EoS and Phase Transition at nonzero baryon density

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1. Introduction. Physical picture.
2. Nonperturbative EoS at nonzero μ.
3. Phase transition: deconfinement and CSB.
4. Equation for $T_c(\mu)$ and numerical solution.
5. Conclusions.

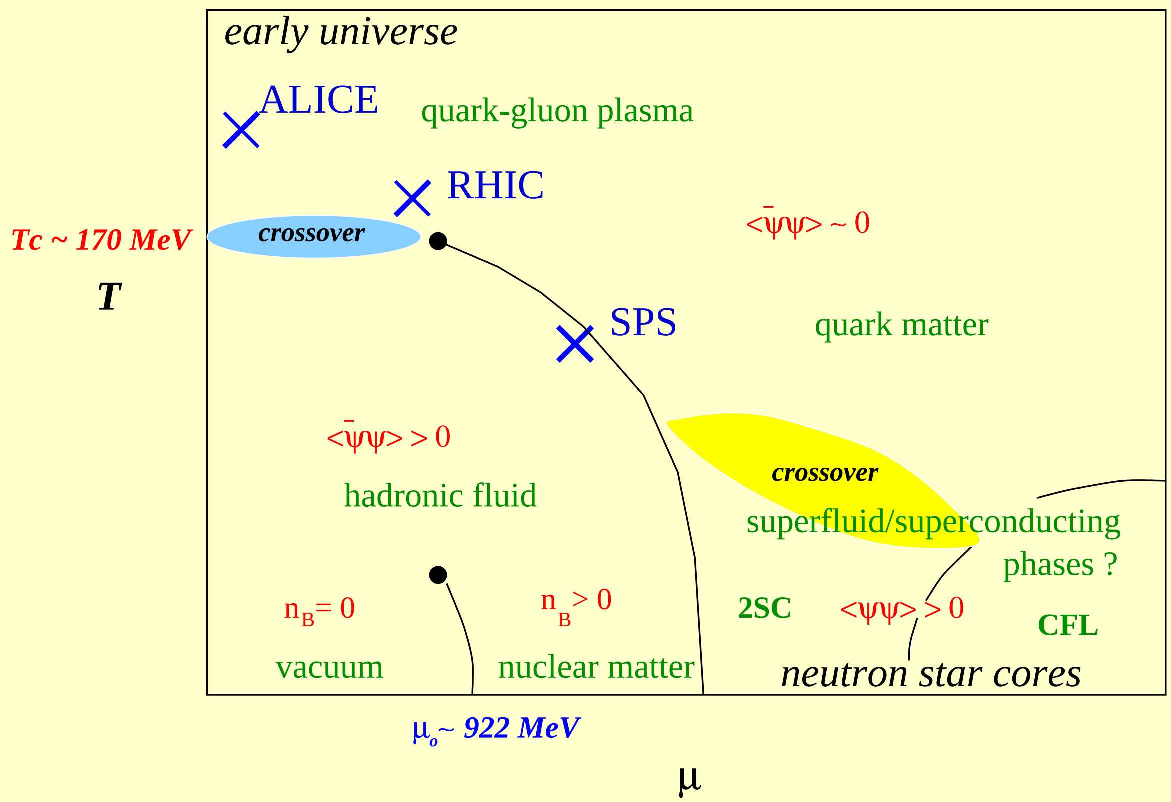


Figure 1: The QCD phase diagram in the temperature-chemical potential plane. First order phase transitions are shown as solid lines, and second order transitions by filled circles. Crosses depict heavy-ion collision experiments. The region of interest in this work is the *crossover* transition at $T_c \approx 170$ MeV.

Physical picture:

General properties of $T > 0$ QCD

1. Gauge invariance also for $T > T_c$

But: Color becomes free – one can see it in the ratio $\frac{P}{P_{SB}}$??

$$P_{SB} = \frac{\pi^2(N_c^2 - 1)}{45} T^4 + \frac{7\pi^2 N_c}{180} n_f T^4$$

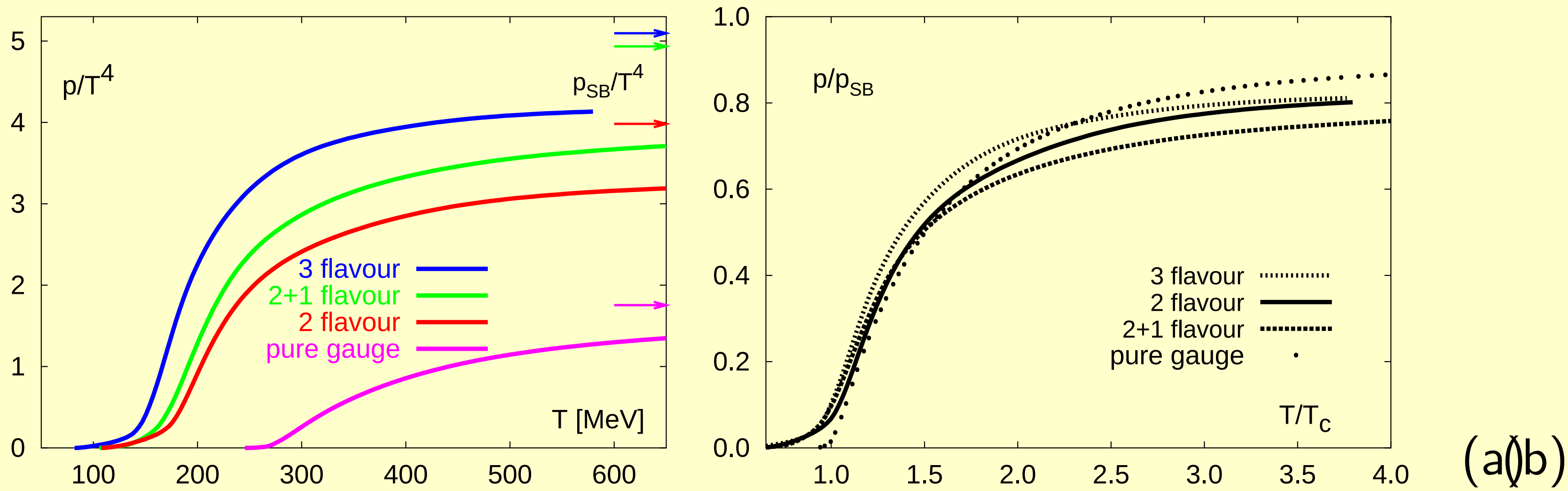


Figure 2: The pressure in QCD with $n_f = 0,\ 2$ and 3 light quarks as well as two light and a heavier (strange) quark. For $n_f \neq 0$ calculations have been performed on a $N_\tau = 4$ lattice using improved gauge and staggered fermion actions. In the case of the SU(3) pure gauge theory the continuum extrapolated result is shown. Arrows indicate the ideal gas pressure p_{SB} as given in Eq. 3.

Vacuum Dominance Scheme of approximation

First step: Vacuum energy density and interaction of individual gluons and quarks with vacuum is taken into account.

Second step: Pair and triple interaction meditated by vacuum is taken into account.

Perturbative to $O(\alpha_s)$.

Higher orders.

Result: EoS in terms of one-particle propagators, modified by vacuum $\rightarrow$ modulus of Polyakov loop.

Qualitative check:

Correlations of quark quantum numbers – QNS.

Physical picture in terms of correlators

I. Confining phase: correlators D^E, D^H, D_1^E, D_1^H are all nonzero.

At $T=0$ $\;D^E = D^H, \;\; D_1^E = D_1^H, \;\; \sigma^E = \sigma^H \equiv \sigma_s$

$$\sigma^{(i)} = \frac{1}{2}\int D^{(i)}(x) d^2x, i = E, H$$

$$D^{(i)}(x) = D^{(i)}(0)\exp\left(-|x|/\lambda\right), \quad \lambda \cong 0.2 \;\; \text{fm}.$$

Gluonic condensate $G_2 \equiv \frac{\alpha_s}{\pi}\langle(F^a_{\mu\nu})^2\rangle = G_2^E + G_2^H$

at $T=0$ $\;G_2^E = G_2^H$ till $T = T_c$.

$G_2 = 0.012$ GeV^4 (SVZ).

S. Narison $G_2 = \frac{1}{\pi}(0.07 \div 0.009)$ GeV^4.

Vacuum energy density

$$\varepsilon = \frac{\beta(\alpha_s)}{16\alpha_s}\langle(F^a_{\mu\nu})^2\rangle \cong -\frac{11 - \frac{2}{3}n_f}{32}G_2 = \varepsilon^E + \varepsilon^H \quad \varepsilon^E = \varepsilon^H, \quad T = 0$$

II Deconfined phase, $T \geq T_c(\mu)$.

Here only D^E vanishes:
New QCD vacuum state with strong colormagnetic forces.
Colorelectric only in $D_1^E \to$ Polyakov loops.
Definitions $(H_k \equiv \frac{1}{4}\varepsilon_{ijk}F_{ij})$

$$g^2\langle \hat{t}r_f[E_i(x)\Phi(x,y)E_k(y)\Phi(y,x)]\rangle$$

$$= \delta_{ik}\left[D^E + D_1^E + u_4^2\frac{\partial D_1^E}{\partial u_4^2}\right] + u_i u_k \frac{\partial D_1^E}{\partial \mathbf{u}^2} \ ,$$

$$g^2\langle \hat{t}r_f[H_i(x)\Phi(x,y)H_k(y)\Phi(y,x)]\rangle$$

$$= \delta_{ik}\left[D^H + D_1^H + \mathbf{u}^2\frac{\partial D_1^H}{\partial \mathbf{u}^2}\right] - u_i u_k \frac{\partial D_1^H}{\partial \mathbf{u}^2} \ ,$$

$$g^2\langle \hat{t}r_f[E_i(x)\Phi(x,y)H_k(y)\Phi(y,x)]\rangle = -\frac{1}{2}\varepsilon_{ikn}u_n\frac{\partial D_1^{HE}}{\partial u_4} \ . \qquad (1)$$

How FC behave below and above T_c?

It is advantageous (minimal $F(T)$) to have vacuum without confining colorelectric fields. Calculation of T_c Simonov, (1991)

$D^E = 0$, $T > T_c$ all others nonzero.

CE gluon condensate vanishes:

Check: Pisa group

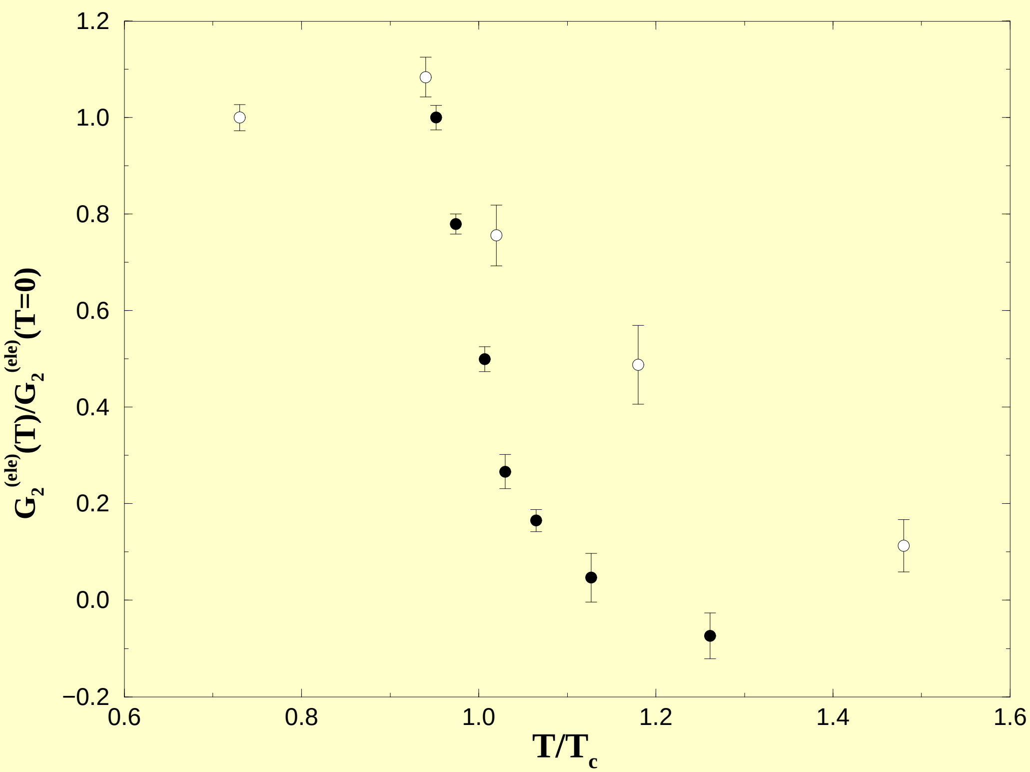


Figure 3: The electric gluon condensate $G_2^{(mag)}(T)$ in units of $G_2^{(mag)}(T = 0)$, versus T/T_c. The black circles refer to the *quenched* case, while the white circles refer to the full–QCD case.

