}{(\tilde{-})} \qquad \overset{03}{(+i)}\overset{12}{(+)} \mid \overset{56}{(+)}\overset{78}{(+)} \mid\mid \overset{910}{(+)}\overset{1112}{(-)}\overset{1314}{(-)} =$$

$$\overset{03}{[+i]}\overset{12}{[+]} \mid \overset{56}{(+)}\overset{78}{(+)} \mid\mid \overset{910}{(+)}\overset{1112}{(-)}\overset{1314}{(-)}$$

Break of symmetries

$$
\underbrace{
\begin{array}{ccccc}
 & & SO(1,13) & & \\
 & & \downarrow & & \\
 & & SO(1,7)\times & & SU(3)\times U(1) \\
 & \swarrow & & \searrow & \\
 & ? & ? & ? & \\
SO(1,3)\times SO(4)\times U(1) & & & & SU(3) \\
 & & SO(1,3)\times SU(2)\times U(1) & & SU(3)
\end{array}
}
$$

$$
\begin{array}{c}
\downarrow \\
? \\
\downarrow \\
\mathrm{SO}(1,3)\times \mathrm{U}(1)\times \mathrm{SU}(3)
\end{array}
$$

Yukawa couplings:

$$
\begin{aligned}
\mathcal{L}_Y = \; & \psi^\dagger \gamma^0 \{ \overset{78}{(+)} \, (\sum_{y=Y,Y'} y A^y_+ + \frac{-1}{2} \sum_{(ab)} \tilde{S}^{ab} \tilde{\omega}_{ab+})) + \\
& \overset{78}{(-)} \, (\sum_{y=Y,Y'} y A^y_- + \frac{-1}{2} \sum_{(ab)} \tilde{S}^{ab} \tilde{\omega}_{ab-}) \\
& \overset{78}{(+)} \sum_{\{(ac)(bd)\},k,l} \overset{ac}{(\tilde{k})} \overset{bd}{(\tilde{l})} \, \tilde{A}^{kl}_+ ((ac),(bd)) \; + \\
& \overset{78}{(-)} \sum_{\{(ac)(bd)\},k,l} \overset{ac}{(\tilde{k})} \overset{bd}{(\tilde{l})} \, \tilde{A}^{kl}_- ((ac),(bd)) \} \psi ,
\end{aligned}
$$

with $k, l = \pm 1$, if $\eta^{aa}\eta^{bb} = 1$ and $\pm i$, if $\eta^{aa}\eta^{bb} = -1$, while $Y = \tau^{21} + \tau^{41}$ and $Y' = -\tau^{21} + \tau^{41}$, $(ab), (cd), \cdots$ **Cartan only**.

We assume:

- Breaking symmetries from $SO(1,13)$ to $SO(1,7) \times U(1) \times SU(3)$ occurs at very high energy scale and left very heavy all the families except one which is left massless.
- There are $2^{8/2-1} = 8$ families.
- Two ways of breaking from $SO(1,7) \times U(1)$ to $SO(1,3) \times U(1)$ in the $\tilde{S}^{ab}\tilde{\omega}_{ab\pm}$ sector.

A. In the ordinary $(S^{ab}\omega_{ab\pm})$ sector $S^{sa}\omega_{sa\pm}$, with $s = 5, 6$ and $a \neq 5, 6$, would contribute a term, which would transform u-quarks into d-quarks. We forbid such terms in the $\tilde{S}^{ab}\omega_{ab\pm}$ sector as well.

Eight families split into two times four families, well separated in masses. We study properties of the lower energy four families with the assumption that mass matrices are real and symmetric (no CP).

B. First we break in both sectors $SO(1,7) \times U(1)$ into $SO(1,3) \times SO(4) \times U(1)$ (by putting all $\omega_{am\pm}$ and $\tilde{\omega}_{am\pm}, m = 0,1,2,3,\; a = 5,6,7,8$ equal to zero).

Then we break $SO(4) \times U(1)$ in two successive breaks first into $SU(2) \times U(1)$ and then to $U(1)$.

The assumption that the first break occurs at much higher energy then the second one (which occurs at the weak scale) makes the eight families two times four families, well separated in masses. We then study the properties of the lower four families.

