Chiral symmetry breaking and the Lorentz nature of confinement

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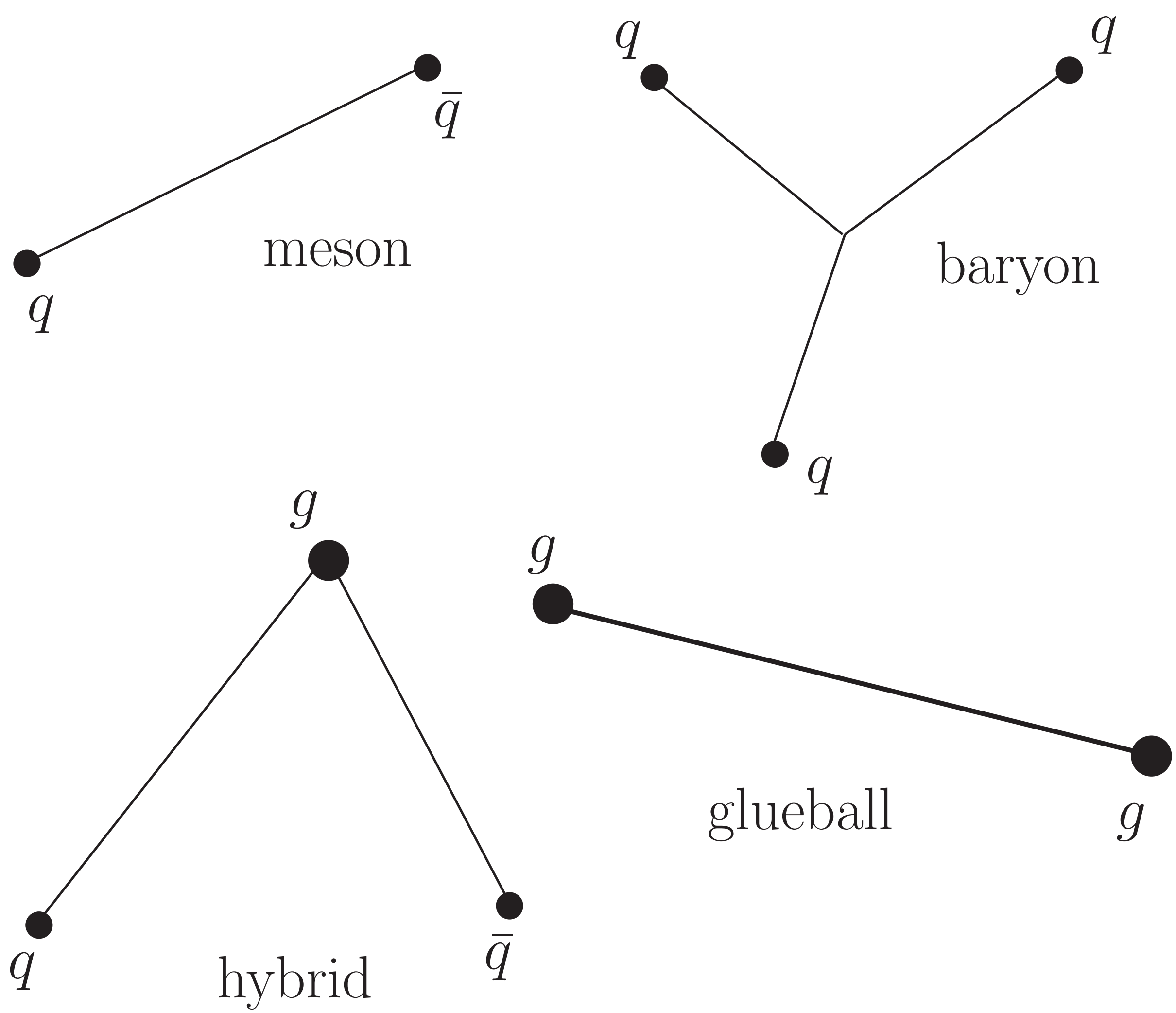
The QCD string picture of hadrons