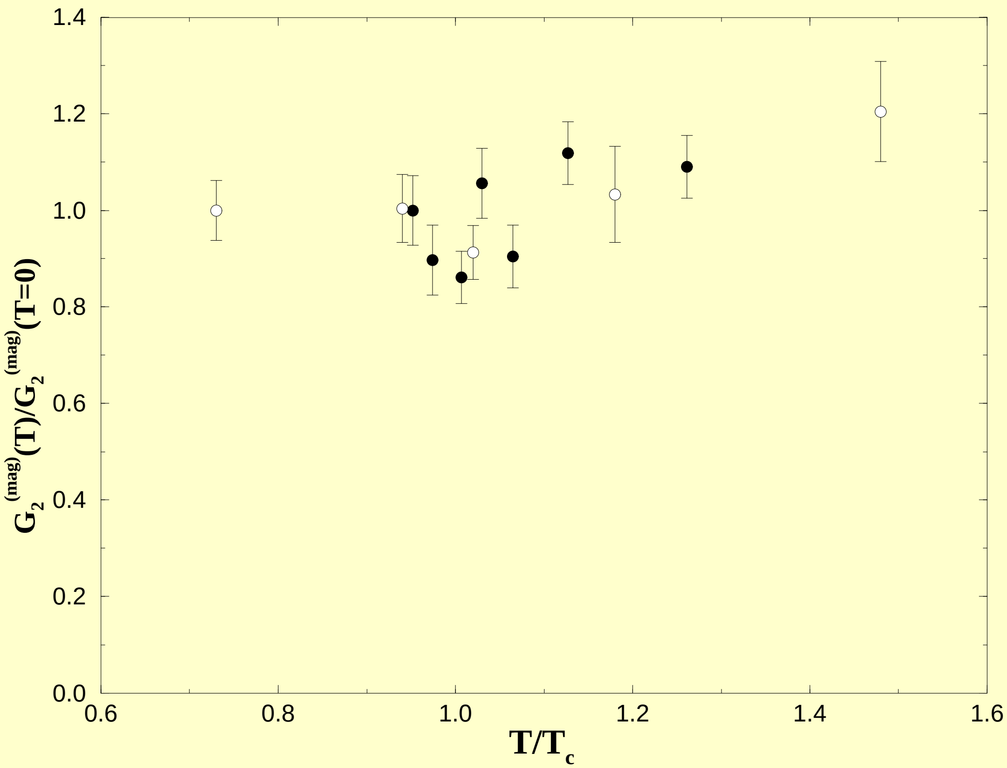


Figure 4: The magnetic gluon condensate $G_2^{(ele)}(T)$ in units of $G_2^{(ele)}(T = 0)$, versus T/T_c. The notation is the same as in Fig. 9.

7. Nonperturbative EoS in Single Line Approximation

Background Perturbation Theory and Field Correlators

Splitting into vacuum background field B_μ and valence gluon a_μ

$$A_\mu = B_\mu + a_\mu, \tag{2}$$

$$B_\mu(z_4, z_i) = B_\mu(z_4 + n\beta, z_i); \quad a_\mu(z_4, z_i) = a_\mu(z_4 + n\beta, z_i), \tag{3}$$

$$Z(V,T) = \langle Z(B)\rangle_B \tag{4}$$

with

$$Z(B) = N \int D\phi \exp\left(-\int_0^\beta d\tau \int d^3x L_{tot}(x,\tau)\right) \tag{5}$$

$$Z(B) = N'(\det W(B))^{-1/2}_{reg}[\det(-D_\mu(B)D_\mu(B+a))]_{a=\frac{\delta}{\delta J}}$$

$$\times\left\{1 + \sum_{l=1}^{\infty} \frac{S_{int}\left(a = \frac{\delta}{\delta J}\right)^l}{l!}\right\} \exp\left(-\frac{1}{2}JGJ\right)_{J_\mu = D_\mu(B)F_{\mu\nu}(B)}, \tag{6}$$

$$L_{tot}(x,\tau) = \sum_{i=1}^{8} L_i + L(j^{(n)}, a_\mu, \Psi, \Psi^+), \tag{7}$$

$$\begin{aligned} L_1 = \frac{1}{4}(F^a_{\mu\nu}(B))^2; L_2 = \frac{1}{2} a^a_\mu W^{ab}_{\mu\nu} a^b_\nu, \\ L_3 = \bar{\Theta}^a (D^2(B))_{ab} \Theta^b; L_4 = -ig\bar{\Theta}^a (D_\mu, a_\mu)_{ab} \Theta^b \\ L_5 = \frac{1}{2}\alpha(D_\mu(B) a_\mu)^2; L_6 = L_{int}(a^3, a^4) \\ L_7 = -a_\nu D_\mu(B) F_{\mu\nu}(B);\ L_8 = \Psi^+(m + \mu\gamma_4 + \hat{D}(B+a))\Psi \end{aligned} \tag{8}$$

$$L(j^{(n)}, a_\mu, \Psi, \Psi^+) = \int dx_1 dx_2 \{ j^{(gg)}_{\mu\nu}(x, x_1, x_2) \times$$

$$\times tr(a_\mu(x_1) \tilde{\Phi}(x_1, x_2) a_\nu(x_2)) +$$

$$+ j^{(q\bar{q})}_\lambda (x, x_1, x_2) tr(\Psi^+(x_1) \gamma_\lambda \Phi(x_1, x_2) \Psi(x_2))\} + ... \quad (9)$$

$$W^{ab}_{\mu\nu} = -D^2(B)_{ab} \cdot \delta_{\mu\nu} - 2g F^c_{\mu\nu}(B) f^{acb} \quad (10)$$

$$(D_\lambda)_{ca} = \partial_\lambda \delta_{ca} - ig T^b_{ca} B^b_\lambda \equiv \partial_\lambda \delta_{ca} - g f_{bca} B^b_\lambda \quad (11)$$

To lowest order in α_s, but all background kept (strong background and weak α_s)

$$F(T, \mu) = -T \ln \langle Z(B) \rangle_B . \quad (12)$$

4. Lowest order gluon and quark contribution.

The lowest order gluon contribution.

To lowest order in ga_μ (keeping all dependence on gB_μ explicit) we have

$$Z_0 = e^{-F_0(T)/T} = N' \langle \exp(-F_0(B)/T) \rangle_B, \quad (13)$$

where using Eq. (6) $F_0(B)$ can be written as

$$\begin{aligned} \frac{1}{T} F_0^{gl}(B) &= \frac{1}{2} \ln \det G^{-1} - \ln \det(-D^2(B)) = \\ &= Sp \left\{ -\frac{1}{2} \int_0^\infty \xi(s) \frac{ds}{s} e^{-sG^{-1}} + \int_0^\infty \xi(s) \frac{ds}{s} e^{sD^2(B)} \right\}. \end{aligned} \quad (14)$$

Sp: summation over coord., Lorenz, color.

FFSR: Ghost propagator:

$$(-\tilde{D}^2)^{-1}_{xy} = \langle x| \int_0^\infty ds e^{sD^2(B)} |y\rangle = \int_0^\infty ds (Dz)^w_{xy} e^{-K} \tilde{\Phi}(x,y). \quad (15)$$

$$\tilde{\Phi}(x,y) = P \exp(ig \int \tilde{B}_\mu(z) dz_\mu) \quad (16)$$

$$\begin{aligned}(Dz)^w_{xy} &= \lim_{N\to\infty} \prod_{m=1}^{N} \frac{d^4\zeta(m)}{(4\pi\varepsilon)^2} \\ & \sum_{n=0,\pm,\ldots} \frac{d^4p}{(2\pi)^4} \exp\left[ip_\mu \left(\sum_{m=1}^{N} \zeta_\mu(m) - (x-y)_\mu - n\beta\delta_{\mu 4}\right)\right].\end{aligned} \quad (17)$$

gluon propagator

$$G_{xy} = \int_0^\infty ds (Dz)^w_{xy} e^{-K} \tilde{\Phi}_F(x,y), \tag{18}$$

$$\tilde{\Phi}_F(x,y) = P_F P \exp\left(2ig\int_0^t \tilde{F}(z(\tau))d\tau\right) \exp\left(ig\int_y^x \tilde{B}_\mu dz_\mu\right). \tag{19}$$

$$\langle F_0^{gl}(B)\rangle_B = -T\int \frac{ds}{s}\xi(s) d^4x (Dz)^w_{xx} e^{-K} \times$$
$$\times \left[\frac{1}{2} tr\langle \tilde{\Phi}_F(x,x)\rangle_B - \langle tr \tilde{\Phi}(x,x)\rangle_B\right], \tag{20}$$

$B_\mu \equiv 0$, Stefan-Boltzmann.

$$F_0^{gl}(B=0) = -Tf(B=0) = -(N_c^2-1)V_3 \frac{T^4\pi^2}{45}. \tag{21}$$

The lowest order quark contribution

$$\det(m_q + \hat{D}(B' + a)) = [\det(m_q^2 - \hat{D}^2(B' + a))]^{1/2}, \tag{22}$$

$$\frac{1}{T} F_0^q(B') = \frac{1}{2} \ln \det(m_q^2 - \hat{D}^2(B')) = -\frac{1}{2} Sp \int_0^\infty \xi(s) \frac{ds}{s} e^{-sm_q^2 + s\hat{D}^2(B')}, \tag{23}$$

$$\hat{D}^2 = (D_\mu \gamma_\mu)^2 = D_\mu^2(B') - gF_{\mu\nu}\sigma_{\mu\nu} \equiv D^2 - g\sigma F;$$
$$B'_\mu = B_\mu - \frac{i\mu}{g}\delta_{\mu 4}, \quad \sigma_{\mu\nu} = \frac{1}{4i}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu). \tag{24}$$

$$\frac{1}{T} F_0^q(B') = -\frac{1}{2} tr \int_0^\infty \xi(s) \frac{ds}{s} d^4x \overline{(Dz)}_{xx}^w e^{-K - sm^2} W_\sigma(C_n), \tag{25}$$

$$W_\sigma(C_n) = P_F P_A \exp\left(ig\int_{C_n} B'_\mu dz_\mu\right) \exp g\int_0^s (\sigma_{\mu\nu}F_{\mu\nu})\, d\tau,$$

$$\overline{(Dz)}^w_{xy} = \prod_{m=1}^{N} \frac{d^4\zeta(m)}{(4\pi\varepsilon)^2} \sum_{n=0,\pm1,\pm2,\dots} (-1)^n \frac{d^4p}{(2\pi)^4} \times$$
$$\times \exp\left[ip\left(\sum_{m=1}^{N}\zeta(m) - (x-y) - n\beta\delta_{\mu4}\right)\right]. \qquad (26)$$

For a nonzero μ and n_f massless flavors with $B_\mu \equiv 0$ one has

$$\langle F\rangle = -\frac{4N_c V_3 T^4}{\pi^2} n_f \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4} cosh\left(\frac{\mu n}{T}\right). \qquad (27)$$

Single Line Approximation

Using $P_{gl}V_3 = -\langle F_0(B)\rangle_B$

$$P_{gl} = (N_c^2 - 1)\int_0^\infty \frac{ds}{s}\sum_{n\neq 0} G^{(n)}(s) \tag{28}$$

$$P_q = 2N_c\int_0^\infty \frac{ds}{s}e^{-m_q^2 s}\sum_{n=1}^{\infty}(-1)^{n+1}[S^{(n)}(s) + S^{(-n)}(s)]. \tag{29}$$

$$G^{(n)}(s) = \int (Dz)_{on}^{w} e^{-K}\hat{tr}_a\langle W_\Sigma(C_n)\rangle \tag{30}$$

$$S^{(n)}(s) = \int \overline{(Dz)}_{on}^{w} e^{-K}\hat{tr}_f\langle W_\sigma(C_n)\rangle \tag{31}$$

$$\hat{tr}_f W_\sigma(C_n) = \frac{1}{N_c}tr_f W_\sigma(C_n) \tag{32}$$

and

$$\hat{tr}_a\langle W_\Sigma(C_n)\rangle = \frac{tr_a}{(N_c^2-1)}(\frac{1}{2}\tilde{\Phi}_F(C_n) - \tilde{\Phi}(C_n)) \quad (33)$$

Expansion in Field Correlators –take Gaussian approximation.

$$\hat{tr}_f\langle W(C_n)\rangle = \frac{tr_c}{N_c}\langle P\exp ig\int_{C_n} B_\mu dz_\mu\rangle =$$

$$= \frac{tr_c}{N_c}\exp\left(-\frac{g^2}{2}\int_{S_n}\int_{S_n} d\sigma_{\mu\nu}(u)d\sigma_{\lambda\sigma}(v)\langle F_{\mu\nu}(u)F_{\lambda\sigma}(v)\rangle\right). \quad (34)$$

