Low dimensional manyfolds in lattice QCD

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ITEP (Moscow)

13th Lomonosov Conference on Elementary Particle Physics

Plan of Section

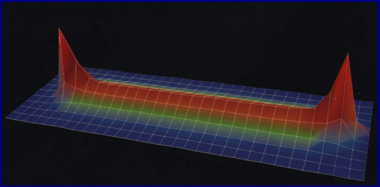
Akira Ukawa, Paul Windey, and Alan H. Guth. Dual variables for lattice gauge theories and the phase structure of Z(N) systems. Physical Review D21, (1980) 1013.

A. Irbäck”, ”A random surface representation of Wilson loops in Z(2) gauge theory”,Phys.Lett B (1988).

E.T. Tomboulis, ”t Hooft loop in SU(2) lattice gauge theories”, Phys.Rev. D (1981).

P.V. Buividovich, M.I.P. arXiv:0705.3745; P.V. Buividovich, M.I.P. and V.I. Zakharov *Contribution to Lattice 2007*; P.V. Buividovich, M.I.P. (2007) unpublished.

Wilson loop,
lattice calculations



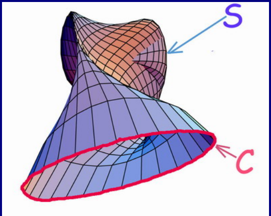
[G.S. Bali, Ch. Schlichter and K. Schilling, (1995)]

Wilson loop,
“theory”

$$< W(\mathcal{C}) >= \frac{1}{\mathcal{Z}} \sum_{\delta\mathcal{S}=\mathcal{C}} e^{A_E(S)}$$

$$\propto e^{-\sigma S_{min}}$$

the sum is over electric strings.



How to get area low from the sum over the surfaces?

- Why the sum over S converges while the entropy of surfaces in 4D is very large?
- Why the minimal area enters the answer?
- Why the phenomenology of ADS/QCD (5D string + classical limit) is successful?

It is possible to get the *exact* formulae:

Wilson loop:

$$< W(\mathcal{C}) >= \frac{1}{\mathcal{Z}} \sum_{\delta\mathcal{S}=\mathcal{C}} \mathcal{P}_E(\mathcal{S}) e^{A_E(\mathcal{S})}$$

't Hooft loop:

$$< H(\mathcal{C}) >= \frac{1}{\mathcal{Z}} \sum_{\delta\mathcal{S}=\mathcal{C}} \mathcal{P}_M(\mathcal{S}) e^{A_M(\mathcal{S})}$$

- For Wilson loop $< W(\mathcal{C}) >$ ('t Hooft loop $< H(\mathcal{C}) >$) the sum is over electric (magnetic) strings.
- The weight $\mathcal{P}(\mathcal{S})$ is not positive definite.
- The expression for 't Hooft loop corresponds to the sum over center vortices.

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Random surface representation for Z(2) gauge theory

Wilson loop

$$< W(\mathcal{C}) >=$$

$$\frac{1}{\mathcal{Z}} \sum_{Z_b=\pm 1} e^{\beta \sum_P Z_P} \prod_{b \in \mathcal{C}} Z_b =$$

$$\frac{1}{\mathcal{Z}} (\mathrm{ch}\, \beta)^{N_P} \sum_{\delta \mathcal{S}=\mathcal{C}} \{\mathrm{th}\, \beta\}^S = \frac{1}{\mathcal{Z}} (\mathrm{ch}\, \beta)^{N_P} \sum_{\delta \mathcal{S}=\mathcal{C}} e^{S \ln \mathrm{th}\, \beta}$$

$$\propto \sum_{\delta \mathcal{S}=\mathcal{C}} e^{-\sigma S}\, ; \;\; \sigma = -\ln \mathrm{th}\, \beta$$

- The sum is over self-avoiding surfaces, $\mathcal{S}$, with the boundary $\mathcal{C}$. S is the area of the surface $\mathcal{S}$.
- By duality we have the same representation for the 't Hooft loop with $\beta \to \beta^* = -\frac{1}{2} \ln \mathrm{th}\, \beta$.