Meson bound–state equation

$$L = -m_1\sqrt{1-\dot{\vec{x}}_1^{\,2}} - m_2\sqrt{1-\dot{\vec{x}}_2^{\,2}} - \sigma r \int_0^1 d\beta \sqrt{1-[\vec{n}\times(\beta\dot{\vec{x}}_1+(1-\beta)\dot{\vec{x}}_2)]^2}$$

$$\vec{r} = \vec{x}_1 - \vec{x}_2 \quad \vec{n} = \vec{r}/r$$

$$H = \sqrt{\vec{p}^2 + m_1^2} + \sqrt{\vec{p}^2 + m_2^2} + \sigma r + \ldots$$

$$H\psi = M\psi$$

Dyson–Schwinger-type approach to heavy–light quarkonium

$$S_{q\bar{Q}}(x,y) = \frac{1}{N_C}\int D\psi D\psi^\dagger DA_\mu\ \psi^\dagger(x) S_{\bar{Q}}(x,y|A)\psi(y)$$

$$\times \exp\left\{-\frac{1}{4}\int d^4x F_{\mu\nu}^{a2} - \int d^4x \psi^\dagger(-i\hat{\partial} - im - \hat{A})\psi\right\}$$

$$\vec{x}\vec{A}(x_4,\vec{x}) = 0 \quad A_4(x_4,\vec{0}) = 0$$

$$\int DA_\mu \exp\left(-\int d^4x\ \psi^\dagger A\psi\right) = \exp\left(-\int d^4x d^4y\ \psi^\dagger\psi^\dagger\langle\langle AA\rangle\rangle\psi\psi + \ldots\right)$$

$$(-i\hat{\partial}_x - im)S(x,y) - i\int d^4z M(x,z)S(z,y) = \delta^{(4)}(x-y)$$

$$-iM(x,z) = K_{\mu\nu}(x,z)\gamma_\mu S(x,z)\gamma_\nu$$

$$K_{\mu\nu} \propto \langle\langle AA \rangle\rangle \propto \langle\langle FF \rangle\rangle$$

Simonov '97

$$K_{44}(x,y) \equiv K(x,y) \approx \sigma(|\vec{x}| + |\vec{y}| - |\vec{x} - \vec{y}|)$$

$$K_{4i}(x,y) = 0 \quad K_{ik}(x,y) = 0$$

A.N., Simonov '05

$$(\vec{\alpha}\hat{\vec{p}} + \beta m)\Psi(\vec{x}) + \beta \int d^3 z M(\vec{x},\vec{z})\Psi(\vec{z}) = E\Psi(\vec{x})$$

$$M(\vec{x},\vec{z}) = -\frac{i}{2}K(\vec{x},\vec{z})\beta\Lambda(\vec{x},\vec{z})$$

$$\Lambda(\vec{x},\vec{z}) = 2i\int \frac{d\omega}{2\pi} S(\omega,\vec{x},\vec{y})\beta \propto \begin{cases} \hat{1},\ \vec{\alpha} & \text{vector confinement} \\ \beta & \text{scalar confinement} \end{cases}$$

$$K(\vec{x},\vec{y}) = V(|\vec{x}|) + V(|\vec{y}|) - V(|\vec{x}-\vec{y}|)$$

Chiral angle

$$\Sigma(\vec{p}) = -i\int \frac{d^4k}{(2\pi)^4} V(\vec{p}-\vec{k})\gamma_0 \frac{1}{\gamma_0 k_0 - \vec{\gamma}\vec{k} - m - \Sigma(\vec{k})}\gamma_0$$

$$\Sigma(\vec{p}) = [A_p - m] + (\vec{\gamma}\hat{\vec{p}})[B_p - p]$$

$$\frac{A_p}{B_p} = \tan\varphi_p$$

$$ps_p - mc_p = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3} V(\vec{p}-\vec{k})\left[c_k s_p - (\hat{\vec{p}}\hat{\vec{k}}) s_k c_p\right]$$