Resulting pressure in SLA is (we neglect colormagnetic contribution and write $G_3^{(0)}(s) = \frac{1}{(4\pi s)^{3/2}} = S_3^{(0)}(s)$

$$P_{gl}^{(0)} = \frac{(N_c^2-1)}{2\sqrt{\pi}}\int_0^\infty \frac{ds}{s^{3/2}}G_3(s)\sum_{n\neq 0} e^{-\frac{n^2}{4T^2 s}} L_{adj}^{(n)} =$$

$$= \frac{(N_c^2-1)}{16\pi^2}\int_0^\infty \frac{ds}{s^3}\sum_{n\neq 0} e^{-\frac{n^2}{4T^2 s}} e^{-\tilde{J}_n^E} \quad (35)$$

$$P_q^{(0)} = \frac{2N_c}{\sqrt{\pi}} n_f \int_0^\infty \frac{ds}{s^{3/2}} e^{-m_q^2 s} S_3(s) \sum_{n=1}^\infty (-1)^{n+1} e^{-\frac{n^2}{4T^2 s}} L_{fund}^{(n)} cosh \frac{\mu n}{T} =$$

$$= n_f \frac{N_c}{4\pi^2} \int_0^\infty \frac{ds}{s^3} e^{-m_q^2 s} \sum_{n=1}^\infty (-1)^{n+1} e^{-\frac{n^2}{4T^2 s}} e^{-J_n^E} cosh \frac{\mu n}{T}, \quad (36)$$

Check: free gas: $m_q = 0, \mu = 0, L_i = 1.$

Using

$$\sum_{n=1}^\infty \frac{1}{n^4} = \frac{\pi^4}{90}, \quad \sum_{n=1}^\infty \frac{(-1)^{n+1}}{n^4} = \frac{7\pi^4}{720}. \quad (37)$$

one has Stefan-Boltzmann result

$$P_{gl}^{SB}(\sigma \to 0) = \frac{\pi^2 (N_c^2 - 1) T^4}{45}, \quad P_q^{SB}(\sigma \to 0) = \frac{7\pi^2 N_c T^4}{180} n_f, \quad (38)$$

Taking L_i into account and $L_i^{(n)} = (L_i)^n$ for $T < 1$ GeV $= \frac{1}{\lambda_{gl}}$

$$P_{gl}^{(0)} = \frac{(N_c^2-1)T^4}{\pi^2} \sum_{n\neq 0} \frac{(L_{adj})^{|n|}}{n^4} \approx \frac{2(N_c^2-1)}{\pi^2} L_{adj} T^4 \tag{39}$$

$$P_q^{(0)} = \frac{4N_c n_f}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^4} \varphi_q^{(n)}(T) cosh \frac{\mu n}{T} \approx$$

$$\approx \frac{4N_c}{\pi^2} n_f L_{fund} T^4 \varphi_q^{(1)}(T) cosh \frac{\mu}{T} \tag{40}$$

$$\varphi_q^{(n)}(T) = \frac{1}{16T^4} \int_0^\infty \frac{ds}{s^3} e^{-m_q^2 s} e^{-\frac{1}{4T^2 s}} \cong$$

$$\cong \frac{n^2 m_q^2}{2T^2} K_2\left(\frac{m_q n}{T}\right), \tag{41}$$

$$P_g = T^4 p_g; \quad P_q = T^4 p_q;$$

$$p_g = \frac{16}{\pi^2} L_{adj}(T) = \frac{16}{\pi^2} \exp\left(-\frac{9V_1(\infty, T)}{8T}\right) \tag{42}$$

$$p_q = \frac{12 n_f}{\pi^2} \sum_{n=1}^{\infty} \frac{(-)^{n+1}}{n^4} (L_f)^n \varphi_q^{(n)}(T) ch \frac{\mu n}{T} \tag{43}$$

$$m_q = 0$$

$$p_q(m_q = 0) = \frac{12 n_f}{\pi^2} \sum_{n=1}^{\infty} \frac{(-)^{n+1}}{n^4} L_f^n ch \frac{\mu n}{T} =$$

$$= \frac{n_f}{\pi^2} \left[\Phi_-^{(3)} \left(\frac{\mu - \frac{V_1}{2}}{T} \right) + \Phi_+^{(3)} \left(\frac{\mu + \frac{V_1}{2}}{T} \right) \right] \tag{44}$$

$$T_c = \left[\frac{|\frac{\varepsilon}{2}| + \chi_1(T)}{p_g + p_q}\right]^{1/4}$$

$$\Phi_-^{(k)}(z) = \int_0^\infty \frac{x^k dx}{e^{x-z}+1}; \quad \Phi_+^{(k)}(z) = \int_0^\infty \frac{x^k dx}{e^{x+z}+1}; \tag{45}$$

$$|L_{fund}| = \exp\left(-\frac{V_1}{2T}\right).$$

Properties of Polyakov loops

$$G^{(n)}(s) = \frac{1}{2\sqrt{\pi s}} e^{-\frac{n^2\beta^2}{4s}} G_3(s) L^{(n)}_{adj} \tag{46}$$

$$L^{(n)}_{adj} \equiv \exp(-\tilde{J}^E_n), \quad \tilde{J}^E_n = \frac{9}{4} J^E_n \tag{47}$$

$$J^E_n = \frac{n\beta}{2} \int_0^{n\beta} d\nu \left(1 - \frac{\nu}{n\beta}\right) \int_0^\infty \xi d\xi D^E_1(\sqrt{\nu^2 + \xi^2}). \tag{48}$$

$$L_{fund} = \exp\left(-\frac{1}{2T} V_1(\infty)\right), \quad \frac{1}{2T} V_1(\infty) \equiv J^E_1 \tag{49}$$

$$D^E_1(x) = D^E_{1pert}(x) + D^E_{1nonp.}(x) \tag{50}$$

$$D^E_{1nonp}(x) \approx const \exp(-M_1|x|), |x| \gtrsim 1/M_1 \tag{51}$$

$$D^E_{1\ pert.} = \frac{4C_2\alpha_s}{\pi x^4} + O(\alpha_s^2)$$

Renormalization (similar to lattice)

$$V_1(r,T) = V_1^{pert}(r,T) + V_1^{(np)}(r,T) + V_1^{(div)}(a), \qquad (52)$$

$$V_1^{(pert)}(r,T) = -\frac{C(f)\alpha_s}{r} e^{-m_D r}(1 + O(rT)), \qquad (53)$$

$$V_1^{(div)}(a) \cong \frac{2C(f)\alpha_s}{\pi}\left(\frac{1}{a} + O(Tlna)\right). \qquad (54)$$

The renormalization amounts to discarding $V_1^{(div)}(a)$ – on the lattice equiv. to keeping $V_1^{(pert)}$ at small r.

$$L^{ren}_{fund} \equiv \exp\left(-\frac{V_1^{(np)}(\infty)}{2T}\right) \qquad (55)$$

Below and above $\mathbf{T_c}$:

(also D^E is nonzero below T_c)

$$V_D(r,T) = 2\int_0^\beta d\nu(1-\nu T)\int_0^r (r-\xi)d\xi D^E(\sqrt{\xi^2+\nu^2}) \quad (56)$$

$$L_{fund} = \exp\{-(V_D(r_1^*,T) + \frac{1}{2}V_1^{np}(\infty,T))/T\}, \quad L_{adj} = (L_{fund})^{9/4} \quad (57)$$

r_1^* – average radius of heavy-light meson (of gluelump for L_{adj})

$r^* \approx 0.5$ fm (0.3 fm) for $fund$ (adj)

$$V_1^{(np)}(\infty,T) = \int_0^\beta d\nu(1-\nu T)\int_0^\infty \xi d\xi D_{1np}(\sqrt{\xi^2+\nu^2});$$

Check 1: Casimir scaling: $(V_D, V_1)_{adj} = \frac{9}{4}(V_D, V_1)_{fund}$;

for higher repr. of $SU(3)_c$; $\frac{9}{4} \to \frac{C_2(j)}{C_2(fund)}$

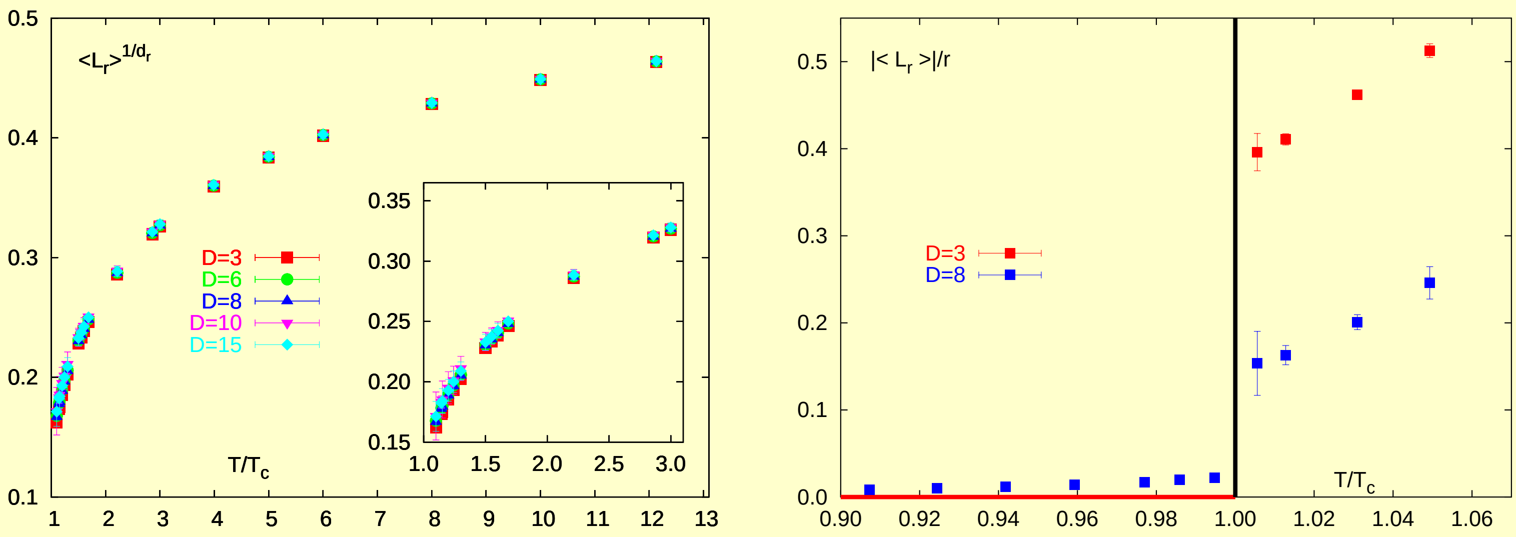
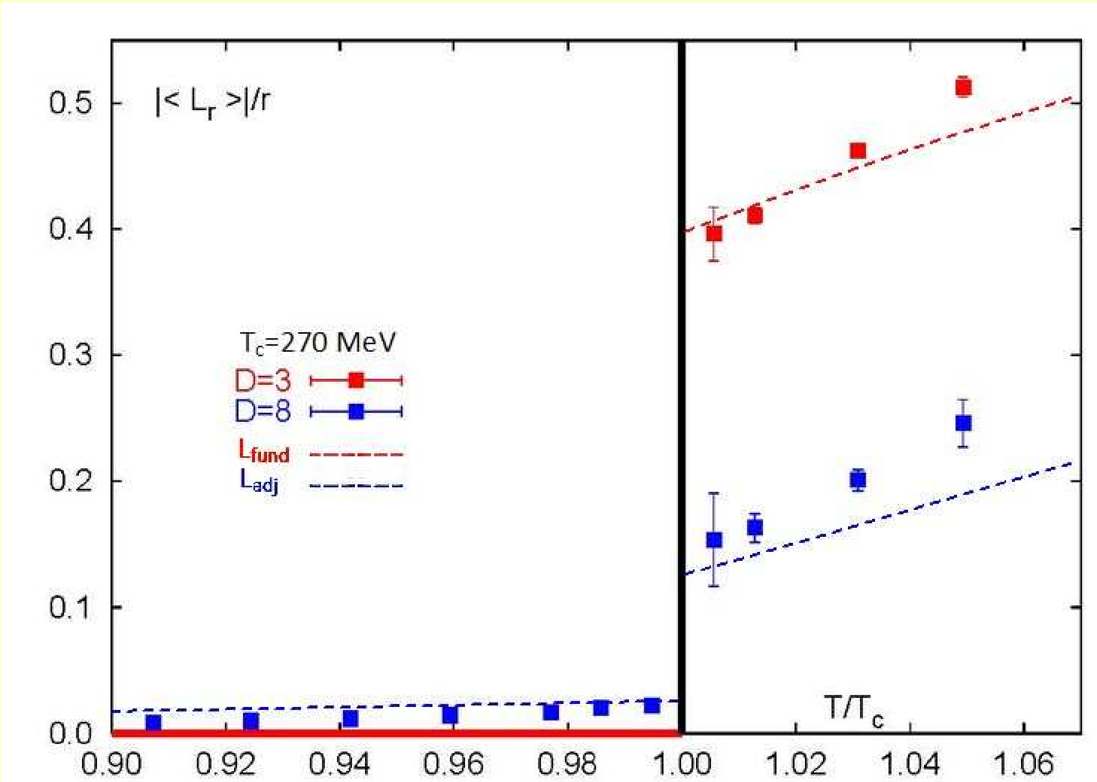