A. Four families of u_R and correspondingly four octets (which can be reached by $((\overset{ac}{\tilde{k}})(\overset{bd}{\tilde{l}}))$

$$
\begin{array}{ll}
I. & \overset{03}{(+i)}\overset{12}{(+)} \mid \overset{56}{(+)}\overset{78}{(+)} \mid\mid \ldots \\
II. & \overset{03}{[+i]}\overset{12}{[+]} \mid \overset{56}{(+)}\overset{78}{(+)} \mid\mid \ldots \\
III. & \overset{03}{[+i]}\overset{12}{(+)} \mid \overset{56}{(+)}\overset{78}{[+]} \mid\mid \ldots \\
IV. & \overset{03}{(+i)}\overset{12}{[+]} \mid \overset{56}{(+)}\overset{78}{[+]} \mid\mid \ldots .
\end{array}
$$

α	I_R	II_R	III_R	IV_R
I_L	a_α	$\frac{\tilde{\omega}_{327\alpha}+\tilde{\omega}_{018\alpha}}{2}$	$\frac{\tilde{\omega}_{387\alpha}+\tilde{\omega}_{078\alpha}}{2}$	$\frac{\tilde{\omega}_{187\alpha}}{2}$
II_L	$\frac{\tilde{\omega}_{327\alpha}+\tilde{\omega}_{018\alpha}}{2}$	$a_\alpha + (\tilde{\omega}_{127\alpha} - \tilde{\omega}_{038\alpha})$	$\frac{\tilde{\omega}_{187\alpha}}{2}$	$\frac{\tilde{\omega}_{387\alpha}-\tilde{\omega}_{078\alpha}}{2}$
III_L	$\frac{\tilde{\omega}_{387\alpha}+\tilde{\omega}_{078\alpha}}{2}$	$\frac{\tilde{\omega}_{187\alpha}}{2}$	$\frac{k_\alpha(\tilde{\omega}_{387\alpha}+\tilde{\omega}_{078\alpha})}{2}$	$\frac{k_\alpha}{2}\tilde{\omega}_{187\alpha}$
IV_L	$\frac{\tilde{\omega}_{187\alpha}}{2}$	$\frac{\tilde{\omega}_{387\alpha}-\tilde{\omega}_{078\alpha}}{2}$	$\frac{k_\alpha}{2}\tilde{\omega}_{187\alpha}$	$\frac{k_\alpha(\tilde{\omega}_{387\alpha}-\tilde{\omega}_{078\alpha})}{2}$

Taking into account that experimental data for the known families suggest twice weakly coupled two and two families, we require that mass matrices have the shape

$$\begin{pmatrix} A & B \\ B & C = A + kB \end{pmatrix}. \tag{1}$$

with $k_u = -k_d$, $k_\nu = -k_e$,
$\tilde{\omega}_{abs_{u,\nu}} = b_{abs_{u,\nu}} \, \tilde{\omega}_{abs_{d,e}}$,
for $(abs) = (018)(078)(127)(387)$.
There are 3 angles determining the diagonalization of the mass matrices,
those for u related uniquely to those for d
and those for ν related to those for e,
which lead to

$$
\begin{aligned}
\tilde{\omega}_{018u} &= \frac{1}{2}[\frac{m_{u2}-m_{u1}}{\sqrt{1+({}^{a}\eta)^2}} + \frac{m_{u4}-m_{u3}}{\sqrt{1+({}^{b}\eta)^2}}],\\
\tilde{\omega}_{078u} &= \frac{1/2}{\sqrt{1+(\frac{k}{2})^2}}[\frac{{}^{a}\eta\,(m_{u2}-m_{u1})}{\sqrt{1+({}^{a}\eta)^2}} - \frac{{}^{b}\eta\,(m_{u4}-m_{u3})}{\sqrt{1+({}^{b}\eta)^2}}],\\
\tilde{\omega}_{127u} &= \frac{1}{2}[\frac{{}^{a}\eta\,(m_{u2}-m_{u1})}{\sqrt{1+({}^{a}\eta)^2}} + \frac{{}^{b}\eta\,(m_{u4}-m_{u3})}{\sqrt{1+({}^{b}\eta)^2}}],\\
\tilde{\omega}_{187u} &= \frac{1}{2\sqrt{1+(\frac{k}{2})^2}}[-\frac{m_{u2}-m_{u1}}{\sqrt{1+({}^{a}\eta)^2}} + \frac{m_{u4}-m_{u3}}{\sqrt{1+({}^{b}\eta)^2}}],\\
\tilde{\omega}_{387u} &= \frac{1}{2\sqrt{1+(\frac{k}{2})^2}}[(m_{u4}+m_{u3})-(m_{u2}+m_{u1})],\\
a_u &= \frac{1}{2}(m_{u1}+m_{u2}-\frac{{}^{a}\eta\,(m_{u2}-m_{u1})}{\sqrt{1+({}^{a}\eta)^2}}), \qquad (2)
\end{aligned}
$$

