Field Correlator Method in QCD:
P-wave baryons

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The spectroscopy of c and b baryons has undergone a great renaissance in recent years.

New results have been appearing in abundance as a result of improved experimental techniques including information on states made of both light (u, d, s) and heavy (c, b) quarks:

Belle,

DØ,
CDF

The spin-parity values have not been measured but have been assigned in accord with expectation of the theory.

A powerful guideline for assigning quantum numbers to new states is required both by theory and experiment.

Table 1: Masses of the heavy baryons.

Baryon	$I(J^P)$	PDG'06	BELLE	Babar	CDF	FCM
$\Lambda_c(2285)$	$0(\frac{1}{2}^+)$	2284.9(6)				
$\Lambda_c(1595)$	$0(\frac{1}{2}^-)$	2595.0(6)				
$\Lambda_c(2630)$	$0(\frac{3}{2}^-)$	2628.1(6)				
$\Lambda_c(2880)^+$	$0(\frac{5}{2}^+?)$		2881 ± 0.5	2881.9 ± 0.6		
$\Lambda_c(2940)^+$			2937 ± 1.8	2938.8 ± 2.1		
$\Sigma_c(2450)$	$1(\frac{1}{2}^+)$	2451.3(7)			2501	
$\Sigma_c^*(2515)$	$1(\frac{3}{2}^+)$	2515.9(2.4)			2547	
$\Xi_c(2470)$	$\frac{1}{2}(\frac{1}{2}^+)$	2466.3(1.4)				
$\Xi_c'(2575)$	$\frac{1}{2}(\frac{1}{2}^+)$	2574.1(3.3)				
$\Xi_c^*(2645)$	$\frac{1}{2}(\frac{3}{2}^+)$	2647.4(2.0)			2631	
Ω_c	$0(\frac{1}{2}^+)$	2697.5(2.6)			2637	
Ω_c^*	$0(\frac{3}{2}^+)$	2752			2668	
Λ_b	$0(\frac{1}{2}^+)$	5624(9)				
Σ_b	$0(\frac{1}{2}^+)$				5816	5830
Ξ_b	$0(\frac{1}{2}^+)$				5794	5806

Belle 2006:

New states decaying into $\Lambda_c K\pi$: $\Xi_c(2645)$ and $\Xi_c(2815)$

Study at Belle using $e^+e^- \to \Lambda_c^+ \pi^+ \pi^- X$:

Angular analysis of $\Lambda_c(2880) \to \Sigma_c \pi$:

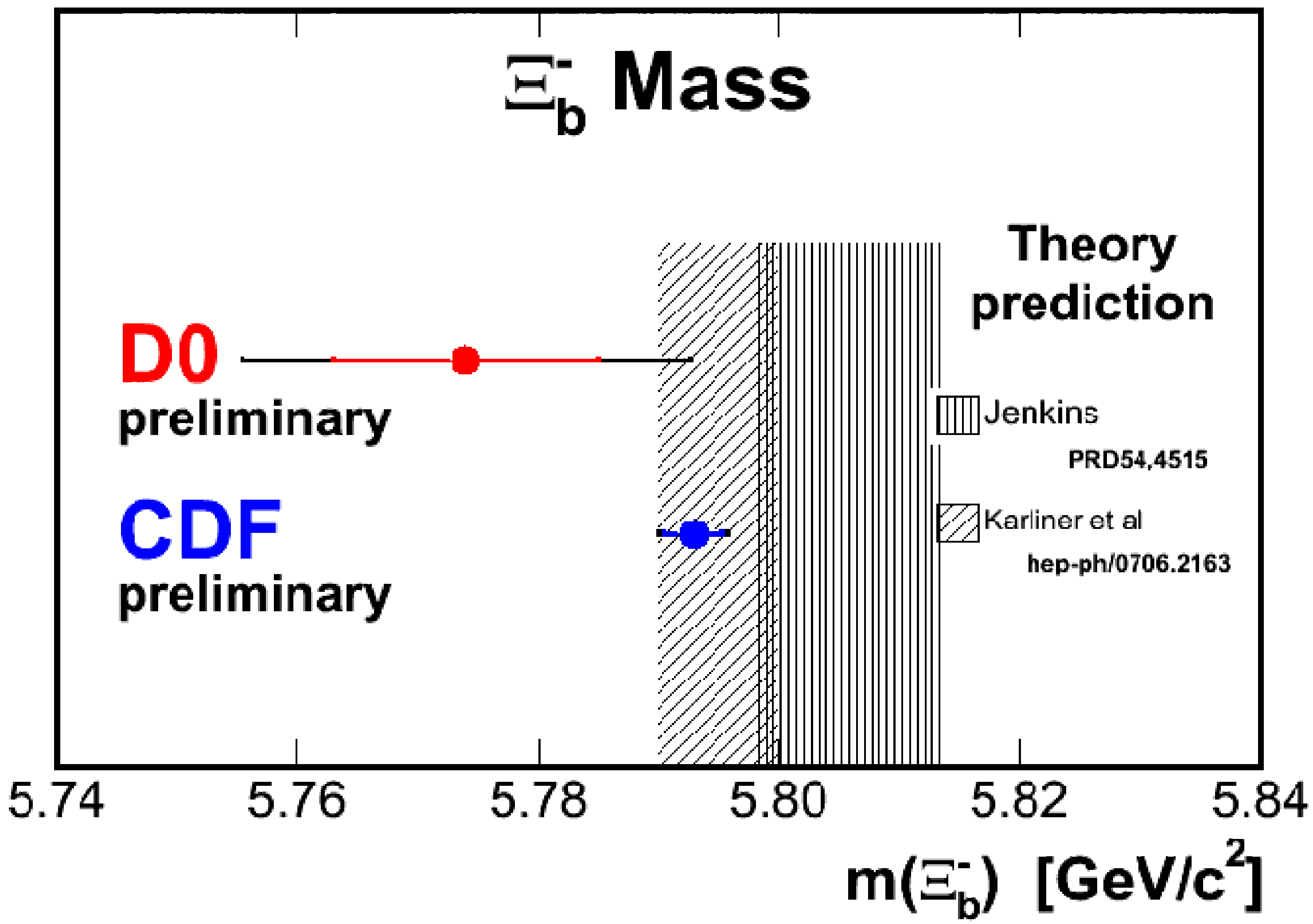
$J^P = 5/2$ hypotethis is strongly favorated over $J^P = 3/2$ and $1/2$

CDF 006:

spin averaged $M(\Sigma_b) - M(\Lambda_b) =$ 192 MeV

$$New!\ 2007 \quad \Xi_b^- \to J/\psi\,\Xi \to J/\psi\Lambda\pi$$

DØ $M(\Xi_b^-) = 5774 \pm 11 \pm 15$ MeV CDF $M(\Xi_b^-) = 5793 \pm 2.4 \pm 1.7$ MeV.



Ξ_b^- Mass
Theory prediction
D0
preliminary
CDF
preliminary
Jenkins
PRD54,4515
Karliner et al
hep-ph/0706.2163
5.74
5.76
5.78
5.80
5.82
5.84
$m(\Xi_b^-)$ [GeV/c^2]