Figure 5: Left: bare Polyakov loops for $r \leq 15$ with cubic spline above T_c. Right: renormalized fundamental and adjoint Polyakov loop for temperatures around T_c.



|< L_r >|/r
T_c=270 MeV
D=3
D=8
L_fund
L_adj
T/T_c
0.0
0.1
0.2
0.3
0.4
0.5
0.90
0.92
0.94
0.96
0.98
1.00
1.02
1.04
1.06

Figure 6:

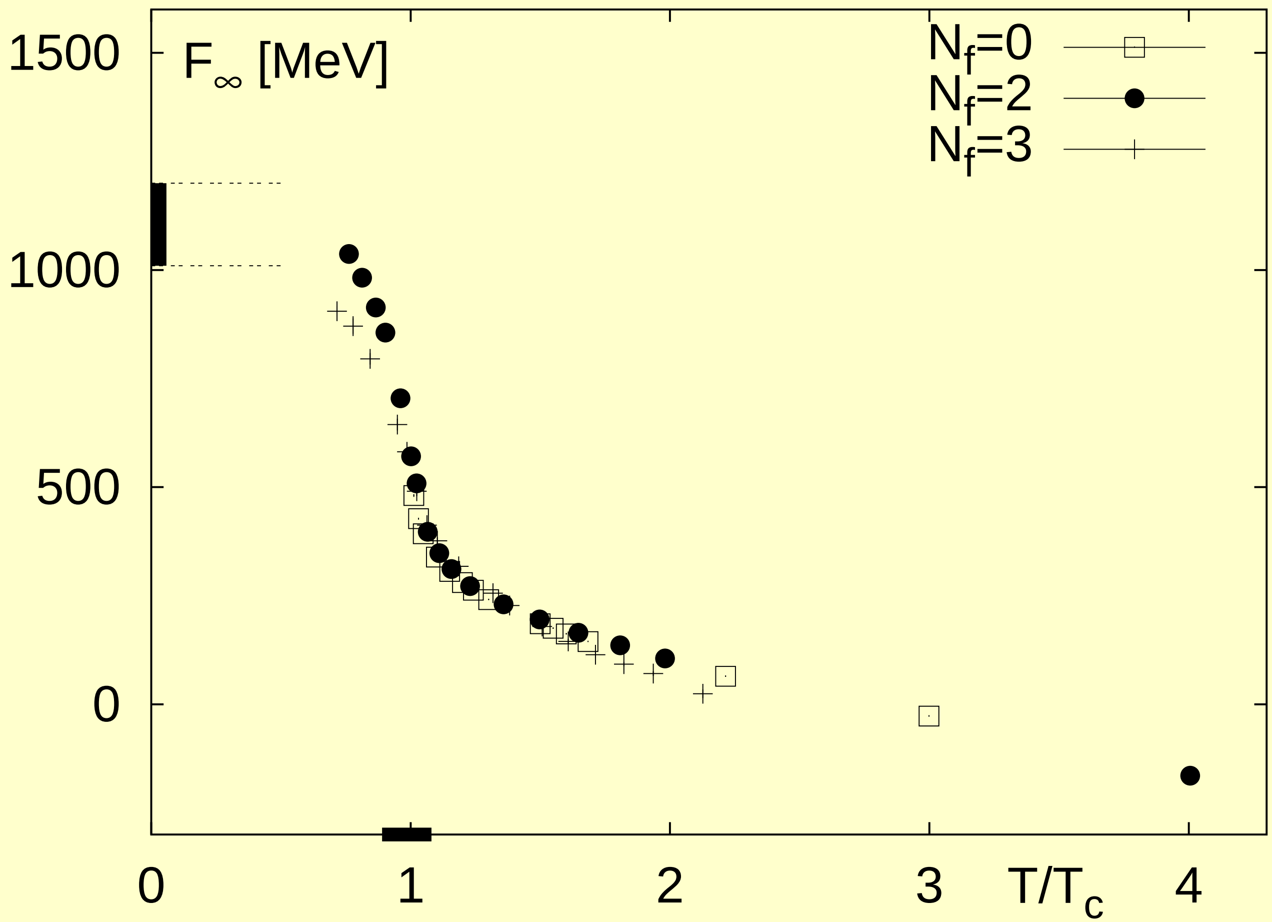


Figure 7:

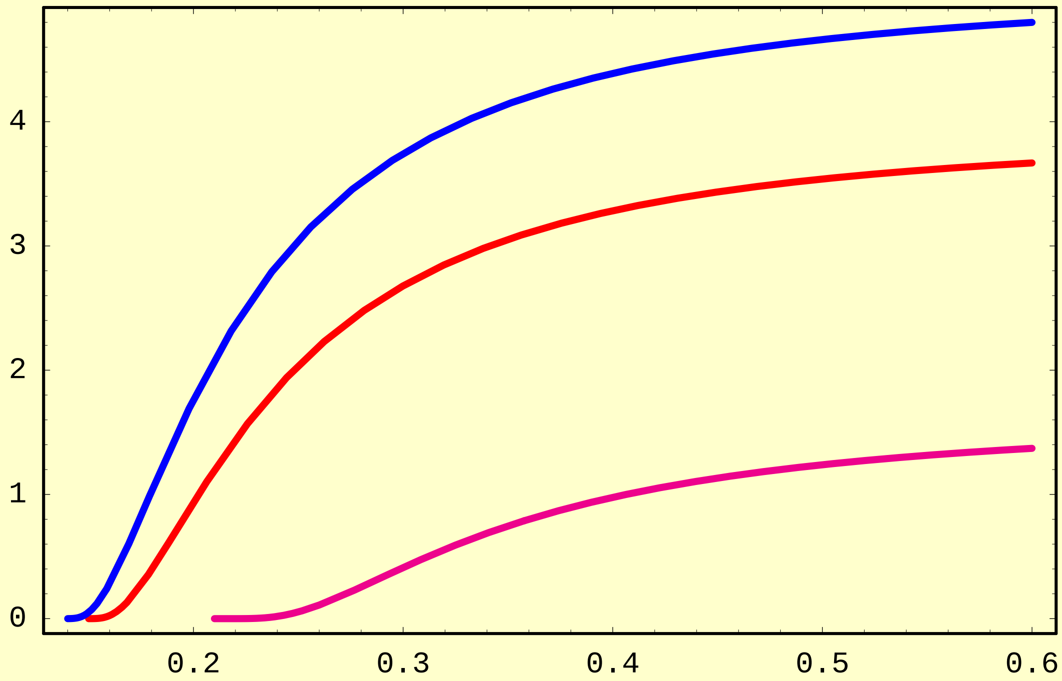
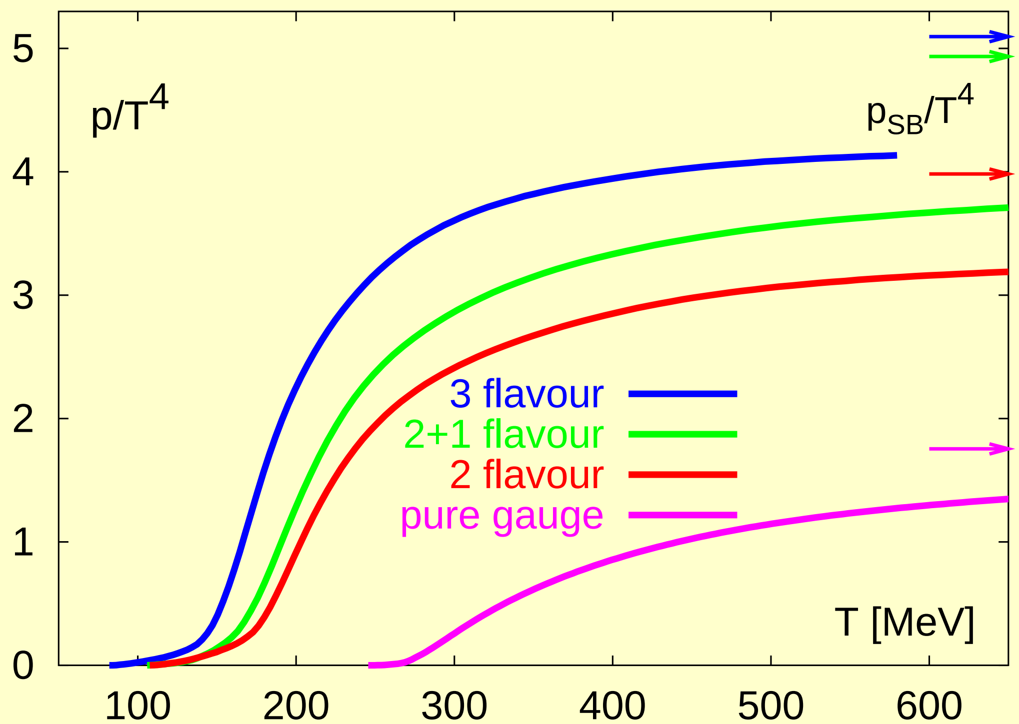
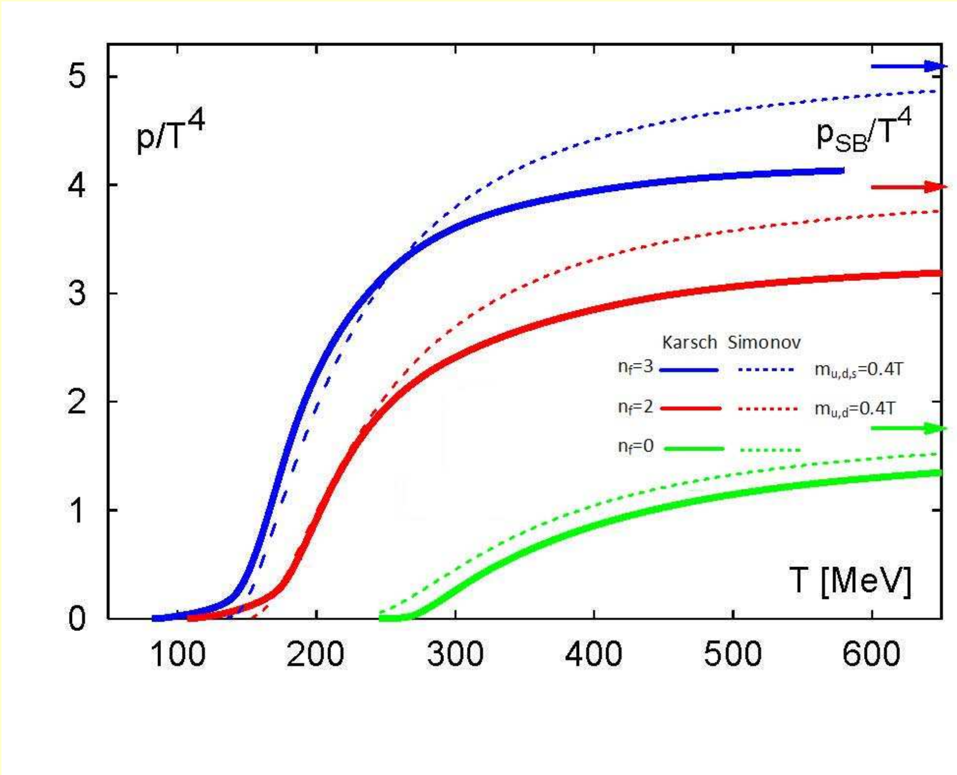


Figure 8: Pressure $\frac{P}{T^4}$ from Eq.(**??**,**??**) as function of temperature T (in GeV) for $n_f = 3, 2, 0$ (top to bottom) and $\Delta G_2 = 0.0034$ GeV^4.