Equivalently for d quarks, neutrinos and electrons. Here
$a_{u.\nu} = A^{I}_{u,\nu} - \frac{1}{2}\tilde{\omega}_{038_{u,\nu}} + \frac{1}{2}(\frac{k_{u,\nu}}{2} - \sqrt{1 + (\frac{k_{u,\nu}}{2})^2})(\tilde{\omega}_{078_{u,\nu}} + \tilde{\omega}_{387_{u,\nu}})$,
and equivalently for the d-quarks and electrons.
No. of parameters 8, no. of data: 9

We use the Monte-Carlo program to fit—within the experimental accuracy—our free parameters $\tilde{\omega}_{abc}$
so that our mass matrices reproduce the experimental data.
We predict—in this very rough estimation because of several assumptions, we made— treating equivalently quarks and leptons
i. masses of the fourth family and
ii. mixing matrices.

We obtain:

	u	d	ν	e
k	-0.085	0.085	-1.25	1.254
${}^{a}\eta$	-0.225	0.225	1.58	-1.584
${}^{b}\eta$	0.420	-0.440	-0.162	0.162

	u	d	u/d	ν	e	ν/e
$\lvert\tilde{\omega}_{018}\rvert$	21205	42547	0.498	10729	21343	0.503
$\lvert\tilde{\omega}_{078}\rvert$	49536	101042	0.490	31846	63201	0.504
$\lvert\tilde{\omega}_{127}\rvert$	50700	101239	0.501	37489	74461	0.503
$\lvert\tilde{\omega}_{187}\rvert$	20930	42485	0.493	9113	18075	0.505
$\lvert\tilde{\omega}_{387}\rvert$	230055	114042	2.017	33124	67229	0.493
a^{a}	94174	6237		1149	1142	

Masses for quarks

$$
\begin{aligned}
m_{u_i}/GeV &= (0.0034, 1.15, 176.5, 285.2),\\
m_{d_i}/GeV &= (0.0046, 0.11, 4.4, 224.0),
\end{aligned}
$$

and the corresponding mixing matrix

$$
\begin{pmatrix}
0.974 & 0.223 & 0.004 & 0.042\\
0.223 & 0.974 & 0.042 & 0.004\\
0.004 & 0.042 & 0.921 & 0.387\\
0.042 & 0.004 & 0.387 & 0.921
\end{pmatrix},
$$

Masses for leptons

$$
\begin{aligned}
m_{\nu_i}/GeV &= (1\ 10^{-12}, 1\ 10^{-11}, 5\ 10^{-11}, 84.0),\\
m_{e_i}/GeV &= (0.0005, 0.106, 1.8, 169.2),
\end{aligned}
$$

and the corresponding mixing matrix

$$
\begin{pmatrix}
0.697 & 0.486 & 0.177 & 0.497\\
0.486 & 0.697 & 0.497 & 0.177\\
0.177 & 0.497 & 0.817 & 0.234\\
0.497 & 0.177 & 0.234 & 0.817
\end{pmatrix}.
$$

Both in agreement with the references
Phys. Lett. **B 592** (2004)1,
Phys.Scripta **T121** (2005) 72, hep-ph/0410030v1
Phys. Lett. **B 529**(2002) 111, hep-ph/0111028,
Phys.Lett. **B 476** (2000) 107, hep-ph/9911535v1
Phys. Atom.Nucl. **66** (2003) 2169; *Yad.Fiz.* **66** (2003) 2219, hep-ph/0301268v2.