$$s_p = \sin\varphi_p \qquad c_p = \cos\varphi_p$$

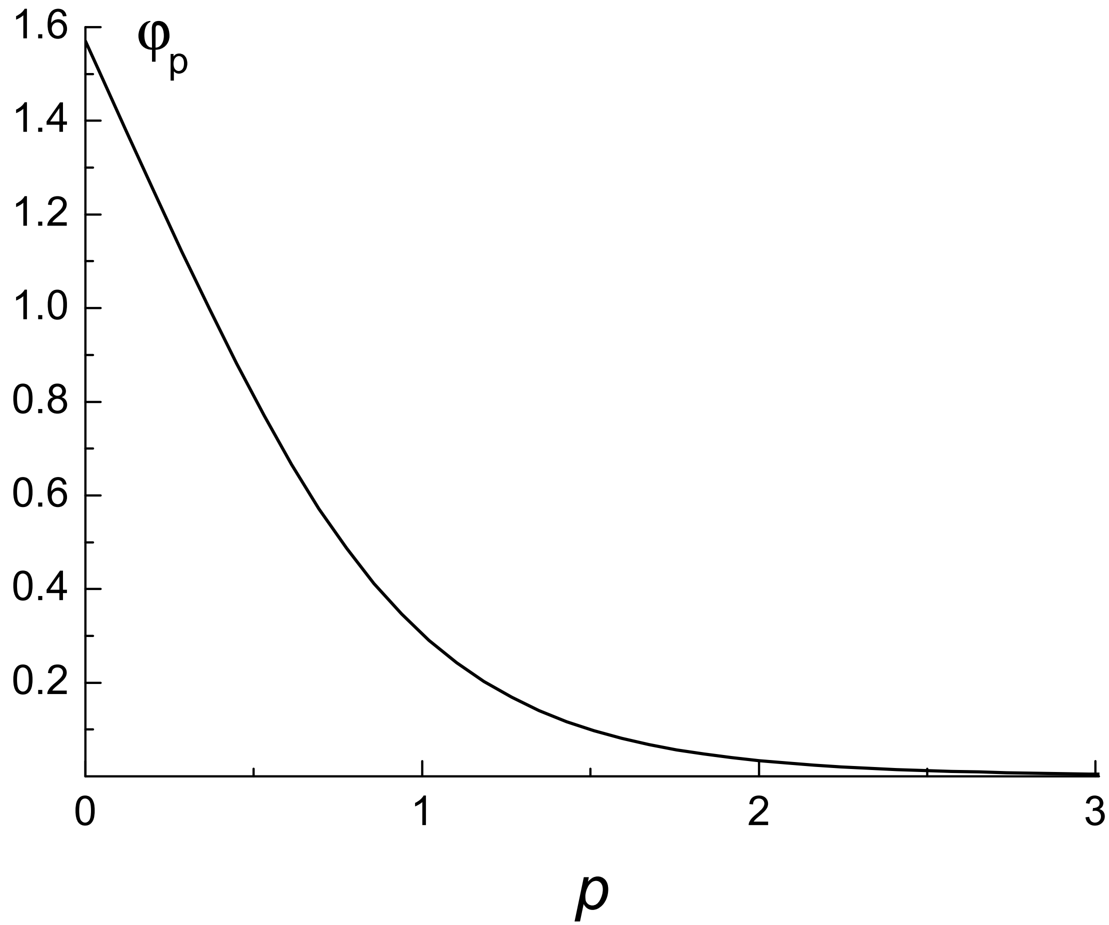


Figure 1: The tipical profile of the chiral angle — solution to the mass–gap equation

The Lorentz nature of confinement

$$\Lambda(\vec{p},\vec{q}) = 2i\int\frac{d\omega}{2\pi}S(\omega,\vec{p},\vec{q})\beta = (2\pi)^3\delta^{(3)}(\vec{p}-\vec{q})U_p$$

$$U_p = \beta\sin\varphi_p - (\vec{\alpha}\hat{\vec{p}})\cos\varphi_p$$

$$\Lambda(\vec{p},\vec{q}) \propto \begin{cases} \beta & \varphi_p \approx \frac{\pi}{2} \\ (\vec{\alpha}\vec{p}) & \varphi_p \to 0 \end{cases}$$

Foldy–Wouthuysen transformation

$$T_p = \exp\left[-\frac{1}{2}(\vec{\gamma}\hat{\vec{p}})\left(\frac{\pi}{2}-\varphi_p\right)\right] \qquad \Psi(\vec{p}) = T_p\begin{pmatrix}\psi(\vec{p})\\ 0\end{pmatrix}$$

$$E_p\psi(\vec{p})+\int\frac{d^3k}{(2\pi)^3}V(\vec{p}-\vec{k})\left[C_pC_k+(\vec{\sigma}\hat{\vec{p}})(\vec{\sigma}\hat{\vec{k}})S_pS_k\right]\psi(\vec{k}) = E\psi(\vec{p})$$

$$C_p = \cos\frac{1}{2}\left(\frac{\pi}{2}-\varphi_p\right) \qquad S_p = \sin\frac{1}{2}\left(\frac{\pi}{2}-\varphi_p\right)$$

$$E_p = A_p\sin\varphi_p + B_p\cos\varphi_p$$

Kalashnikova, A.N., Ribeiro '05

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- Chiral regime

$$\varphi_p \approx \frac{\pi}{2} \implies M(\vec{x},\vec{z}) \propto \beta \quad C_p = 1 \quad S_p = 0$$

$$[E_p + V(r)]\psi = E\psi$$

- Restoration regime

$$\varphi_p \to 0 \implies M(\vec{x},\vec{z}) \propto \vec{\alpha} \quad C_p = S_p = 1/\sqrt{2}$$

$$E_p\psi(\vec{p}) + \frac{1}{2}\int \frac{d^3k}{(2\pi)^3} V(\vec{p}-\vec{k}) \left[1 + (\vec{\sigma}\hat{\vec{p}})(\vec{\sigma}\hat{\vec{k}})\right] \psi(\vec{k}) = E\psi(\vec{p})$$