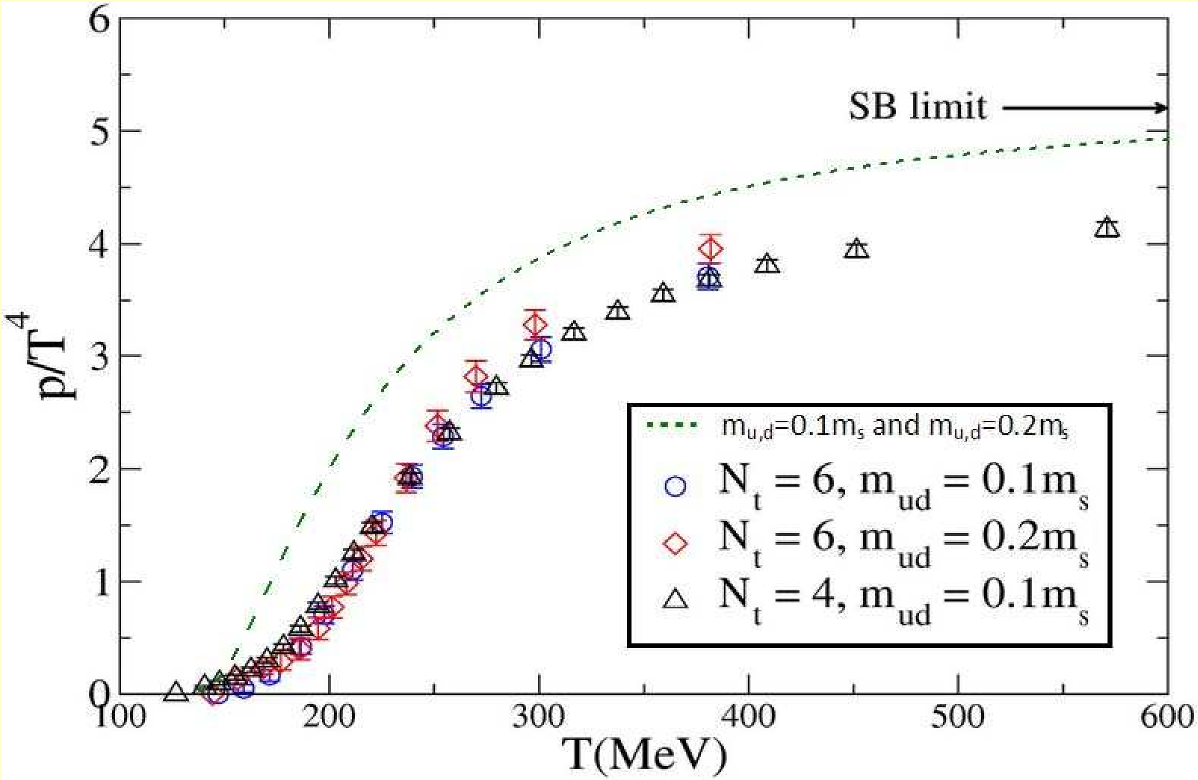
p/T4
pSB/T4
3 flavour
2+1 flavour
2 flavour
pure gauge
T [MeV]
0
1
2
3
4
5
100
200
300
400
500
600

Figure 9:



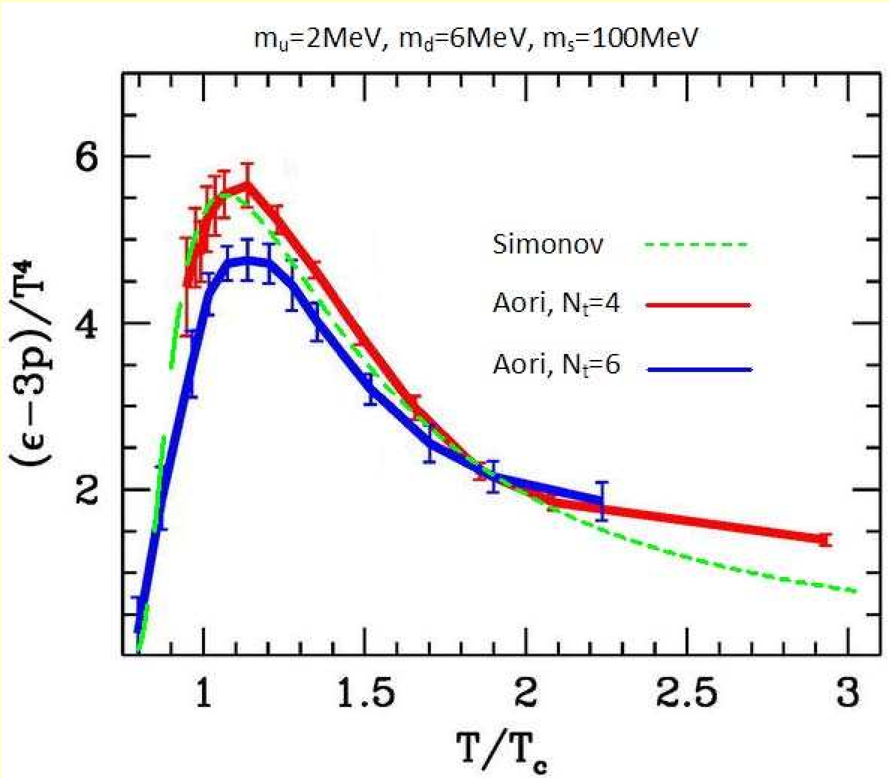
5
4
3
2
1
0
p/T4
pSB/T4
Karsch Simonov
nf=3
mu,d,s=0.4T
nf=2
mu,d=0.4T
nf=0
T [MeV]
100
200
300
400
500
600

Figure 10:



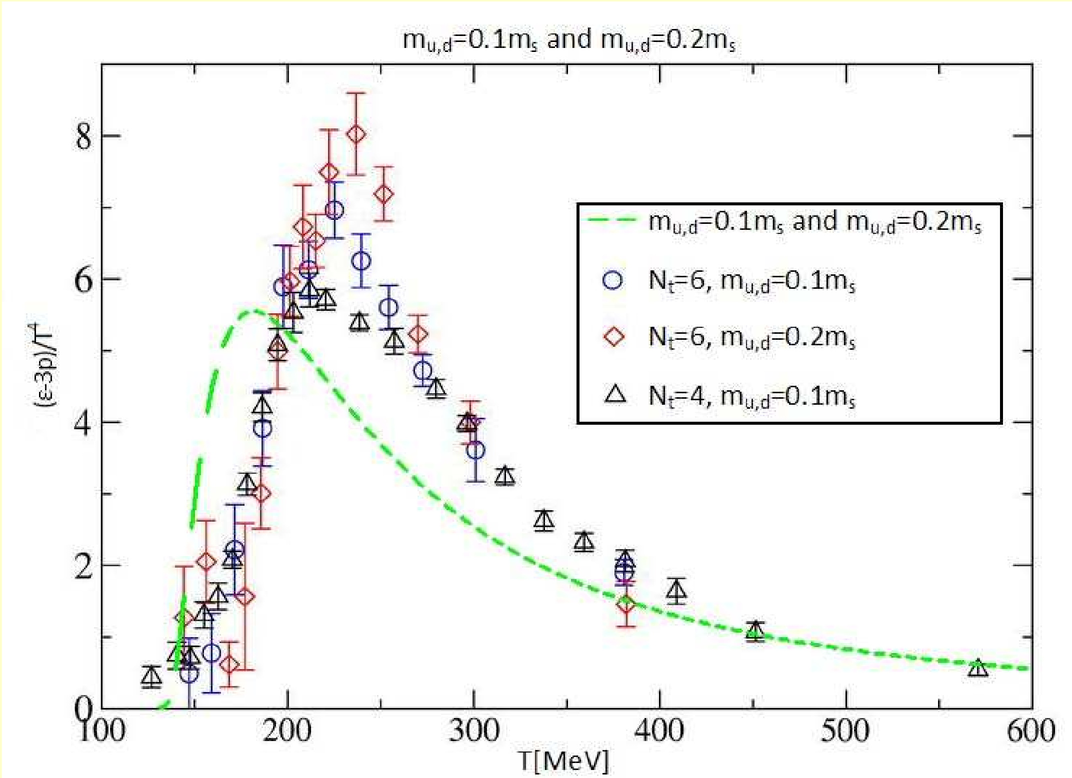
p/T^4
T(MeV)
SB limit
m_{u,d}=0.1m_s and m_{u,d}=0.2m_s
N_t = 6, m_ud = 0.1m_s
N_t = 6, m_ud = 0.2m_s
N_t = 4, m_ud = 0.1m_s

Figure 11:



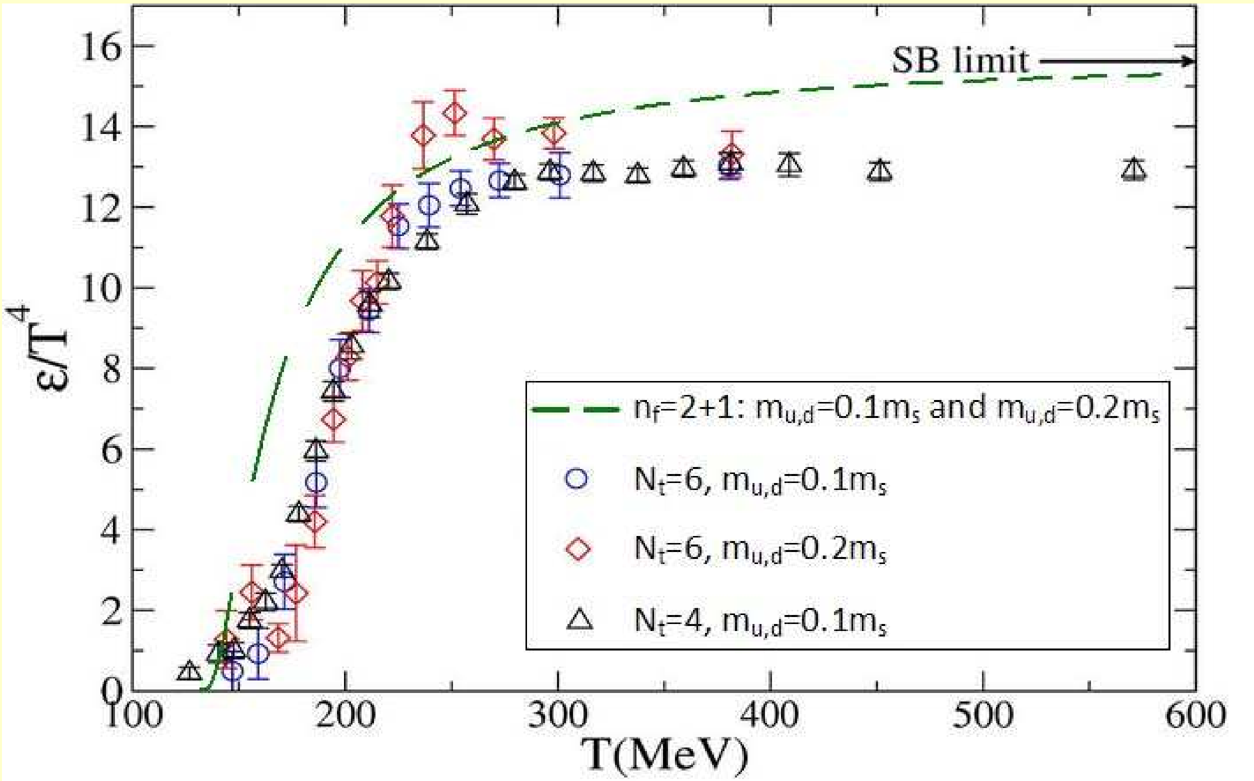
m_u=2MeV, m_d=6MeV, m_s=100MeV
(ε−3p)/T⁴
T/T_c
Simonov
Aori, N_t=4
Aori, N_t=6
6
4
2
1
1.5
2
2.5
3

Figure 12:



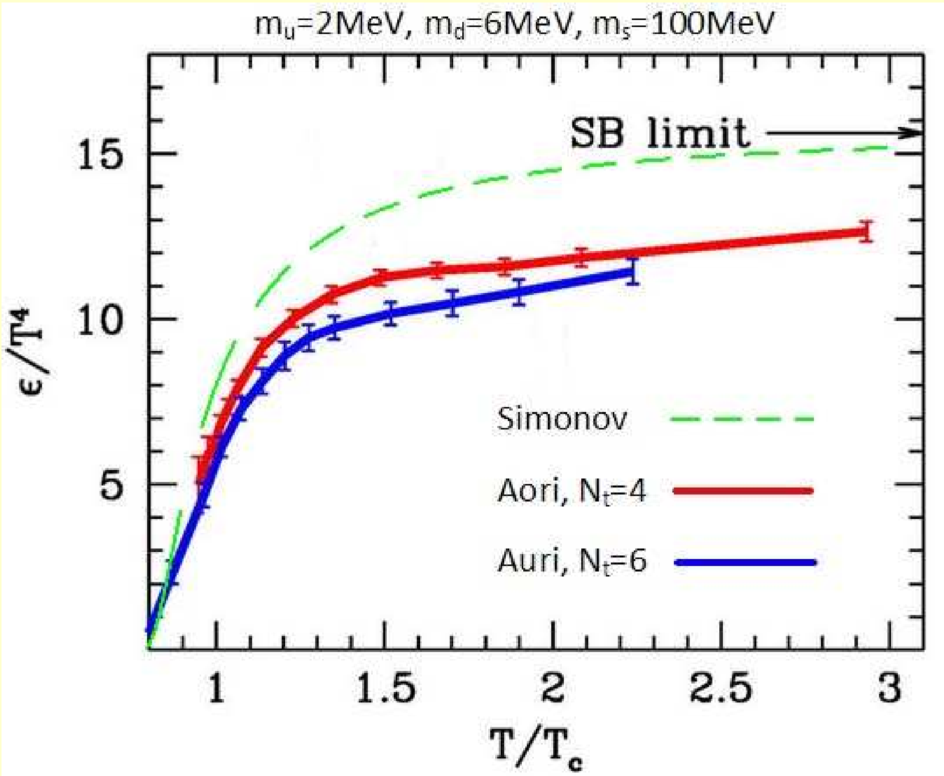
m_{u,d}=0.1m_s and m_{u,d}=0.2m_s
(ε-3p)/T^4
T[MeV]
m_{u,d}=0.1m_s and m_{u,d}=0.2m_s
N_t=6, m_{u,d}=0.1m_s
N_t=6, m_{u,d}=0.2m_s
N_t=4, m_{u,d}=0.1m_s
0
2
4
6
8
100
200
300
400
500
600