Particle	$I(J^P)$	rating	Particle	$I(J^P)$	rating
$\Xi(1318)$	$\frac{1}{2}(\frac{1}{2}^+)$	****	$\Omega(1672)$	$0(\frac{3}{2}^+)$	****
$\Xi(1530)$	$\frac{1}{2}(\frac{3}{2}^+)$	****	$\Omega(2250)$	$0(?^?)$	***
$\Xi(1620)$	$\frac{1}{2}(?^?)$	*	$\Omega(2380)$	$?(?^?)$	**
$\Xi(1690)$	$\frac{1}{2}(?^?)$	***	$\Omega(2470)$	$?(?^?)$	**
$\Xi(1820)$	$\frac{1}{2}(\frac{3}{2}^-)$	***			
$\Xi(1950)$	$\frac{1}{2}(?^?)$	***			
$\Xi(2030)$	$\frac{1}{2}(\geq\frac{5}{2}^?)$	***			
$\Xi(2120)$	$\frac{1}{2}(?^?)$	*			
$\Xi(2250)$	$\frac{1}{2}(?^?)$	**			
$\Xi(2370)$	$\frac{1}{2}(?^?)$	**			
$\Xi(2500)$	$\frac{1}{2}(?^?)$	*			

Table 2: Ξ and Ω baryons listed in the PDG.

The investigation of the cascade resonances is an essential part of the JLab baryon spectroscopy program.

P-wave baryons within the field correlator method (FCM) in QCD (Yu.A.Simonov).

Masses and wave functions of light mesons (A.M.Badalyan *et al.*)
Heavy quarkonia system (Yu.S.Kalashnikova, A.M.B. *et al.*)

Only a few calculations for S-wave baryons (M.A.Trusov. and I.M.N.)

The present investigation was initially motivated as an attempt to extend the approach for the $P-$ wave low-lying orbitally excited baryons.

We do not consider the hyperfine splitting *e.g.* between Λ and Σ which is known to be shared between the pseudoscalar forces and the perturbative QCD contributions, provided by the one-gluon exchange.

We also discard all spin-orbit forces. Experimentally, there is evidence for the feebleness of the spin-orbit forces in the baryon spectra. This is clearly indicated *e.g.* in the degeneracy of

$$N(1535)S_{11} \text{ and } N(1520)D_{13}$$

nucleon resonances. The same is observed around the hyperon spectra except for the particular problem of the relatively large separation between

$$\Lambda(1405) \text{ and } \Lambda(1520),$$

presumably reflecting the influence of the nearby KN threshold.

Our conservative estimation is that for heavy baryons the effect of the spin-orbit splitting does not exceed 10-20 MeV.

$$H = \sum_{i=1}^{3} \left(\frac{\overline{m}_i^2}{2\,\mu_i} + \frac{\mu_i}{2} \right) + H_0 + V.$$

$$V \;=\; V_{\rm string}(\mathbf{r}_1,\,\mathbf{r}_2,\,\mathbf{r}_3)\;(=\;\sigma\,l_{\rm min})\;+\;V_{\rm OGE}\left(= -\frac{2}{3}\sum_{i<j}\frac{\alpha_s(r_{ij})}{r_{ij}}\right)$$

α_s is the strong coupling constant, r_{ij} are the distances between quarks.

$\overline{m}_i$ are the bare (pole) quark masses,

μ_i are the constant auxiliary einbein fields.

$$M_B = \min_{m_i}\langle H\rangle \;+\; C,$$

C is the perturbative quark self-energy correction which is created by the color magnetic moment of a quark propagating through the vacuum background field (Yu.A.S. and A. Digiacomo).

This correction, adds an overall negative constant to the hadron masses.

Standard quark models include $\sim$ 20 fitting parameters

In the FCM the parameters are $\bar{m}_i$ (~ 0 for u, d, ~ 1400 MeV for c and ~ 4800 MeV for b).

$$\alpha_s(q) = \frac{4\pi}{\beta_0\, t}\left(1 - \frac{\beta_1}{\beta_0^2}\frac{q^2 + M_B^2}{\Lambda_B^2}\right) \quad t = \frac{q^2 + M_B^2}{\Lambda_B^2}$$

$$m_B \approx 1\,\mathrm{GeV}, \quad \Lambda \approx 400\,\mathrm{MeV}$$

Freezing of $\alpha_s(r)$ at large r.