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Plan of Section

Synthesis of two ideas

1. Random surface representation for Z(2) gauge theory
2. Center vortices in SU(2) theory are made from center variables, Z_b:

$$U_b = Z_b\,\widetilde{U}_b,\;\; \mathrm{Tr}\,\widetilde{U}_b > 0\,,\;\; U_b = e^{iaA_\mu(x)}$$

Conserving center monopole current

$$U_b = Z_b\,\widetilde{U}_b\,,\ \ \mathrm{Tr}\,\widetilde{U}_b > 0\,,$$

$$U_P = Z_P\,\widetilde{U}_P(-1)^{m_P}\,,\ \ Z_P = Z_1 Z_2 Z_3 Z_4\,,\ \ \mathrm{Tr}\,U_P > 0\,,$$

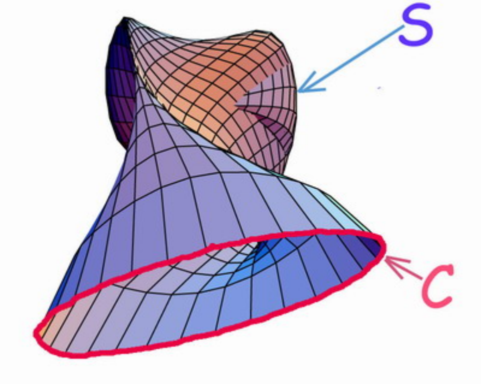
Under gauge transformations $m_P \to m_P + d\,m_b$, thus $j_{b^*} = \delta^* m_P$ is the conserving center monopole current.

$\sigma_{Z(2)} = \sigma_{SU(2)}$

Z(2) Wilson loop, $W_{Z(2)}(\mathcal{C}) = \prod_{b\in\mathcal{C}} Z_b$ gives exact string tension *[Chernodub, M.I.P. unpublished; Faber, Greensite, Olejnik (1998)]*.

String representation for Wilson loop in SU(2) LGT

$$< W_{Z(2)}(\mathcal{C}) >= \sum_{\delta\mathcal{S}=\mathcal{C}} \int \mathcal{D}\widetilde{U}_b \operatorname{ch}(\beta \operatorname{Tr} \widetilde{U}_P) \prod_{P\in\mathcal{S}} \operatorname{th}(\beta \operatorname{Tr} \widetilde{U}_P)(-1)^{m_P}$$



$$< W_{Z(2)}(\mathcal{C}) >= \sum_{\delta\mathcal{S}=\mathcal{C}} \int \mathcal{D}\widetilde{U}_b \mathrm{ch}\,(\beta \mathrm{Tr}\,\widetilde{U}_P) \prod_{P\in\mathcal{S}} \mathrm{th}\,(\beta\mathrm{Tr}\,\widetilde{U}_P)(-1)^{m_P}$$

THUS WE FOUND ELECTRIC STRINGS IN GLUODYNAMICS

- The summation $\sum_{\delta\mathcal{S}=\mathcal{C}}$ is over self-avoiding surfaces.
- The weight is not positive definite due to monopole contribution $(-1)^{m_P}$.
- The integration, $\int \mathcal{D}\widetilde{U}_b$, is over SO(3) (not over SU(2)) group.
- The expressions are exact.

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Non-positivity of the weight is very important. If we neglect center monopoles m_P then we loose confinement.

$$< W_{Z(2)}(\mathcal{C}) >= \sum_{\delta\mathcal{S}=\mathcal{C}} \int \mathcal{D}\widetilde{U}_b \mathrm{ch}\,(\beta \mathrm{Tr}\,\widetilde{U}_P) \prod_{P\in\mathcal{S}} \mathrm{th}\,(\beta \mathrm{Tr}\,\widetilde{U}_P)(-1)^{m_P}$$

$m_P = 0$

$$< W_{Z(2)}(\mathcal{C}) >= \sum_{\delta\mathcal{S}=\mathcal{C}} \mathrm{ch}\,(\beta\epsilon) \prod_{P\in\mathcal{S}} \mathrm{th}\,(\beta\epsilon)\,,$$

But this is the expression for Z(2) gauge theory and for sufficiently large β in this theory we have deconfinement; ($\epsilon > 0$ is the minimal value of $\mathrm{Tr}\,\widetilde{U}_P$).