Figure 13:



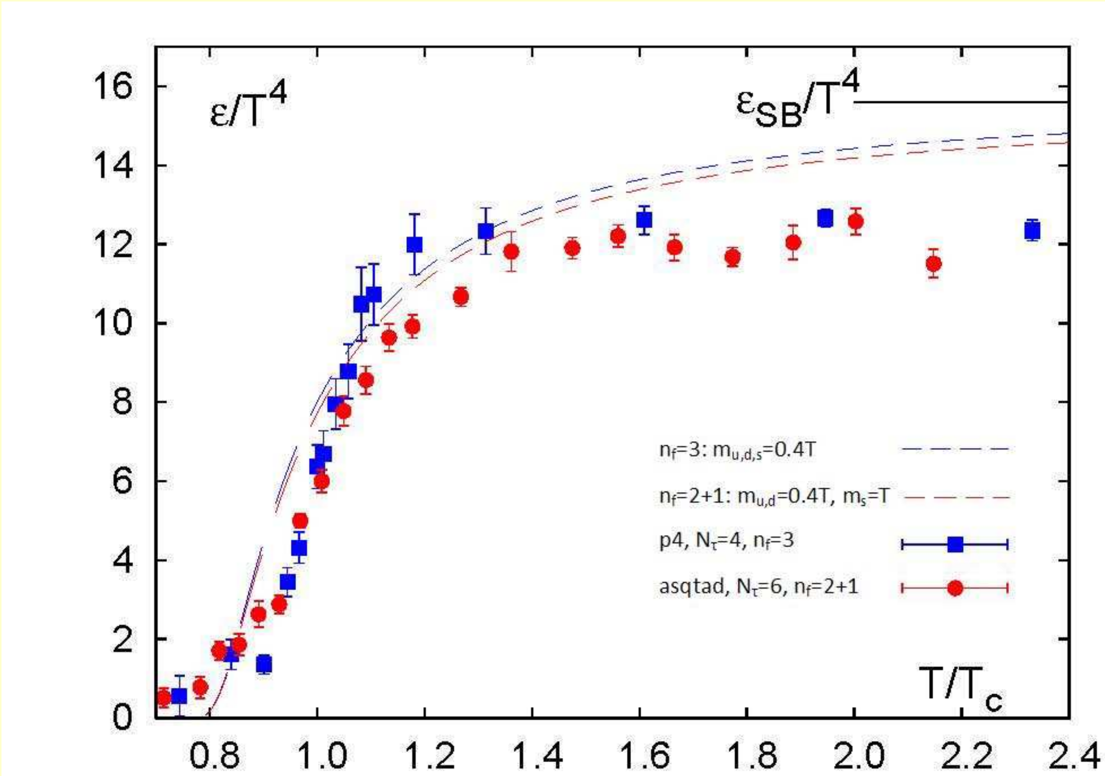
SB limit
n_f=2+1: $m_{u,d}$=0.1m_s and $m_{u,d}$=0.2m_s
N_t=6, $m_{u,d}$=0.1m_s
N_t=6, $m_{u,d}$=0.2m_s
N_t=4, $m_{u,d}$=0.1m_s
ε/T^4
T(MeV)

Figure 14:



m_u=2MeV, m_d=6MeV, m_s=100MeV
SB limit
15
10
5
ε/T⁴
Simonov
Aori, N_t=4
Auri, N_t=6
1
1.5
2
2.5
3
T/T_c

Figure 15:



ε/T4
εSB/T4
nf=3: mu,d,s=0.4T
nf=2+1: mu,d=0.4T, ms=T
p4, Nτ=4, nf=3
asqtad, Nτ=6, nf=2+1
T/Tc

Figure 16:

Phase transition: Equation for $T_c(\mu)$ M.A.Trusov and Yu.S.

$$SVZ \quad \varepsilon_{vac} = 1/4\theta_{\mu\mu} = \frac{\beta(\alpha_s)}{16\alpha_s}\langle (F^a_{\mu\nu})^2\rangle \cong -\frac{(11-\frac{2}{3}n_f)}{32} G_2^{(n_f)} \qquad (58)$$

$$(NSVZ) \quad G_2^{(n_f=2)} \approx \left(\frac{1}{3} \div \frac{1}{4}\right) G_2^{(n_f=0)} \qquad (59)$$

$$G_2(0.02 \pm 0.005)\ GeV^4 \ \text{S.Narison}$$
$$G_2(0.01 \pm 0.002)\ GeV^4 \ \text{Andreev, Zakharov} \qquad (60)$$

$$P_1(T) = |\varepsilon_{vac}| + \frac{\pi^2}{30}T^4 + T\sum_k \frac{(2m_k T)^{3/2}}{8\pi^{3/2}} e^{-m_k/T} \equiv |\varepsilon_{vac}| + T^4\chi_1(T). \qquad (61)$$

In the deconfined phase one can assume (later confirmed by lattice) (Yu.S. JETP Lett.'92), that

$$D^E(x) = 0 = \sigma_E; \quad D^H(x), D_1^H, D_1^E \neq 0. \qquad (62)$$

$$P_2(T) = |\varepsilon_{vac}^{dec}| + T^4(p_{gl} + p_q) \tag{63}$$

Critical line $T_c(\mu)$

$$P_I = |\varepsilon_{vac}| + \chi_1(T) \to \frac{11}{32} G_2$$

$$P_{II} = \frac{11}{32} G_2^{dec} + (p_{gl} + p_q) T^2;$$

$$P_I(T_c) = P_{II}(T_c)$$

$$T_c(\mu) = \left(\frac{\frac{11}{32} \Delta G_2}{p_{gl} + p_q} \right)^{1/4},$$

within 10% $\Delta G_2 \approx \frac{1}{2} G_2$

$$P_{gl}(T_c) = \frac{16}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^4} L_{adj}^n \to \frac{16}{\pi^2} L_{adj}$$

$$p_q(T_c) = \frac{n_f}{\pi^2} \left[\Phi_\nu \left(\frac{\mu - \frac{V_!(T_C)}{2}}{T_c} \right) + \Phi_\nu \left(- \frac{\mu + \frac{V_!(T_C)}{2}}{T_c} \right) \right]$$

$$\Phi_\nu(a) = \int_0^\infty \frac{z^4 dz}{\sqrt{z^2 + \nu^2} \left(e^{\sqrt{z^2+\nu^2} - a} + 1 \right)}$$

$\nu = \frac{m_q}{T}$. Take $m_q = 0$.

Only 2 parameters (input)

ΔG_2 and $V_1(T_c)$.

$$1.\Delta G_2 = G_2(E,H) - G_2(0,H) \approx \frac{1}{2} G_2(E,H)(10\% acc)$$

$$G_2 = \frac{\pi^2}{36}(D^E(0) + D_1^E(0) + D^H(0) + D_1^H(0));$$

$$D_1^E(0) \approx 0.2 D^E(0) \quad (T=0)$$

2. $V_1(T_c) \cong 0.5$ GeV (lattice, analytic) within 10%.

Possible dependence on μ: weak for $\mu \ll$ dilaton mass ≈ 1.5 GeV

lattice data: $V_1 \Rightarrow F^1_{Q\bar{Q}}$

Two limiting cases for $T_c(\mu)$

1). $T_c(\mu \to 0)$: Expanding in $\frac{V_1(T_c)}{8T_c}$

$$T_c = T^{(0)} \left(1 + \frac{V_1(T_c)}{8T_c} + O\left(\frac{V_1(T_c)}{8T_c}\right)^2 \right)$$

with 3% accuracy

$$T_c(0) \approx \frac{1}{2} T^{(0)} \left(1 + \sqrt{1 + \frac{\kappa}{T^{(0)}}} \right), \quad T^{(0)} = \left(\frac{(11 - \frac{2}{3} n_f) \pi^2 \Delta G_2}{384 n_f} \right)^{1/4}. \tag{64}$$

$$\kappa \equiv \frac{1}{2} V_1(\infty, T_c) \cong \frac{1}{2} F^1_{Q\bar{Q}}(\infty, T_c) = 0.25 \;\; GeV.$$

2). End-point: $\mu_c(T \to 0)$

Using asymptotics

$$\Phi_0(a \to \infty) = \frac{a^4}{4} + \frac{\pi^2}{2}a^2 + \frac{7\pi^4}{60} + ...$$

one has

$$\mu_c(T \to 0) = \frac{V_1(T_c)}{2} + (48)^{1/4} T^{(0)} \left(1 - \frac{\pi^2}{2} \frac{T^2}{(\mu_c - \frac{V_1(T_c)}{2})} + \right)$$

For $V_1(T_c) = 0.5$ GeV and $n_f = 2$ one has.

$\frac{\Delta G_2}{0.01\ \mathrm{GeV}^4}$	0.191	0.341	0.57	1
$T_c(\ \mathrm{GeV}) \quad n_f = 0$	0.246	0.273	0.298	0.328
$T_c(\ \mathrm{GeV}) \quad n_f = 2$	0.168	0.19	0.21	0.236
$T_c(\ \mathrm{GeV}) \quad n_f = 3$	0.154	0.172	0.191	0.214
$\mu_c(\ \mathrm{GeV}) \quad n_f = 2$	0.576	0.626	0.68	0.742
$\mu_c(\ \mathrm{GeV}) \quad n_f = 3$	0.539	0.581	0.629	0.686

Check of predictions vs other data

Possible order parameters are

1. $$\langle\bar{\psi}\psi\rangle,\ \chi_{chiral} = \frac{\partial^2}{\partial m_q^2}\frac{T}{V}\ln Z \approx \frac{\partial}{\partial m_q}\langle\bar{\psi}\psi\rangle \tag{65}$$

2. $$\sigma_E(T) \tag{66}$$

3. $$L_{fund},\quad \chi_L \equiv \langle L_{fund}^2\rangle - \langle L_{fund}\rangle^2$$

$$L_{fund} = \frac{1}{N_c}\langle tr_c P\exp(ig\int_0^{1/T} A_4 dz_4)\rangle \tag{67}$$

Predictions: $\sigma_E(T), \langle\bar{\psi}\psi\rangle$ nonzero for $T < T_c$.

A $V_D(\infty, T) = \infty \to L_{fund} = 0$ for $T < T_c(n_f = 0)$.

B For $T > T_c$ $\sigma_E(T), \langle\bar{\psi}\psi\rangle$ are zero, L_{fund} nonzero.

C $L_{adj} = (L_{fund})^{9/4}$ since $D^E, P_1^E \sim C_2(adj, fund)$

D First order transition; $T_{deconf} = T_{chiral} = T_c$

comparison to lattice

$n_f = 0$

Kaczmarek et al:calculation of L_{fund}, L_{adj}

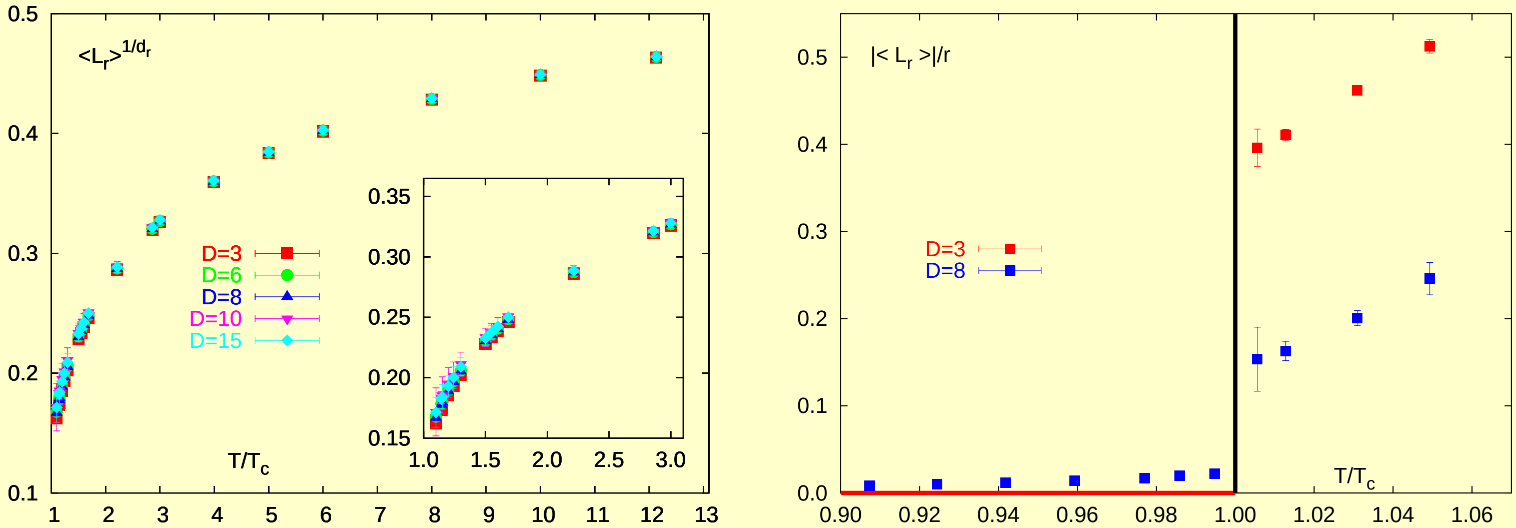


Figure 17: Left: bare Polyakov loops for $r \leq 15$ with cubic spline above T_c. Right: renormalized fundamental and adjoint Polyakov loop for temperatures around T_c.