B. At $SO(1,7) \times U(1)$ to $SO(1,3) \times SO(4) \times U(1)$
we require $\tilde{\omega}_{sm\pm} = 0$, with $s = 5,6,7,8$; $m = 0,1,2,3$.
The eight families break into two decoupled four families.
At the break $SO(4) \times U(1)$ into $SU(2) \times U(1)$ (at some large scale) new fields $\tilde{A}^{Y}_{\pm}$ and $\tilde{A}^{Y'}_{\pm}$ are formed:

$$\begin{aligned}
\tilde{A}^{23}_{\pm} &= \tilde{A}^{Y}_{\pm} \sin\tilde{\theta}_2 + \tilde{A}^{Y'}_{\pm} \cos\tilde{\theta}_2, \\
\tilde{A}^{41}_{\pm} &= \tilde{A}^{Y}_{\pm} \cos\tilde{\theta}_2 - \tilde{A}^{Y'}_{\pm} \sin\tilde{\theta}_2,
\end{aligned} \tag{3}$$

the gauge fields of the new operators:

$$\tilde{Y} = \tilde{\tau}^{41} + \tilde{\tau}^{23}, \quad \tilde{Y\prime} = \tilde{\tau}^{23} - \tilde{\tau}^{41} \tan\tilde{\theta}_2,$$

with $\tilde{\tau}^{23} = \frac{1}{2}(\tilde{S}^{56} + \tilde{S}^{78})$, $\tilde{\tau}^{41} = -\frac{1}{3}(\tilde{S}^{9\,10} + \tilde{S}^{11\,12} + \tilde{S}^{13\,14})$.
Small enough $\tilde{\theta}_2$ makes the two four families be well separated in masses.

At the weak scale $SU(2) \times U(1)$ breaks into $U(1)$.

New fields $\tilde{A}_{\pm}, \tilde{Z}_{\pm}$ appear

$\tilde{A}^{13}_{\pm} = \tilde{A}_{\pm} \sin\tilde{\theta}_1 + \tilde{Z}_{\pm} \cos\tilde{\theta}_1$,

$\tilde{A}^{Y}_{\pm} = \tilde{A}_{\pm} \cos\tilde{\theta}_1 - \tilde{Z}_{\pm} \sin\tilde{\theta}_1$,

the gauge fields of

$\tilde{Q} = \tilde{\tau}^{13} + \tilde{Y} = \tilde{S}^{56} + \tilde{\tau}^{41}$,

$\tilde{Q}' = -\tilde{Y} \tan^2\tilde{\theta}_1 + \tilde{\tau}^{13}$,

with $\tilde{e} = \tilde{g}^{Y} \cos\tilde{\theta}_1, \tilde{g'} = \tilde{g}^{1} \cos\tilde{\theta}_1$, $\tan\tilde{\theta}_1 = \frac{\tilde{g}^{Y}}{\tilde{g}^{1}}$.

	I	II	III	IV
I	$a_\pm$	$\frac{\tilde{g}^m}{\sqrt{2}}\tilde{A}^{+N_+}_\pm$	$-\frac{\tilde{g}^1}{\sqrt{2}}\tilde{A}^{1+}_\pm$	0
II	$\frac{\tilde{g}^m}{\sqrt{2}}\tilde{A}^{-N_+}_\pm$	$a_\pm + \frac{1}{2}\tilde{g}^m(\tilde{A}^{3N_-}_\pm + \tilde{A}^{3N_+}_\pm)$	0	$-\frac{\tilde{g}^1}{\sqrt{2}}\tilde{A}^{1+}_\pm$
III	$\frac{\tilde{g}^1}{\sqrt{2}}\tilde{A}^{1-}_\pm$	0	$a_\pm + \tilde{e}\tilde{A}_\pm + \tilde{g'}\tilde{Z}_\pm$	$\frac{\tilde{g}^m}{\sqrt{2}}\tilde{A}^{+N_+}_\pm$
IV	0	$\frac{\tilde{g}^1}{\sqrt{2}}\tilde{A}^{1-}_\pm$	$\frac{\tilde{g}^m}{\sqrt{2}}\tilde{A}^{-N_+}_\pm$	$a_\pm + \tilde{e}\tilde{A}_\pm + \tilde{g'}\tilde{Z}_\pm$ $+\frac{1}{2}\tilde{g}^m(\tilde{A}^{3N_-}_\pm + \tilde{A}^{3N_+}_\pm)$

The mass matrix for the lower four families of u-quarks $(-)$ and d-quarks $(+)$ is not assumed to be real and symmetric.
We parameterize

$$\begin{pmatrix} a_\pm & b_\pm & -c_\pm & 0 \\ b_\pm & a_\pm + d_{1\pm} & 0 & -c_\pm \\ c_\pm & 0 & a_\pm + d_{2\pm} & b_\pm \\ 0 & c_\pm & b_\pm & a_\pm + d_{3\pm} \end{pmatrix}$$