We now have a lot of new operators, e.g. chemical potential for vortex world sheet

Partition function with surface chemical potential μ is:

$$\mathcal{Z} = \sum_{\delta\mathcal{S}=0} W(S) \to \mathcal{Z}[\mu] = \sum_{\delta\mathcal{S}=0} e^{-\mu S} W(S)$$

or

$$\mathcal{Z}[\mu] = \int \mathcal{D}U_b [\mathrm{ch}\,(\beta \mathrm{Tr}\, \widetilde{U}_P) + e^{\mu} \mathrm{sh}\,(\beta \mathrm{Tr}\, \widetilde{U}_P)]$$

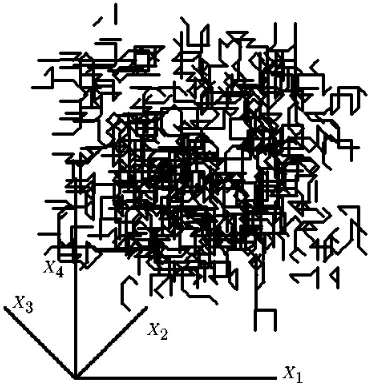
For $\mu \to \infty$ we have SO(3) LGT, for $\mu = 0$ we have SU(2) LGT.

The derivative of $\mathcal{Z}[\mu]$ over μ yields the expectation value of total area of the electric strings:

$$\langle |S| \rangle = -\mathcal{Z}^{-1} \frac{\partial}{\partial \mu} \mathcal{Z} = \frac{N_p}{2} \left\langle \, (1 - \exp\left(-\beta \mathrm{Tr}\, U_p\right)) \, \right\rangle$$

$\langle \, \exp\left(-\beta \mathrm{Tr}\, U_p\right) \, \rangle \to 0$ at $\beta \to \infty$. Thus the surfaces dual to center vortices occupy half of all plaquettes and are in a creased phase with Hausdorf dimension $d_H \to \infty$.
The same we have for magnetic strings in SU(2) LGT if we do not fix central gauge.

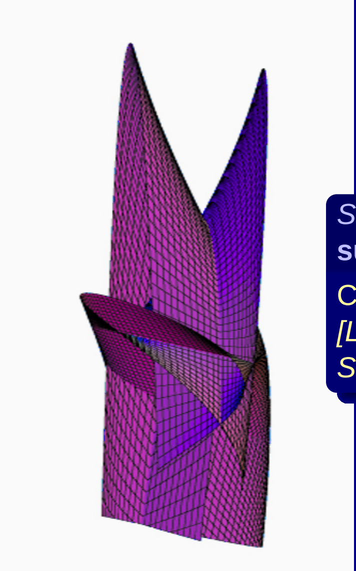
Plan of Section



$SU(2) \rightarrow U(1)$ Monopole Current (Closed lines on 4D lattice)

Confinement $\Leftrightarrow$ Monopoles
[H. Shiba and T. Suzuki (1994)]

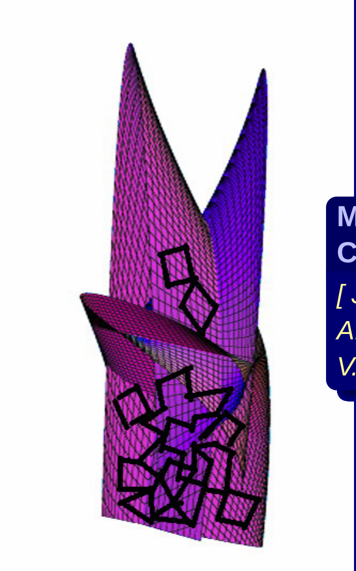
Monopoles and center vortices



$SU(2) \to Z(2)$ Center Vortices (Closed surfaces on 4D lattice)

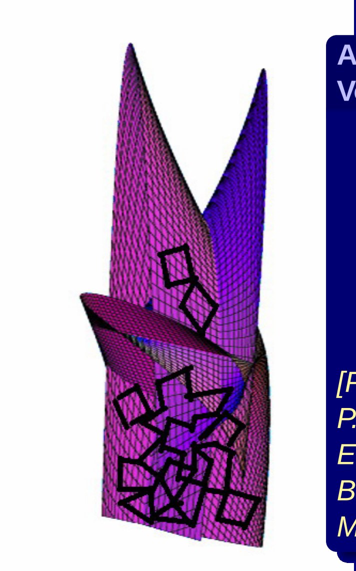
Confinement $\Leftrightarrow$ Center Vortices
[L. Del Debbio, M. Faber, J. Greensite, S. Olejnik (1997)]

Monopoles and center vortices



Monopole Currents are lying on Center Vortices

[J. Ambjorn, J. Giedt, J. Greensite (2000); A. V. Kovalenko, M. I. P., S. N. Syritsyn, V. I. Zakharov (2004)]



Abelian Monopoles and Center Vortices are interrelated

- Removing monopoles
  (or removing center vortices) $\Rightarrow$
- Removing center vortices
  (or removing monopoles) $\Rightarrow$
- remove confinement and chiral symmetry breaking.