Agreement: First order transition; Casimir scaling observed

$n_f = 2$ (Di Giacomo et al.)

$$C_V - C_0 = L_s^{\frac{\alpha}{\nu}} \Phi_C(\tau L_s^{\frac{1}{\nu}}, m L_s^{y_h}) \quad (68)$$

and

$$\chi_{\langle\bar{\psi}\psi\rangle} - \chi_0 = L_s^{\frac{\gamma}{\nu}} \Phi_{\langle\bar{\psi}\psi\rangle}(\tau L_s^{\frac{1}{\nu}}, m L_s^{y_h}) \quad (69)$$

	y_h	ν	α	γ	δ
$O(4)$	2.487(3)	0.748(14)	-0.24(6)	1.479(94)	4.852(24)
$O(2)$	2.485(3)	0.668(9)	-0.005(7)	1.317(38)	4.826(12)
MF	$9/4$	$2/3$	0	1	3
$1^{st} Order$	3	$1/3$	1	1	∞

$\tau = 1 - \frac{T}{T_c}, \alpha, \gamma, \nu, y_h$ -crit. indices

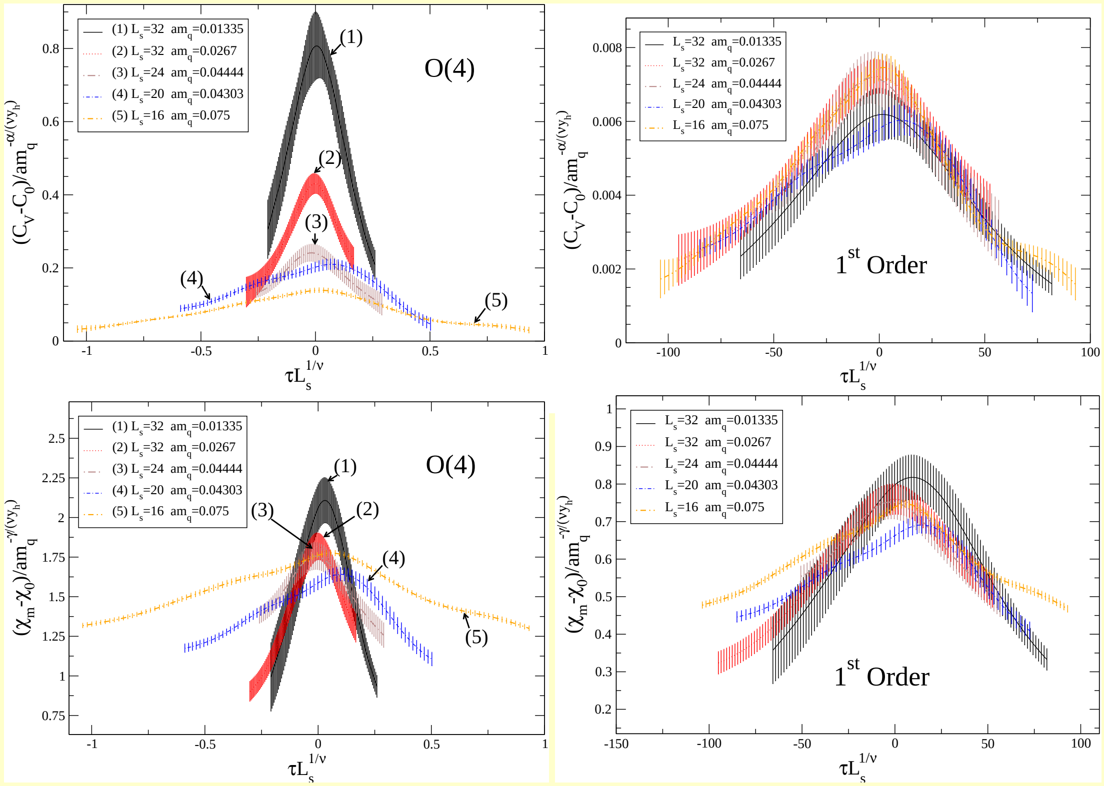


Figure 18: Scaling Eqs. (19),(20) for second order $O(4)$ (left) and first order (right)

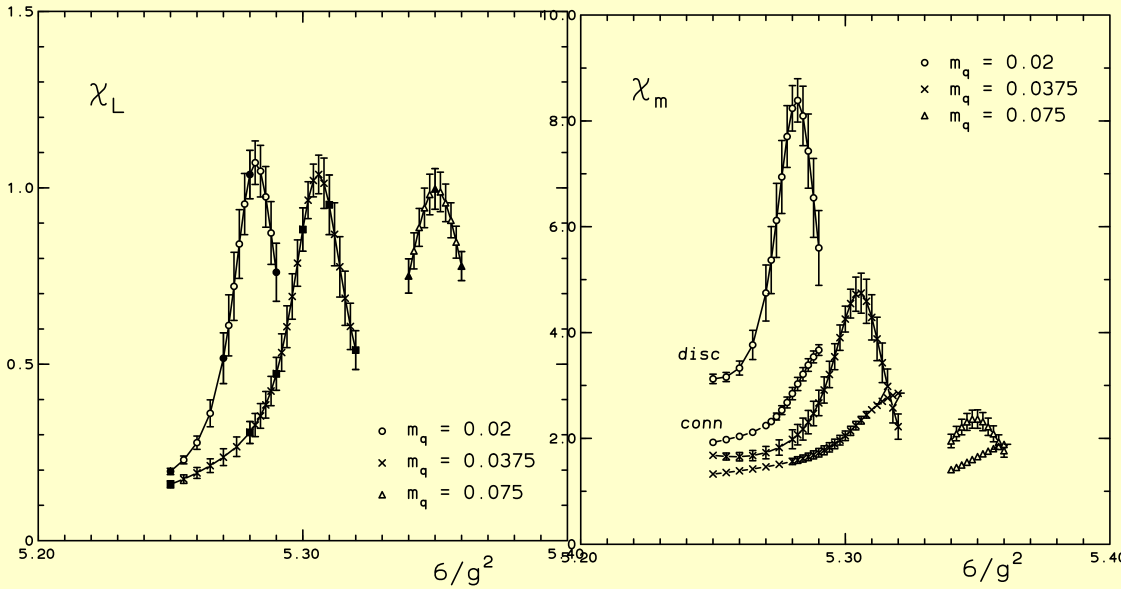


Figure 19: Polyakov loop and chiral susceptibilities versus $\beta = 6/g^2$ in 2-flavour QCD for several values of the quark mass.

One can see coincidence of T_{crit} for each m_q

$n_f = 3$

$m_q = 0$ 1st order

$m_q \neq 0$ crossover

$m_q \to \infty$ 1st order

Conclusions depend on type of quarks used (Wilson, staggered) and discretization.

The slope of $T_c(\mu)$

$$T_c(\mu_B) = T_c(0) - C\frac{\mu_B^2}{T_c^2}, \quad \mu_B = 3\mu_q$$

From general Eq. for $\mu \to 0$

$$p_q(\mu) = \frac{12}{\pi^2} L_{fund} ch\frac{\mu}{T} \cong p_q(0) - (\frac{\mu}{T})^2 \frac{6}{\pi^2} L_{fund}$$

$$T_c(\mu) = \left(\frac{|\Delta \varepsilon_{vac}|}{p_{gl} + p_q(\mu)} \right)^{1/4}$$

One obtains $\kappa \equiv \frac{1}{2} V_1(T_c) \cong 0.25$ GeV

$$C = \frac{1 + \sqrt{1 + \frac{\kappa}{T^{(0)}}}}{144 \sqrt{1 + \frac{\kappa}{T^{(0)}}}} = 0.0110(3) \quad \text{for } n_f = 2, 3, 4.$$

Comparison to lattice data

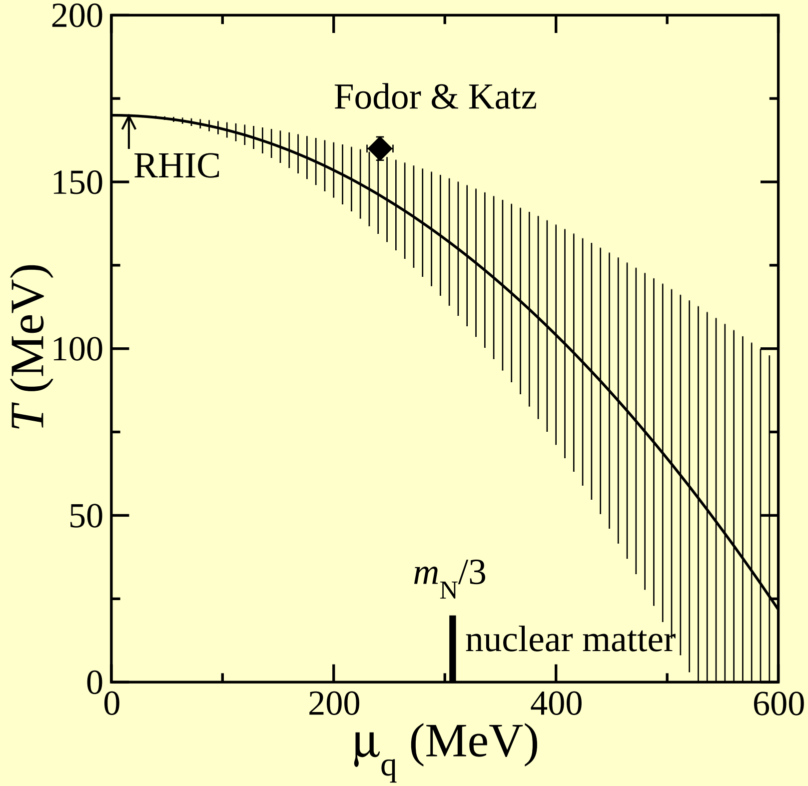


Figure 20: The phase diagram obtained from our reweighting technique. The errors shown are statistical. The diamond is the endpoint of the first order phase transition obtained by Fodor and Katz

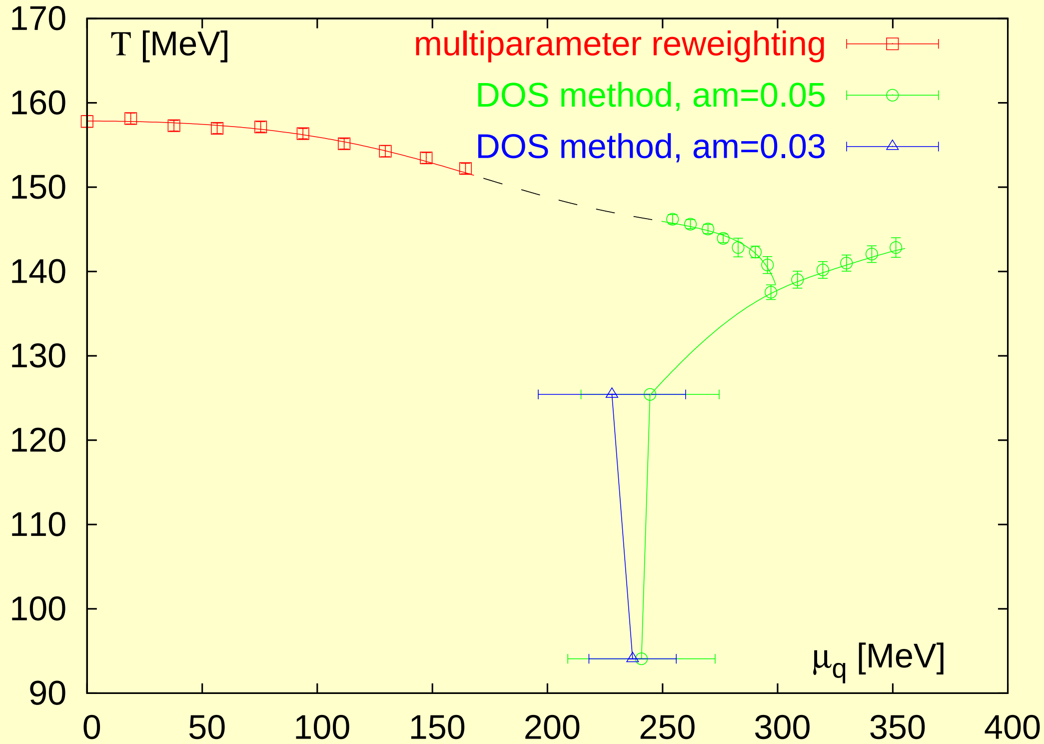


Figure 21: The phase diagram in physical units from $N_t = 6$ and 8 lattices.