Fitting these parameters with the Monte-Carlo program to the experimental data within the known accuracy and to the assumed values for the fourth family masses we get for the u-quarks the mass matrix

$$\begin{pmatrix} (9,22) & (-150,-83) & 0 & (-306,304) \\ (-150,-83) & (1211,1245) & (-306,304) & 0 \\ 0 & (-306,304) & (171600,176400) & (-150,-83) \\ (-306,304) & 0 & (-150,-83) & 200000 \\ \end{pmatrix}$$

and for the d-quarks the mass matrix

$$\begin{pmatrix} (5,11) & (8.2,14.5) & 0 & (174,198) \\ (8.2,14.5) & (83,115) & (174,198) & 0 \\ 0 & (174,198) & (4260,4660) & (8.2,14.5) \\ (174,198) & 0 & (8.2,14.5) & 200000 \\ \end{pmatrix}.$$

This corresponds to the following values for the masses of the u and the d quarks

$$
\begin{aligned}
m_{u_i}/GeV &= (0.005, 1.220, 171., 215.),\\
m_{d_i}/GeV &= (0.008, 0.100, 4.500, 285.),
\end{aligned}
$$

and the mixing matrix for the quarks

$$
\begin{pmatrix}
-0.974 & -0.226 & -0.00412 & 0.00218\\
0.226 & -0.973 & -0.0421 & -0.000207\\
0.0055 & -0.0419 & 0.999 & 0.00294\\
0.00215 & 0.000414 & -0.00293 & 0.999
\end{pmatrix}.
$$

Concluding remarks

:

- We started with a simple Lagrange density suggested by **the approach unifying spins and charges** with one Weyl spinor in $d = 1 + 13$, which carries only the spin (no charges) and interacts with only the gravity through vielbeins and the two kinds of the spin connection fields, which are the gauge fields of S^{ab} and $\tilde{S}^{ab}$, respectively.
- There are S^{ab}, which determine in $d = (1 + 3)$ the spin and all the charges. One Weyl spinor representation includes (if analyzed with respect to the Standard model groups) the left handed weak charged quarks and leptons and the right handed weak chargeless quarks and leptons.
- $\tilde{S}^{ab}$ generate an even number of families.

- It is a part of a simple starting action which manifests as Yukawa couplings of the Standard model $\psi^\dagger\gamma^0\gamma^s p_{0s}\psi,\ s=7,8$, with $p_{0s} = -\frac{1}{2}S^{ab}\omega_{abs} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abs}$ contributing to diagonal and off diagonal elements of mass matrices.
- Making assumptions about possible ways of breaking symmetries, each of the two assumed ways leads at "low energy sector" to four times four mass matrices for four families of quarks and leptons, two by two weakly coupled and all three also weakly coupled to the fourth one.

- The way A. of breaking symmetries leads to the masses of the fourth family in agreement with the experimental data, predicting that the fourth family appears at low enough energies to be measured with new accelerators. The three families are weakly coupled to the fourth one.
- The way B., although leading to mass matrices with only two off diagonal elements for each type of quarks and leptons, has too many free parameters to predict the masses of the fourth family. For chosen masses of the fourth family, however, the way B. predicts the couplings of the fourth to the first three families. Letting the fourth family mass growing, the fourth family very slowly decouples from the first three. The way B. also predicts, for example, the changed values for $|V_{31}|/|V_{32}| = 0.128 - 0.149$, acceptable since four instead of three families at weak scale contribute to this value.

- The higher four families might be the candidates for the dark matter in both proposed ways.
- What is the right way? To answer this question we should go beyond the tree level.
- Is the Approach unifying spins and charges the right way to understand the Yukawa couplings and accordingly going beyond the Standard model?