[Ph. de Forcrand, M. D'Elia, (1999); P. Yu. Boyko, V. G. Bornyakov, E.-M. Ilgenfritz, A. V. Kovalenko, B. V. Martemyanov, M. Müller-Preussker, M. I. P., A. I. Veselov (2005)]

1. The total area of center vortices scales: $A_{center\ \ vortex} \approx 24 \frac{V_4}{fm^2}$
2. There are long range correlations $< \mathrm{Tr}\, F^2_{\mu\nu}(x)\, \mathrm{Tr}\, F^2_{\mu\nu}(y) >$ on the surface of center vortex *[V.G. Bornyakov, P.Yu. Boyko, M.I.P., V.I. Zakharov (2005), P.V. Buividovich, M.I.P. (2007)]*.

These facts can be explained if we suppose that there exists some field living on the center vortex, the natural candidate is the monopole field.

There exists something nontrivial (long range correlations) on the center vortex world sheet.

Fields on the world sheet (monopoles) can induce long range correlations and nontrivial action for vortices:

$$\exp\left(-A\left[S\right]\right)=\int\mathcal{D}\phi\exp\left(-\int_{S}d^{2}\xi\sqrt{g}\,L\left[\phi\right]\right)$$

$$A=\sigma\times Area+\gamma\times(internal\ curvature)+\kappa\times(extrinsic\ curvature)+\ldots$$

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We calculated numerically coefficients σ, γ and κ for SU(2) lattice gauge theory.

$$A[S] = \int_S d^2\xi \sqrt{g} \left(\sigma_0(a) a^{-2} + \gamma(a) R + \kappa(a) K \right)$$

$\sqrt{g} = \sqrt{\det g_{ab}}$, $g_{ab} = \frac{\partial X^\mu}{\partial \xi^a} \frac{\partial X^\mu}{\partial \xi^b}$ is the induced metric on the surface, $a = \Lambda_{UV}^{-1}$ is the lattice spacing. R (K) is the internal (extrinsic) curvature.

Extrapolation to continuum limit $a \to 0$

$$\sigma_0(0) = 0.192 \pm 0.006$$
$$\kappa(0) = 0.066 \pm 0.003$$
$$\gamma(0) = 0.08 \pm 0.02$$

$$A[S] = \int_S d^2\xi \sqrt{g} \left(\sigma_0(a) a^{-2} + \gamma(a) R + \kappa(a) K \right)$$

- Self consistency check. Our result is

$$\sigma = a^{-2}\sigma_0 \approx a^{-2}A + a^{-1}B,$$

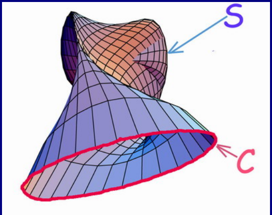
$A \approx 0.192$, $B \approx 2.2$.
If divergence of σ corresponds to self-energy of percolating one-dimensional object (monopole trajectory) on the world sheet *[J. Ambjorn (1994)]* then density of monopoles on the world sheet is $\rho_{1D} = \frac{B}{\ln 4} \approx 1.5 fm^{-1}$ which corresponds to the monopole bare mass $m_{bare} = a^{-1} \ln 4$. The density of vortices is $\rho_v \approx 24 fm^{-2}$, thus density of monopoles in four-dimensional space is $\rho'_{1D} = 37.(9) fm^{-3}$, which should be compared with the density of Abelian monopoles obtained by numerical calculations: $\rho_m \approx 31 fm^{-3}$.

Plan of Section

Discussion: electric strings in gluodynamics

1. The random surface for Wilson and 't Hooft loop for SU(2) LGT is obtained,

$$< W(\mathcal{C}) >= \frac{1}{\mathcal{Z}} \sum_{\delta\mathcal{S}=\mathcal{C}} \mathcal{P}_E(\mathcal{S}) e^{A_E(\mathcal{S})}$$



2. A lot of new operators(chemical potential, etc.) related to electric strings are defined.

Discussion: magnetic strings in gluodynamics

1. The world sheet action with intrinsic and extrinsic curvature naturally arises after integration over fields (monopoles?) living on the vortex.
2. Intrinsic and external rigidity terms in the effective center vortex action was found. Rigidity of center vortices was first suggested by Engelhardt and Reinhardt in 1999 from phenomenological analysis.