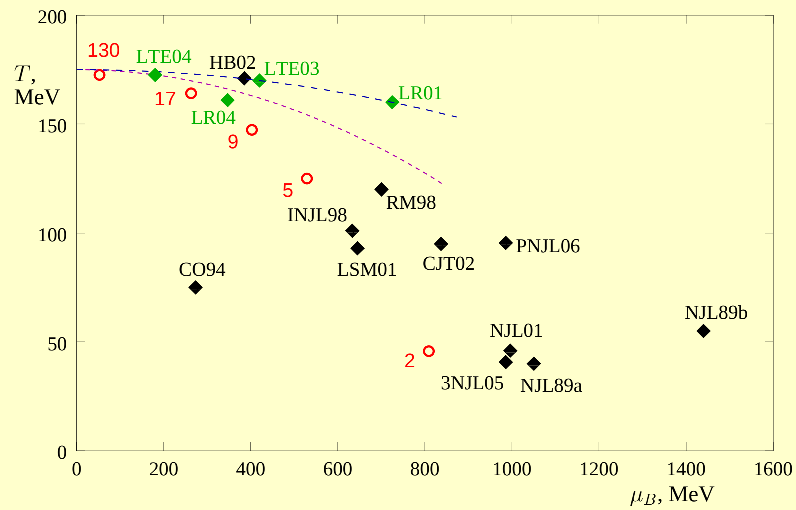


Figure 22: Comparison of predictions for the location of the QCD critical point on the phase diagram. Black points are model predictions: NJLa89, NJLb89, CO94, INJL98, RM98, LSM01, NJL01, HB02, CJT02, 3NJL05, PNJL06. Green points are lattice predictions: LR01, LR04, LTE03, LTE04. The two dashed lines are parabolas with slopes corresponding to lattice predictions of the slope $dT/d\mu_B^2$ of the transition line at $\mu_B = 0$. The red circles are locations of the freezeout points for heavy ion collisions at corresponding center of mass energies per nucleon (indicated by labels in GeV).

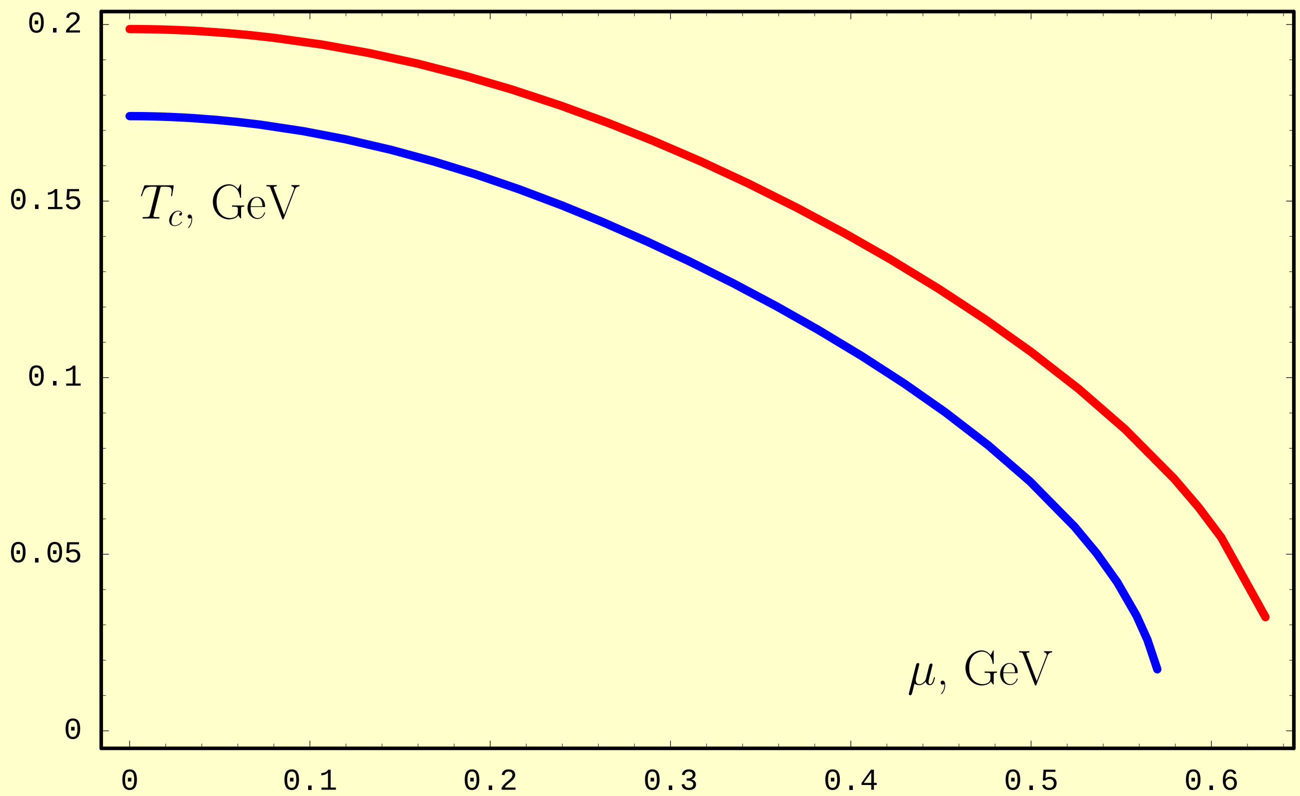
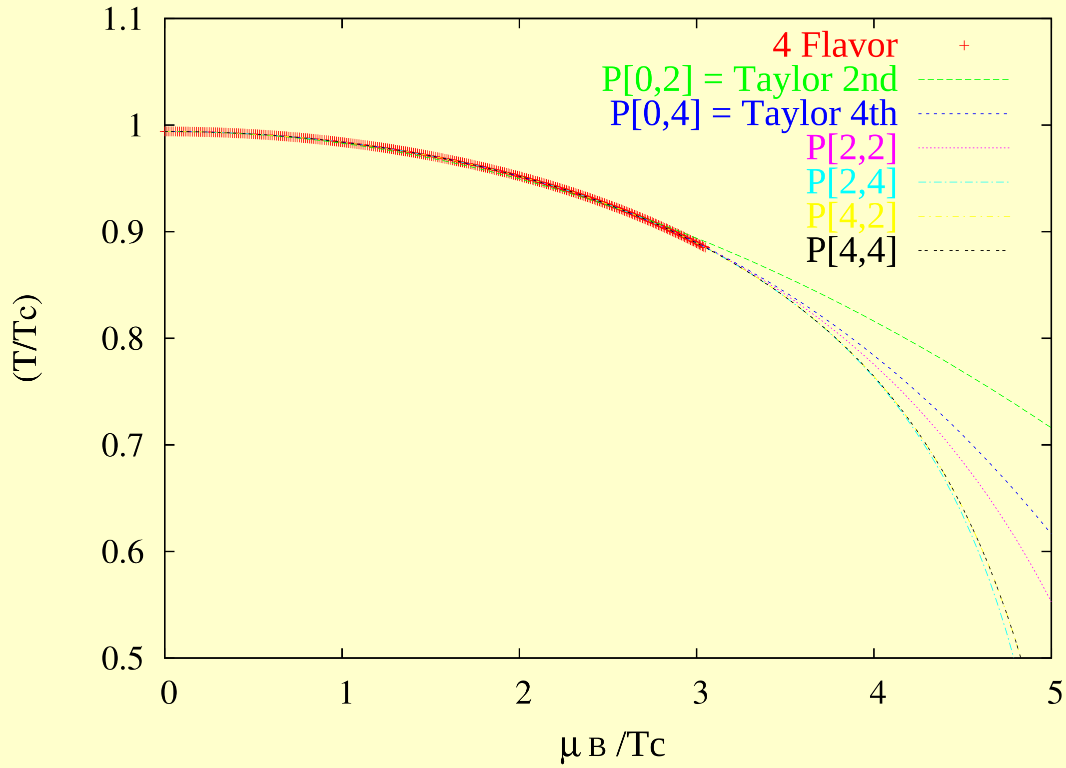
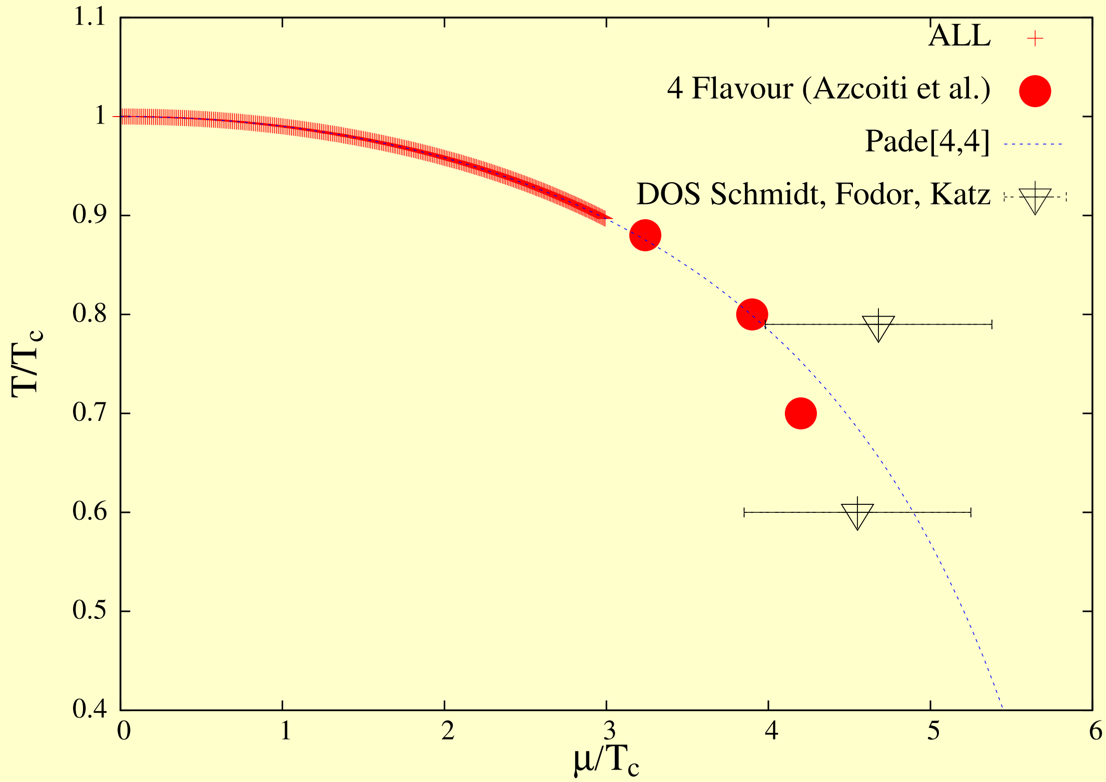


Figure 23:



4 Flavor
P[0,2] = Taylor 2nd
P[0,4] = Taylor 4th
P[2,2]
P[2,4]
P[4,2]
P[4,4]
(T/Tc)
μ B /Tc
1.1
1
0.9
0.8
0.7
0.6
0.5
0
1
2
3
4
5

Figure 24:



ALL
4 Flavour (Azcoiti et al.)
Pade[4,4]
DOS Schmidt, Fodor, Katz
T/T_c
μ/T_c
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0
1
2
3
4
5
6

Figure 25:

Conclusions

1. Nonperturbative theory of quark-gluon plasma yields relativistic pictere of dynamics already in the lowest approximation – Vacuum Dominated picture when $q\bar{q}, qqqg, gg$ correlations are neglected. this can be seen in the curves $P(T)$ for $n_f = 0, 2, 3, ...$ and $\mu = 0$.

Moreover, vanishing baryon number/strangeness correlations on the lattice support independent particle picture (Single Line Approximation (SLA) at $T \gtrsim 1.2 T_c$.

2. The Vacuum Dominated picture of QCD phase transition gives relativistic values for $T_c(\mu)$, which depend only on fundamental parameters– ΔG_2 and $V_1(T_c)$ (expressed via D_1^E or taken from lattice, $V_1(T) \cong F^1_{q\bar{Q}}(\infty, T)$. For $G_2 \sim 0.01$ GeV^4 one has $T_c(0) \sim 0.2$ GeV in good agreement with lattice data.

3. The phase diagram $T_c(\mu)$ obtained in the SLA agrees with majority of lattice data, however behaviour at $\mu \sim \mu_c \sim 0.6$ GeV is not attainable by lattice. Here effects of high baryon density for diquark picture vs $3q, 6q, 9q, ...$ clusters should be elaborated.