α	I_R	II_R	III_R	IV_R	V_R	VI_R	VII_R	$VIII_R$
I_L	$XXXX$	$-\tilde{A}^{++}_{-}$ ((03),(12))	$-\tilde{A}^{++}_{-}$ ((56),(78))	0	$-\tilde{A}^{++}_{-}$ ((03),(78))	$-\tilde{A}^{++}_{-}$ ((12),(56))	$\tilde{A}^{++}_{-}$ ((03),(56))	$\tilde{A}^{++}_{-}$ ((12),(78)
II_L	$-\tilde{A}^{--}_{-}$ ((03),(12))	$XXXX$	0	$-\tilde{A}^{++}_{-}$ ((56),(78))	$-\tilde{A}^{-+}_{-}$ ((12),(78))	$-\tilde{A}^{-+}_{-}$ ((03),(56))	$\tilde{A}^{-+}_{-}$ ((12),(56))	$\tilde{A}^{-+}_{-}$ ((03),(78)
III_L	$\tilde{A}^{--}_{-}$ ((56),(78))	0	$XXXX$	$-\tilde{A}^{++}_{-}$ ((03),(12))	$\tilde{A}^{+-}_{-}$ ((03),(56))	$-\tilde{A}^{+-}_{-}$ ((12),(78))	$\tilde{A}^{+-}_{-}$ ((03),(78))	$-\tilde{A}^{+-}_{-}$ ((12),(56)
IV_L	0	$\tilde{A}^{--}_{-}$ ((56),(78))	$-\tilde{A}^{--}_{-}$ ((03),(12))	$XXXX$	$\tilde{A}^{--}_{-}$ ((12),(56))	$-\tilde{A}^{--}_{-}$ ((03),(78))	$\tilde{A}^{--}_{-}$ ((12),(78))	$-\tilde{A}^{--}_{-}$ ((03),(56)
V_L	$-\tilde{A}^{--}_{-}$ ((03),(78))	$\tilde{A}^{+-}_{-}$ ((12),(78))	$\tilde{A}^{-+}_{-}$ ((03),(56))	$-\tilde{A}^{++}_{-}$ ((12),(56))	$XXXX$	0	$-\tilde{A}^{+-}_{-}$ ((56),(78))	$-\tilde{A}^{-+}_{-}$ ((03),(12)
VI_L	$\tilde{A}^{--}_{-}$ ((12),(56))	$-\tilde{A}^{+-}_{-}$ ((03),(56))	$\tilde{A}^{-+}_{-}$ ((12),(78))	$-\tilde{A}^{++}_{-}$ ((03),(78))	0	$XXXX$	$-\tilde{A}^{+-}_{-}$ ((03),(12))	$\tilde{A}^{-+}_{-}$ ((56),(78)
VII_L	$\tilde{A}^{--}_{-}$ ((03),(56))	$-\tilde{A}^{+-}_{-}$ ((12),(56))	$\tilde{A}^{-+}_{-}$ ((03),(78))	$-\tilde{A}^{++}_{-}$ ((12),(78))	$\tilde{A}^{-+}_{-}$ ((56),(78))	$-\tilde{A}^{-+}_{-}$ ((03),(12))	$XXXX$	0
$VIII_L$	$-\tilde{A}^{--}_{-}$ ((12),(78))	$\tilde{A}^{+-}_{-}$ ((03),(78))	$\tilde{A}^{-+}_{-}$ ((12),(56))	$-\tilde{A}^{++}_{-}$ ((03),(56))	$-\tilde{A}^{+-}_{-}$ ((03),(12))	$-\tilde{A}^{+-}_{-}$ ((56),(78))	0	$XXXX$

I_R	$\overset{03}{(+i)}\overset{12}{(+)}\overset{56}{(+)}\overset{78}{(+)}$
II_R	$\overset{03}{[+i]}\overset{12}{[+]}\overset{56}{(+)}\overset{78}{(+)}$
III_R	$\overset{03}{(+i)}\overset{12}{(+)}\overset{56}{[+]}\overset{78}{[+]}$
IV_R	$\overset{03}{[+i]}\overset{12}{[+]}\overset{56}{[+]}\overset{78}{[+]}$
V_R	$\overset{03}{[+i]}\overset{12}{(+)}\overset{56}{(+)}\overset{78}{[+]}$
VI_R	$\overset{03}{(+i)}\overset{12}{[+]}\overset{56}{[+]}\overset{78}{(+)}$
VII_R	$\overset{03}{[+i]}\overset{12}{(+)}\overset{56}{[+]}\overset{78}{(+)}$
$VIII_R$	$\overset{03}{(+i)}\overset{12}{[+]}\overset{56}{(+)}\overset{78}{[+]}$