The baryon wave function depends on the Jacobi coordinates

$$\rho_{ij} = \sqrt{\frac{\mu_{ij}}{\mu}}(\mathbf{r}_i - \mathbf{r}_j), \quad \lambda_{ij} = \sqrt{\frac{\mu_{ij,\,k}}{\mu}}\left(\frac{m_i\mathbf{r}_i + m_j\mathbf{r}_j}{m_i + m_j} - \mathbf{r}_k\right)$$

$$R^2 = \rho^2 + \lambda^2, \quad \rho \,=\, R\,\sin\theta, \quad \lambda \,=\, R\,\cos\theta,$$

where R is the six-dimensional hyperradius

$$H_0 = -\frac{1}{2\mu}\left(\frac{\partial^2}{\partial R^2} + \frac{5}{R}\frac{\partial}{\partial R} + \frac{\mathbf{L}^2(\Omega)}{R^2}\right)$$

where Ω denotes five residuary angular coordinates, $\mathbf{L}^2(\Omega)$ is an angular operator

$$\mathbf{L}^2 \;=\; \frac{\partial^2}{\partial\theta^2} \,+\, 4\cot\,2\theta\,\frac{\partial}{\partial\theta} - \frac{\mathbf{l}_\rho^2}{\sin^2\theta} \,-\, \frac{\mathbf{l}_\lambda^2}{\cos^2\theta},$$

whose eigenfunctions (the hyperspherical harmonics) satisfy

$$\mathbf{L}^2(\Omega)\,Y_{[K]}(\theta,\mathbf{n}_\rho,\mathbf{n}_\lambda) \;=\; -K(K+4)Y_{[K]}(\theta,\mathbf{n}_\rho,\mathbf{n}_\lambda)$$

$$\Psi_\nu(R,\Omega) \;=\; \frac{u_\nu(R)}{R^{5/2}} \cdot \mathbf{Y}_\nu(\Omega)$$

$$\frac{d^2 u_\nu}{dR^2} \;+\; 2\left(E_0 - \frac{(K+3/2)(K+5/2)}{2\,R^2} - V_\nu(R) \right) u_\nu(R)$$

$$V_{\rm OGE} \;=\; -\,\frac{2}{3}\,\alpha_s \int \,|Y_{\nu\,i}\,(\theta,\chi)|^2 \sum_{i\,<\,j} \frac{1}{r_{ij}}\;\, d\Omega \;=\; -\,\frac{2}{3}\,\alpha_s\,\frac{a_\nu}{R},$$

$$V_{\rm string}(x) \;=\; \int \,|Y_{\nu\,i}\,(\theta,\chi)|^2\, V_{\rm string}(\mathbf{r}_1,\, \mathbf{r}_2,\, \mathbf{r}_3)\, d\Omega \;=\; \sigma\, b_\nu\, R$$

$$\mathbf{Y}^2_{\rho\,i} \;=\; \frac{6}{\pi^3}\;\sin^2\theta \qquad\qquad \mathbf{Y}^2_{\lambda\,i} \;=\; \frac{6}{\pi^3}\;\cos^2\theta$$