α	I_R	II_R	III_R	IV_R	V_R	VI_R	VII_R	$VIII_R$
I_L	$XXXX$	$-\tilde{A}^{++}_{+}$ ((03),(12))	$\tilde{A}^{++}_{+}$ ((56),(78))	0	$\tilde{A}^{++}_{+}$ ((03),(78))	$-\tilde{A}^{++}_{+}$ ((12),(56))	$\tilde{A}^{++}_{+}$ ((03),(56))	$-\tilde{A}^{++}_{+}$ ((12),(78)
II_L	$-\tilde{A}^{--}_{+}$ ((03),(12))	$XXXX$	0	$\tilde{A}^{++}_{+}$ ((56),(78))	$\tilde{A}^{-+}_{+}$ ((12),(78))	$-\tilde{A}^{-+}_{+}$ ((03),(56))	$\tilde{A}^{-+}_{+}$ ((12),(56))	$-\tilde{A}^{-+}_{+}$ ((03),(78)
III_L	$-\tilde{A}^{--}_{+}$ ((56),(78))	0	$XXXX$	$-\tilde{A}^{++}_{+}$ ((03),(12))	$\tilde{A}^{+-}_{+}$ ((03),(56))	$\tilde{A}^{+-}_{+}$ ((12),(78))	$-\tilde{A}^{+-}_{+}$ ((03),(78))	$-\tilde{A}^{+-}_{+}$ ((12),(56)
IV_L	0	$-\tilde{A}^{--}_{+}$ ((56),(78))	$-\tilde{A}^{--}_{+}$ ((03),(12))	$XXXX$	$\tilde{A}^{--}_{+}$ ((12),(56))	$\tilde{A}^{--}_{+}$ ((03),(78))	$-\tilde{A}^{--}_{+}$ ((12),(78))	$-\tilde{A}^{--}_{+}$ ((03),(56)
V_L	$\tilde{A}^{--}_{+}$ ((03),(78))	$-\tilde{A}^{+-}_{+}$ ((12),(78))	$\tilde{A}^{-+}_{+}$ ((03),(56))	$-\tilde{A}^{++}_{+}$ ((12),(56))	$XXXX$	0	$\tilde{A}^{+-}_{+}$ ((56),(78))	$-\tilde{A}^{-+}_{+}$ ((03),(12)
VI_L	$\tilde{A}^{--}_{+}$ ((12),(56))	$-\tilde{A}^{+-}_{+}$ ((03),(56))	$-\tilde{A}^{-+}_{+}$ ((12),(78))	$\tilde{A}^{++}_{+}$ ((03),(78))	0	$XXXX$	$-\tilde{A}^{+-}_{+}$ ((03),(12))	$-\tilde{A}^{-+}_{+}$ ((56),(78)
VII_L	$\tilde{A}^{--}_{+}$ ((03),(56))	$-\tilde{A}^{+-}_{+}$ ((12),(56))	$-\tilde{A}^{-+}_{+}$ ((03),(78))	$\tilde{A}^{++}_{+}$ ((12),(78))	$-\tilde{A}^{-+}_{+}$ ((56),(78))	$-\tilde{A}^{-+}_{+}$ ((03),(12))	$XXXX$	0
$VIII_L$	$\tilde{A}^{--}_{+}$ ((12),(78))	$-\tilde{A}^{+-}_{+}$ ((03),(78))	$\tilde{A}^{-+}_{+}$ ((12),(56))	$-\tilde{A}^{++}_{+}$ ((03),(56))	$-\tilde{A}^{+-}_{+}$ ((03),(12))	$\tilde{A}^{+-}_{+}$ ((56),(78))	0	$XXXX$

I_R	$\overset{03}{(+i)}\overset{12}{(+)}\overset{56}{[-]}\overset{78}{[-]}$
II_R	$\overset{03}{[+i]}\overset{12}{[+]}\overset{56}{[-]}\overset{78}{[-]}$
III_R	$\overset{03}{(+i)}\overset{12}{(+)}\overset{56}{(-)}\overset{78}{(-)}$
IV_R	$\overset{03}{[+i]}\overset{12}{[+]}\overset{56}{(-)}\overset{78}{(-)}$
V_R	$\overset{03}{[+i]}\overset{12}{(+)}\overset{56}{[-]}\overset{78}{(-)}$
VI_R	$\overset{03}{(+i)}\overset{12}{[+]}\overset{56}{(-)}\overset{78}{[-]}$
VII_R	$\overset{03}{[+i]}\overset{12}{(+)}\overset{56}{(-)}\overset{78}{[-]}$
$VIII_R$	$\overset{03}{(+i)}\overset{12}{[+]}\overset{56}{[-]}\overset{78}{(-)}$