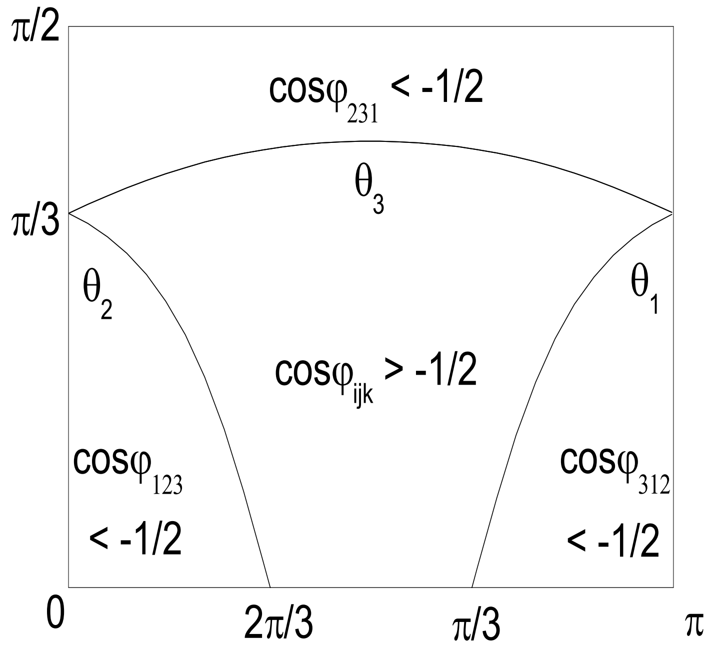


Figure 2: 4 regions in the $\theta - \chi$ plane. $\cos\chi \; = \; \mathbf{n}_\rho \mathbf{n}_\lambda$

$$b_\nu \; = \; \frac{1}{R} \left(\int_I + \int_{II} + \int_{III} + \int_{IV} \right) r_{\min} \, d\Omega_\nu$$

$$m_1 = m_2 = m, \quad m_3 = \kappa m$$

$$a_\rho = \frac{32}{15\,\pi}\left(\sqrt{2} + \sqrt{\frac{\kappa}{\kappa+1}}\,\frac{5\kappa+6}{\kappa+1}\right)$$

$$a_\lambda = \frac{32}{5\,\pi}\left(\frac{1}{\sqrt{2}} + \frac{1}{3}\sqrt{\frac{\kappa}{1+\kappa}}\,\frac{4+5\kappa}{1+\kappa}\right)$$

$$b_\nu = \frac{1}{R}\left(\int_I + \int_{II} + \int_{III} + \int_{IV}\right) r_{\min}\, d\Omega_\nu$$

$$b_\rho = \frac{64}{35\,\pi}\left(\sqrt{\frac{2}{\kappa(\kappa+2)}} + \frac{1}{3}\sqrt{\frac{\kappa+1}{\kappa+2}}\,\frac{7\kappa+8}{\kappa+1}\right),$$

$$b_\lambda = \frac{64}{105\,\pi}\left(4\sqrt{2}\sqrt{\frac{1}{\kappa\,(2+\kappa)}} + \sqrt{\frac{1}{(1+\kappa)(2+\kappa)}}\,(6+7\kappa)\right)$$

For the Λ and the Σ we use the uds basis in which the strange quark is singled out as quark 3 but in which the non strange quarks are still antisymmetrized.

The uds basis states diagonalize the confinement problem with eigenfunctions that correspond to separate excitations of the non strange and strange quarks.

In the same way, for the Ξ we use the ssq basis, in which the non strange quark is singled out as quark 3.

Hyperon	Excitation	$m_1 = m_2$	m_3	E_0	M
$\Lambda,\ \Sigma$	ρ	479	431	1627	1724
	λ	438	509	1629	1717
Ξ	ρ	491	419	1621	1745
	λ	452	500	1620	1752

Table 3: Masses of the ρ and λ hyperon excitations.

Ξ	CI86	GR96	Large N_c	SR	Skyrme	This work	PDG
$\frac{1}{2}^+$	1305	1320		1320		1335	$\Xi(1318)$
$\frac{1}{2}^-$	1755	1758	1780	1550	1660	1780	$\Xi(1690)$?
$\frac{3}{2}^-$	1785	1758	1815	1840	1820	1780	$\Xi(1820)$?

Table 4: Low-lying Ξ spectrum: Capstick and Isgur (CI86), Glozman-Riska (GR94), large N_c, QCDSR, and the Skirm model.

Summary

$Baryon$	$\Delta L = 1$	$\Delta L = 2$	Experiment
Λ, Σ	400		$300 - 400$
Ξ	450		$400 - 500$
Λ_c	320		300-330
		570	590
Σ_c	360		350

Table 5: P wave and D wave excitation energies of Ξ, Λ_c and Σ_c.